Model Solutions

OCR
Oxford Cambridge and RSA

AS Level Further Mathematics B (MEI)
Y411/01 Mechanics a
Question Paper

Thursday 17 May 2018 – Afternoon
Time allowed: 1 hour 15 minutes

You must have:
• Printed Answer Booklet
• Formulae Further Mathematics B (MEI)

You may use:
• a scientific or graphical calculator

INSTRUCTIONS
• Use black ink. HB pencil may be used for graphs and diagrams only.
• Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
• Answer **all** the questions.
• **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
• Do **not** write in the barcodes.
• You are permitted to use a scientific or graphical calculator in this paper.
• Final answers should be given to a degree of accuracy appropriate to the context.
• The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
• The total number of marks for this paper is **60**.
• The marks for each question are shown in brackets **[ ]**.
• You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
• The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

1   Forces of magnitude 4 N, 3 N, 5 N and R N act on a particle in the directions shown in Fig. 1.
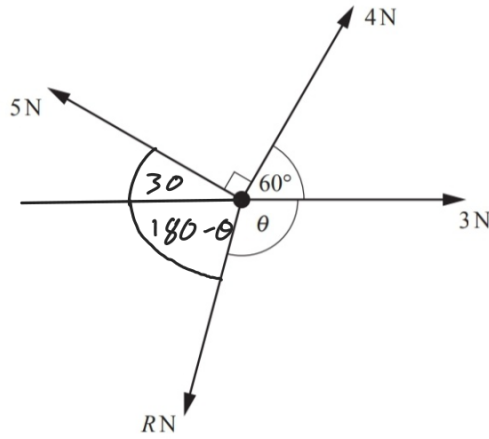

Fig. 1

The particle is in equilibrium.

Find each of the following.

- The value of R.
- The value of θ.

[6]

Horizontal: $3 + 4\cos 60 = R\cos(180 - \theta) + 5\cos 30$

$$3 + 2 = -R\cos\theta + \frac{5\sqrt{3}}{2}$$

$$R\cos\theta = \frac{5\sqrt{3}}{2} - 5 \quad ①$$

Vertical: $4\sin 60 + 5\sin 30 = R\sin(180 - \theta)$

$$R\sin\theta = 2\sqrt{3} + \frac{5}{2} \quad ②$$

$\frac{②}{①} \quad \therefore \quad \frac{R\sin\theta}{R\cos\theta} = \frac{2\sqrt{3} + 5/2}{\frac{5}{2}\sqrt{3} - 5} = -8.903$

$\tan\theta = -8.903$

$\theta = -83.59$

$\theta = -83.59 + 180 = 96.408° = 96.4° \ (3sf)$

$R = \frac{2\sqrt{3} + 2.5}{\sin\theta} = \frac{2\sqrt{3} + 2.5}{\sin 96.408} = 6.0016$

$R = 6.0 \ N$

2 A car of mass 1350 kg travels along a straight horizontal road. Throughout this question the resistance force to the motion of the car is modelled as constant and equal to 920 N.

(i) Calculate the power, in kW, developed by the car when the car is travelling at a constant speed of 25 m s^{-1}. [2]
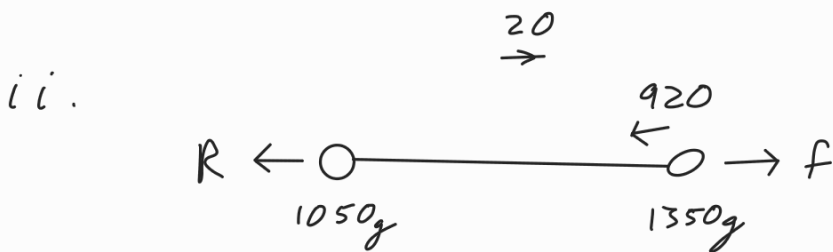
The car is now used to tow a caravan of mass 1050 kg along the same road. When the car tows the caravan at a constant speed of 20 m s^{-1} the power developed by the car is 45 kW.

