

**OCR**

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# A Level Further Mathematics B (MEI)

## Y436 Further Pure with Technology

### Sample Question Paper

## Date – Morning/Afternoon

Time allowed: 1 hour 45 minutes

#### OCR supplied materials:

- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

#### You must have:

- Printed Answer Book
- Formulae Further Mathematics B (MEI)
- Scientific or graphical calculator
- Computer with appropriate software



#### INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.**
- Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

#### COMPUTING RESOURCES

- Candidates will require access to a computer with a computer algebra system, a spreadsheet, a programming language and graph-plotting software throughout the examination.

#### INFORMATION

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [ ].
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages.

## 2

Answer **all** the questions.

- 1 A family of curves has polar equation  $r = \cos^n\left(\frac{\theta}{n}\right)$ ,  $0 \leq \theta < n\pi$ , where  $n$  is a positive *even* integer.
- (i) (A) Sketch the curve for the cases  $n = 2$  and  $n = 4$ . [2]
- (B) State two points which lie on every curve in the family. [1]
- (C) State one other feature common to all the curves. [1]
- (ii) (A) Write down an integral for the length of the curve for the case  $n = 4$ . [2]
- (B) Evaluate the integral. [2]
- (iii) (A) Using  $t = \theta$  as the parameter, find a parametric form of the equation of the family of curves. [1]
- (B) Show that  $\frac{dy}{dx} = \frac{\sin t \sin\left(\frac{t}{n}\right) - \cos t \cos\left(\frac{t}{n}\right)}{\sin t \cos\left(\frac{t}{n}\right) + \cos t \sin\left(\frac{t}{n}\right)}$ . [2]
- (iv) Hence show that there are  $n + 1$  points where the tangent to the curve is parallel to the  $y$ -axis. [6]
- (v) By referring to appropriate sketches, show that the result in part (iv) is true in the case  $n = 4$ . [2]

## 3

- 2 (i) (A) Create a program to find all the solutions to  $x^2 \equiv -1 \pmod{p}$  where  $0 \leq x < p$ .  
Write out your program in full in the Printed Answer Booklet. [5]
- (B) Use the program to find the solutions to  $x^2 \equiv -1 \pmod{p}$  for the primes
- $p = 809$ ,
  - $p = 811$  and
  - $p = 444001$ . [3]
- (ii) State Wilson's Theorem. [1]
- (iii) The following argument shows that  $(4k)! \equiv ((2k)!)^2 \pmod{p}$  for the case  $p = 4k + 1$ .
- $$(4k)! \equiv 1 \times 2 \times 3 \times \dots \times (2k-1) \times 2k \times (2k+1) \times (2k+2) \times \dots \times (4k-1) \times 4k \pmod{p} \quad (1)$$
- $$\equiv 1 \times 2 \times 3 \times \dots \times (2k-1) \times 2k \times (-2k) \times (-(2k-1)) \times \dots \times (-2) \times (-1) \pmod{p} \quad (2)$$
- $$\equiv ((2k)!)^2 \pmod{p} \quad (3)$$
- (A) Explain why  $(2k+2)$  can be written as  $(-(2k-1))$  in line (2). [1]
- (B) Explain how line (3) has been obtained. [2]
- (C) Explain why, if  $p$  is a prime of the form  $p = 4k + 1$ , then  $x^2 \equiv -1 \pmod{p}$  will have at least one solution. [1]
- (D) Hence find a solution of  $x^2 \equiv -1 \pmod{29}$ . [2]
- (iv) (A) Create a program that will find all the positive integers  $n$ , where  $n < 1000$ , such that  $(n-1)! \equiv -1 \pmod{n^2}$ . Write out your program in full. [3]
- (B) State the values of  $n$  obtained. [2]
- (C) A Wilson prime is a prime  $p$  such that  $(p-1)! \equiv -1 \pmod{p^2}$ . Write down all the Wilson primes  $p$  where  $p < 1000$ . [1]

- 3 This question explores the family of differential equations  $\frac{dy}{dx} = \sqrt{1+ax+2y}$  for various values of the parameter  $a$ . Fig. 3 shows the tangent field in the case  $a = 1$ .

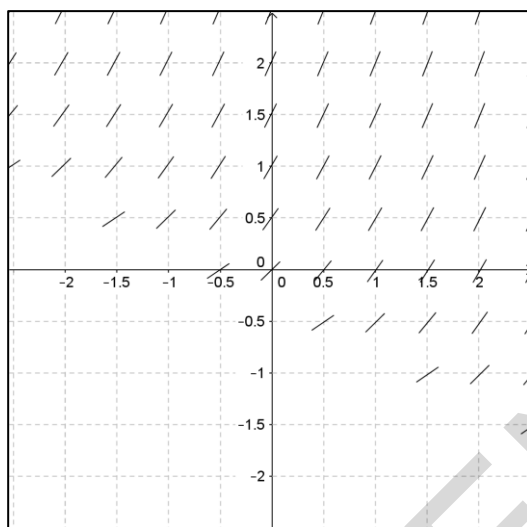


Fig. 3

- (i) (A) Sketch the tangent field in the case  $a = -2$ . [2]  
 (B) Explain why the tangent field is not defined for the whole coordinate plane. [1]  
 (C) Give an inequality which describes the region in which the tangent field is defined. [1]  
 (D) Find a value of  $a$  such that the region for which the tangent field is defined includes the entire  $x$ -axis. [1]
- (ii) (A) For the case  $a = 1$ , with  $y = 1$  when  $x = 0$ , construct a spreadsheet for the Runge-Kutta method of order 2 with formulae as follows, where  $f(x, y) = \frac{dy}{dx}$ .

$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf(x_n + h, y_n + k_1)$$

$$y_{n+1} = y_n + \frac{1}{2}(k_1 + k_2)$$

State the formulae you have used in your spreadsheet. [3]

- (B) Use your spreadsheet to obtain the value of  $y$  correct to 4 decimal places when  $x = 1$  for
- $h = 0.1$
- and
- $h = 0.05$ . [2]

- (iii) (A) For the case  $a = 0$  find the analytical solution that passes through the point  $(0, 1)$ . [1]
- (B) Verify that the solution in part (iii) (A) is a solution to the differential equation. [2]
- (C) Use the solution in part (iii) (A) to find the value of  $y$  correct to 4 decimal places when  $x = 1$ . [1]
- (iv) (A) Verify that  $y = -\frac{a}{2}x + \frac{a^2}{8} - \frac{1}{2}$  is a solution for all cases when  $a \leq 0$ . [2]
- (B) Show that this is the only straight line solution in these cases. [4]

**END OF QUESTION PAPER**

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