



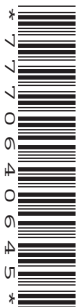
Oxford Cambridge and RSA

Tuesday 25 June 2019 – Morning

A Level Further Mathematics B (MEI)

Y436/01 Further Pure with Technology

Time allowed: 1 hour 45 minutes



You must have:

- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)
- Computer with appropriate software

You may use:

- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

COMPUTING RESOURCES

- Candidates will require access to a computer with a computer algebra system, a spreadsheet, a programming language and graph-plotting software throughout the examination.

INFORMATION

- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets [].
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

Answer **all** the questions.

1 A family of curves is given by the parametric equations

$$x(t) = \cos(t) - \frac{\cos((m+1)t)}{m+1} \quad \text{and} \quad y(t) = \sin(t) - \frac{\sin((m+1)t)}{m+1}$$

where $0 \leq t < 2\pi$ and m is a positive integer.

(a) **(i)** Sketch the curves in the cases $m = 3$, $m = 4$ and $m = 5$ on separate axes in the Printed Answer Booklet. **[3]**

(ii) State one common feature of these three curves. **[1]**

(iii) State a feature for the case $m = 4$ which is absent in the cases $m = 3$ and $m = 5$. **[1]**

(b) **(i)** Determine, in terms of m , the values of t for which $\frac{dx}{dt} = 0$ but $\frac{dy}{dt} \neq 0$. **[4]**

(ii) Describe the tangent to the curve at the points corresponding to such values of t . **[1]**

(c) **(i)** Show that the curve lies between the circle centred at the origin with radius

$$1 - \frac{1}{m+1}$$

and the circle centred at the origin with radius

$$1 + \frac{1}{m+1}. \quad \text{[2]}$$

(ii) Hence, or otherwise, show that the area A bounded by the curve satisfies

$$\frac{m^2\pi}{(m+1)^2} < A < \frac{(m+2)^2\pi}{(m+1)^2}. \quad \text{[1]}$$

(iii) Find the limit of the area bounded by the curve as m tends to infinity. **[1]**

(d) The arc length of a curve defined by parametric equations $x(t)$ and $y(t)$ between points corresponding to $t = c$ and $t = d$, where $c < d$, is

$$\int_c^d \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt.$$

Use this to show that the length of the curve is independent of m . **[6]**

- 2 (a) Prove that if x and y are integers which satisfy $x^2 - 2y^2 = 1$, then x is odd and y is even. [3]
- (b) Create a program to find, for a fixed positive integer s , all the positive integer solutions (x, y) to the equation $x^2 - 2y^2 = 1$ where $x \leq s$ and $y \leq s$. Write out your program in the Printed Answer Booklet. [3]
- (c) Use your program to find all the positive integer solutions (x, y) to the equation $x^2 - 2y^2 = 1$ where $x \leq 600$ and $y \leq 600$. Give the solutions in ascending order of the value of x . [1]
- (d) By writing the equation $x^2 - 2y^2 = 1$ in the form $(x + \sqrt{2}y)(x - \sqrt{2}y) = 1$ show how the first solution (the one with the lowest value of x) in your answer to part (c) can be used to generate the other solutions you found in part (c). [4]
- (e) What can you deduce about the number of positive integer solutions (x, y) to the equation $x^2 - 2y^2 = 1$? [1]

In the remainder of this question T_m is the m^{th} triangular number, the sum of the first m positive integers, so that $T_m = \frac{m(m+1)}{2}$.

- (f) Create a program to find, for a fixed positive integer t , all pairs of positive integers m and n which satisfy $T_m = n^2$ where $m \leq t$ and $n \leq t$. Write out your program in the Printed Answer Booklet. [2]
- (g) Use your program to find all pairs of positive integers m and n which satisfy $T_m = n^2$ where $m \leq 300$ and $n \leq 300$. Give the pairs in ascending order of the value of m . [1]
- (h) By comparing your answers to part (c) and part (g), or otherwise, prove that there are infinitely many triangular numbers which are perfect squares. [5]

3 This question concerns the family of differential equations

$$\frac{dy}{dx} = 1 - x^a y \quad (*)$$

where a is -1 , 0 or 1 .

(a) Determine and describe geometrically the isoclines of $(*)$ when

(i) $a = -1$, [2]

(ii) $a = 0$, [2]

(iii) $a = 1$. [2]

(b) In this part of the question $a = 0$.

(i) Write down the solution to $(*)$ which passes through the point $(0, b)$ where $b \neq 1$. [1]

(ii) Write down the equation of the asymptote to this solution. [1]

(c) In this part of the question $a = -1$.

(i) Write down the solution to $(*)$ which passes through the point (c, d) where $c \neq 0$. [1]

(ii) Describe the relationship between c and d when the solution in part (i) has a stationary point. [4]

(d) In this part of the question $a = 1$.

(i) The standard Runge-Kutta method of order 4 for the solution of the differential equation

$$f(x, y) = \frac{dy}{dx} \text{ is as follows.}$$

$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

$$k_4 = hf(x_n + h, y_n + k_3)$$

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4).$$

Construct a spreadsheet to solve (*) in the case $x_0 = 0$ and $y_0 = 1.5$. State the formulae you have used in your spreadsheet. [4]

(ii) Use your spreadsheet with $h = 0.05$ to find an approximation to the value of y when $x = 1$. [1]

(iii) The solution to (*) in which $x_0 = 0$ and $y_0 = 1.5$ has a maximum point (r, s) with $0 < r < 1$. Use your spreadsheet with suitable values of h to estimate r to two decimal places. Justify your answer. [2]

END OF QUESTION PAPER

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