(ii) Find the additional resistance force due to the caravan. [2]

i.

$$\xrightarrow{25}$$

$$920 \longleftarrow \bigcirc \longrightarrow f$$
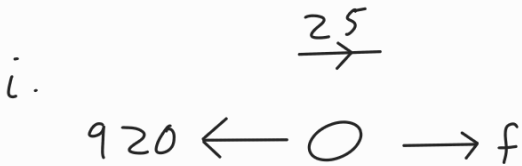
Constant speed $\Rightarrow \Sigma f = 0$ $\therefore$ $f = 920$ N

Power $= f v$

$\quad\quad = 920 \times 25$

$\quad\quad = 23$ kW

ii.

$$\xrightarrow{20}$$

$$R \leftarrow \bigcirc \underset{1050g}{\underline{\hspace{3cm}}} \overset{920}{\underset{1350g}{\nwarrow}} \bigcirc \rightarrow f$$

$P = \Sigma f$

$45000 = \Sigma f \times 20$

$\Sigma f = 2250$ N

$\therefore 2250 - 920 - R = 0$

$R = 1330$ N

In the remaining parts of this question the power developed by the car is constant and equal to 68 kW and the resistance force due to the caravan is modelled as constant and equal to the value found in part (ii).

When the car and caravan pass a point A on the same straight horizontal road the speed of the car and caravan is 20 m s^{-1}.

(iii) Find the acceleration of the car and caravan at point A.   [3]

$$\text{Power} = fv$$
$$68000 = 20f$$
$$f = 3400 \, N$$

Using $f = ma$:
$$3400 - 920 - 1330 = (1050 + 1350)a$$
$$1150 = 2400 \, a$$
$$a = 0.479 \, ms^{-2} \quad (3sf)$$

The car and caravan later pass a point B on the same straight horizontal road with speed 28 m s^{-1}. The distance AB is 1024 m.

(iv) Find the time taken for the car and caravan to travel from point A to point B.   [4]

$$KE \text{ at } A = \frac{1}{2} \times 2400 \times 20^2 = 480000$$

$$KE \text{ at } B = \frac{1}{2} \times 2400 \times 28^2 = 940800$$

WD against friction $= fd$
$$= (920 + 1330) \times 1024$$
$$= 2304000$$

$WD = PE$
$$= 68000t$$

KE at $A + WD = KE$ at $B + WD$ against friction
$$480000 + 68000t = 940800 + 2304000$$
$$68000t = 2764800$$
$$t = 40.7 \, sec$$

(v) Suggest one way in which any of the modelling assumptions used in this question could have been improved. **[1]**

Resistance could have been modelled to vary with speed.

3 Jodie is doing an experiment involving a simple pendulum. The pendulum consists of a small object tied to one end of a piece of string. The other end of the string is attached to a fixed point O and the object is allowed to swing between two fixed points A and B and back again, as shown in Fig. 3.
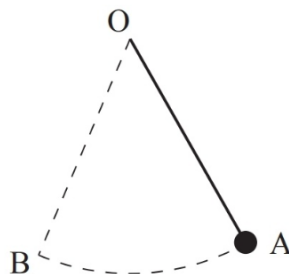

Fig. 3

Jodie thinks that P, the time the pendulum takes to swing from A to B and back again, depends on the mass, m, of the small object, the length, l, of the piece of string, and the acceleration due to gravity g. She proposes the formula $P = km^{\alpha}l^{\beta}g^{\gamma}$.

(i) What is the significance of k in Jodie's formula? **[1]**

(ii) Use dimensional analysis to determine the values of α, β and γ. **[3]**

i. k is a constant

ii. $T = [M]^{\alpha} [L]^{\beta} \left[\dfrac{L}{T^2}\right]^{\gamma}$

$M: \quad \alpha = 0$

$T: \quad 1 = -2\gamma$

$\quad\quad \gamma = -\frac{1}{2}$

$L: \quad 0 = \beta + \gamma$

$\quad\quad 0 = \beta - \frac{1}{2}$

$\quad\quad \beta = \frac{1}{2}$

Jodie finds that when the mass of the object is 1.5 kg and the length of the string is 80 cm the time taken for the pendulum to swing from A to B and back again is 1.8 seconds.

(iii) Use Jodie's formula and your answers to part **(ii)** to find each of the following.

    (A) The value of k **[2]**

    (B) The time taken for the pendulum to swing from A to B and back again when the mass of the object is 0.9 kg and the length of the string is 1.4 m   **[2]**

A) $P = k L^{1/2} g^{-1/2}$

$$1.8 = \frac{k \times \sqrt{0.8}}{\sqrt{g}}$$

$$k = 6.3$$

B) $$P = \frac{6.3 \times \sqrt{1.4}}{\sqrt{g}}$$

$$= 2.38 \text{ sec}$$

(iv) Comment on the assumption made by Jodie that the formula for the time taken for the pendulum to swing from A to B and back again is dependent on *m*, *l* and *g*.   **[1]**

The formula is actually independant of mass.

4 A uniform lamina ABDE is in the shape of an equilateral triangle ABC of side 12 cm from which an equilateral triangle of side 6 cm has been removed from corner C. The lamina is situated on coordinate axes as shown in Fig. 4.
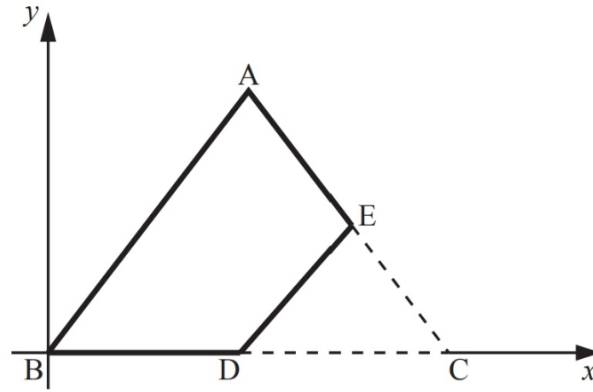

Fig. 4

(i) Explain why angle BDA = 90°. [1]

(ii) Find the coordinates of the centre of mass of the lamina ABDE. [5]

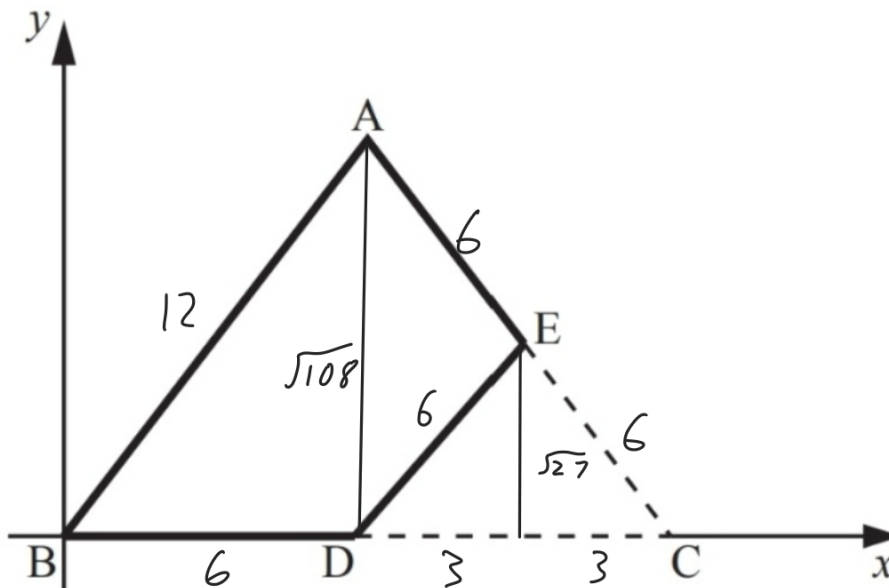i. D is the midpoint of BC, so lies directly below A.

ii.

$$\sqrt{108} = 6\sqrt{3}$$

$$\sqrt{27} = 3\sqrt{3}$$

Fig. 4

$$COM \ of \ BAC = \begin{pmatrix} 6 \\ \frac{1}{3}(0+0+6\sqrt{3}) \end{pmatrix} = \begin{pmatrix} 6 \\ 2\sqrt{3} \end{pmatrix}$$

$\underbrace{}_{\sqrt{108}}$

$$Area \ of \ BAC = \frac{12 \times 6\sqrt{3}}{2} = 36\sqrt{3}$$

$$COM \text{ of } CDE = \begin{pmatrix} 9 \\ \frac{1}{3}(0+0+3\sqrt{3}) \end{pmatrix} = \begin{pmatrix} 9 \\ \sqrt{3} \end{pmatrix}$$

$$\text{Area of } CDE = \frac{6 \times 3\sqrt{3}}{2} = 9\sqrt{3}$$

Let $(\bar{x}, \bar{y})$ be the COM of $ABDE$

$$(36\sqrt{3} - 9\sqrt{3})\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = 36\sqrt{3}\begin{pmatrix} 6 \\ 2\sqrt{3} \end{pmatrix} - 9\sqrt{3}\begin{pmatrix} 9 \\ \sqrt{3} \end{pmatrix}$$

$$27\sqrt{3}\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 216\sqrt{3} \\ 216 \end{pmatrix} - \begin{pmatrix} 81\sqrt{3} \\ 27 \end{pmatrix}$$
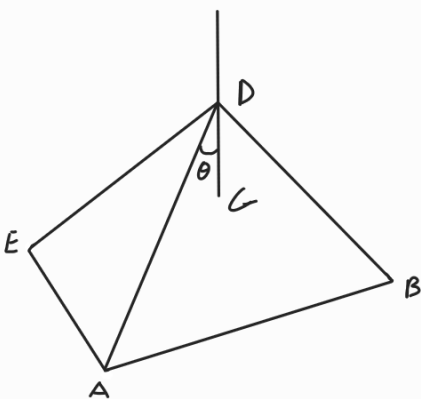
$$27\sqrt{3}\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 135\sqrt{3} \\ 189 \end{pmatrix}$$

$$\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 5 \\ \frac{7}{3}\sqrt{3} \end{pmatrix}$$

The lamina ABDE is now freely suspended from D and hangs in equilibrium.

(iii) Calculate the angle DE makes with the downward vertical. **[3]**

$$A\hat{D}E = 30°$$
$$\tan\theta = \frac{1}{\frac{7}{3}\sqrt{3}}$$
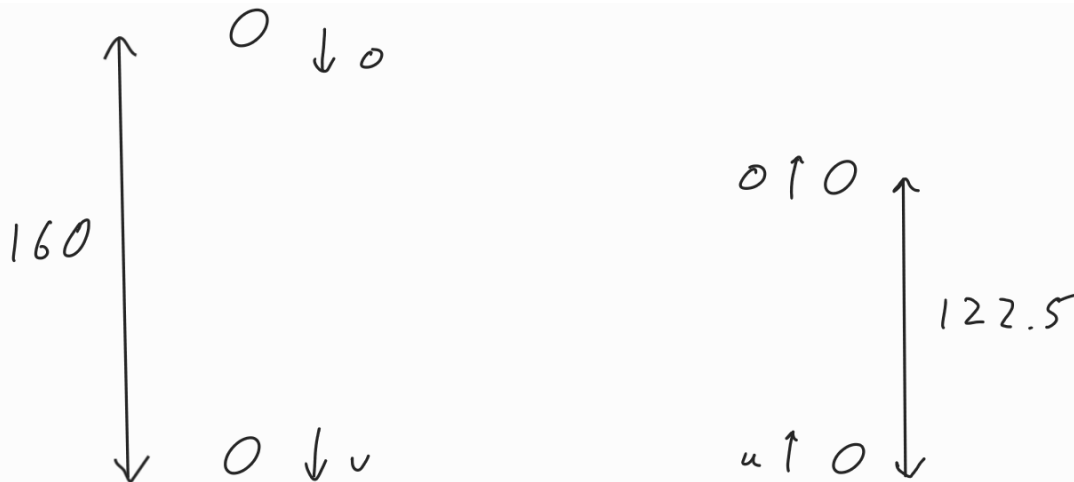$$= \frac{3}{7\sqrt{3}}$$
$$\therefore \theta = 13.9$$

$$G\hat{D}E = \theta + A\hat{D}E$$
$$= 13.9 + 30$$
$$= 43.9°$$

5 A small ball is held at a height of 160 cm above a horizontal surface. The ball is released from rest and rebounds from the surface. After its first bounce on the surface the ball reaches a height of 122.5 cm.

(i) Find the height reached by the ball after its second bounce on the surface. **[8]**

$O \downarrow o$

160

$O \downarrow v$

$o \uparrow O$

122.5

$u \uparrow O \downarrow$

down: $S = 1.60$
$u = 0$
$v = v$
$a = 9.8$
$t =$

$v^2 = u^2 + 2as$
$v^2 = 0 + 2 \times 9.8 \times 1.6$
$v^2 = 31.36$
$v = 5.6 \ ms^{-1}$ down

up: $S = 1.225$
$u = u$
$v = 0$
$a = -9.8$
$t =$

$v^2 = u^2 + 2as$
$0 = u^2 + 2 \times -9.8 \times 1.225$
$u^2 = 24.01$
$u = 4.9 \ ms^{-1}$ up

So $e = \dfrac{u}{v} = \dfrac{4.9}{5.6} = 0.875$

$O \downarrow o$

122.5

$o \uparrow O$

d

$O \downarrow 4.9$

$x \uparrow o \downarrow$
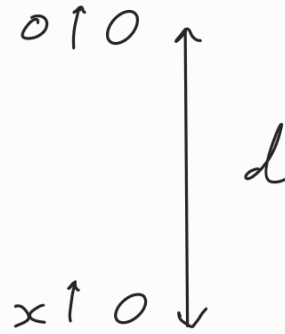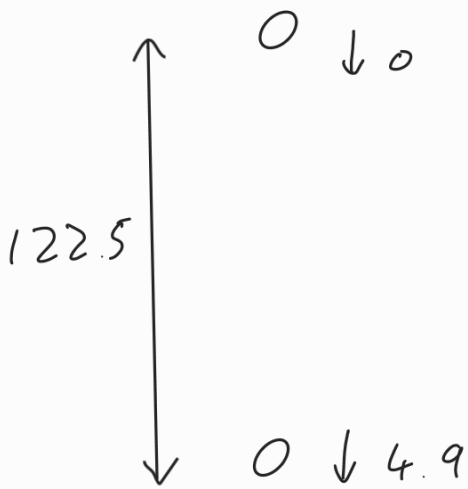
$e = \dfrac{x}{4.9}$

$x = 0.875 \times 4.9 = 4.2875 \text{ ms}^{-1}$

up: $\quad S = d$

$u = 4.2875$

$v = 0$

$a = -9.8$

$t = 0$

$v^2 = u^2 + 2ax$

$0 = 4.2875^2 + 2 \times -9.8 \times d$

$19.6 d = 18.3827$

$d = 0.938 \text{ m} \quad (3.\text{sf})$

After *n* bounces the height reached by the ball is less than 10 cm.

(ii) Find the minimum possible value of *n*. [3]

(iii) State what would happen if the same ball is released from rest from a height of 160 cm above a different horizontal surface and

    (*A*) the coefficient of restitution between the ball and the new surface is 0, [1]

    (*B*) the coefficient of restitution between the ball and the new surface is 1. [1]

Height of bounces forms a geometric sequence:

$\quad 160, \ 122.5, \ 93.8$

$a = 160$

$r = \dfrac{122.5}{160} = \dfrac{93.8}{122.5} = 0.765625$

$\therefore$ Height on nth bounce $= 160 \times \left(\dfrac{122.5}{160}\right)^n$

$\Rightarrow 160\left(\dfrac{122.5}{160}\right)^n < 10$

$\left(\dfrac{122.5}{160}\right)^n < \dfrac{1}{16}$

$n \log\left(\dfrac{122.5}{160}\right) < \log\left(\dfrac{1}{16}\right)$

$n > \dfrac{\log\left(\frac{1}{16}\right)}{\log\left(\frac{122.5}{160}\right)}$

$n > 10.38$

$\therefore$ Minimum value for $\underline{n = 11}$

iii. A) The ball will stay on the surface after reaching it.

B) The ball will repeatedly bounce back up to 160 cm.

6 A uniform rod AB has length 2a and weight W. The rod is in equilibrium in a horizontal position. The end A rests on a smooth plane which is inclined at an angle of 30° to the horizontal. The force exerted on AB by the plane is R. The end B is attached to a light inextensible string inclined at an angle of θ to AB as shown in Fig. 6. The rod and string are in the same vertical plane, which also contains the line of greatest slope of the plane on which A lies. The tension in the string is T.
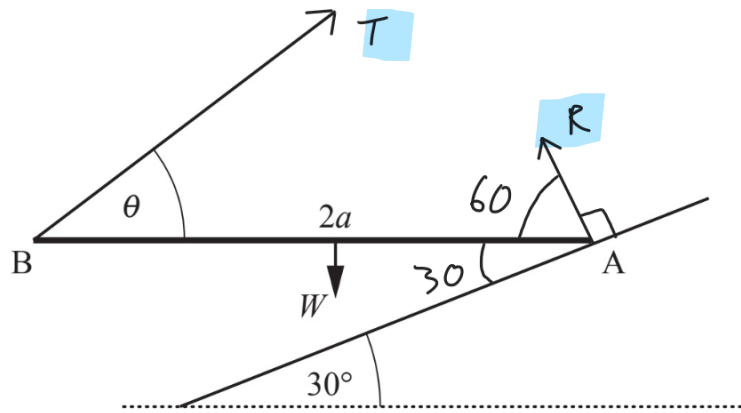
Fig. 6

(i) Add the forces R and T to the copy of Fig. 6 in the Printed Answer Booklet. **[1]**

(ii) By taking moments about B, find an expression for R in terms of W. **[2]**

ii. $aW = 2a \times R \sin 60$

$W = \sqrt{3} R$

$R = \dfrac{W}{\sqrt{3}} = \dfrac{W\sqrt{3}}{3}$

(iii) By resolving horizontally, show that $6T\cos\theta = W\sqrt{3}$. **[3]**

$T\cos\theta = R\cos 60$

$T\cos\theta = R/2$

$T\cos\theta = \dfrac{W\sqrt{3}}{6}$

$6T\cos\theta = W\sqrt{3}$

(iv) By finding a second equation connecting

- the value of θ,

- an expression for T in terms of W. **[5]**

Resolving vertically : $T \sin \theta + R \sin 60 = W$

$$T \sin \theta + \frac{\sqrt{3}}{2} R = W$$

$$T \sin \theta + \frac{\sqrt{3}}{2} \left(\frac{W \sqrt{3}}{3} \right) = W$$

$$T \sin \theta + \frac{W}{2} = W$$

$$T \sin \theta = \frac{W}{2}$$

$$W = 2 T \sin \theta$$

Divide by $6 T \cos \theta = W \sqrt{3}$ from iii :

$$\frac{2 T \sin \theta}{6 T \cos \theta} = \frac{W}{W \sqrt{3}}$$

$$\frac{\tan \theta}{3} = \frac{\sqrt{3}}{3}$$

$$\underline{\theta = 60°}$$

$$W = 2 T \sin 60$$
$$W = \sqrt{3} T$$

$$T = \frac{W \sqrt{3}}{3}$$