



Oxford Cambridge and RSA

Thursday 22 October 2020 – Afternoon

A Level Further Mathematics B (MEI)

Y435/01 Extra Pure

Time allowed: 1 hour 15 minutes



You must have:

- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [].
- This document has **4** pages.

ADVICE

- Read each question carefully before you start your answer.

Answer **all** the questions.

1 The matrix \mathbf{A} is given by $\mathbf{A} = \begin{pmatrix} 0 & 2 \\ 3 & -1 \end{pmatrix}$.

Find

- the eigenvalues of \mathbf{A} ,
- an eigenvector associated with each eigenvalue.

[5]

2 A sequence is defined by the recurrence relation $t_{n+1} = \frac{t_n}{n+3}$ for $n \geq 1$, with $t_1 = 8$.

Verify that the particular solution to the recurrence relation is given by $t_n = \frac{a}{(n+b)!}$ where a and b are constants whose values are to be determined.

[5]

3 A sequence is defined by the recurrence relation $u_{n+2} = 4u_{n+1} - 5u_n$ for $n \geq 0$, with $u_0 = 0$ and $u_1 = 1$.

(a) Find an exact real expression for u_n in terms of n and θ , where $\tan \theta = \frac{1}{2}$.

[7]

A sequence is defined by $v_n = a^{\frac{1}{2}n}u_n$ for $n \geq 0$, where a is a positive constant.

(b) In each of the following cases, describe the behaviour of v_n as $n \rightarrow \infty$.

- $a = 0.1$
- $a = 0.2$
- $a = 1$

[5]

- 4 (a) In each of the following cases, a set G and a binary operation \circ are given. The operation \circ may be assumed to be associative on G .

Determine which, if any, of the other three group axioms are satisfied by (G, \circ) and which, if any, are not satisfied.

(i) $G = \{2n + 1 : n \in \mathbb{Z}\}$ and \circ is addition. [3]

(ii) $G = \{a + b\sqrt{2} : a, b \in \mathbb{Z}\}$ and \circ is multiplication. [3]

(iii) G is the set of all real numbers and \circ is multiplication. [3]

- (b) A group M consists of eight 2×2 matrices under the operation of matrix multiplication. Five of the eight elements of M are as follows.

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \quad \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & i \\ i & -1 \end{pmatrix} \quad \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

(i) Find the other three elements of M . [3]

$(N, *)$ is another group of order 8, with identity element e . You are given that $N = \langle a, b, c \rangle$ where $a * a = b * b = c * c = e$.

- (ii) State whether M and N are isomorphic to each other, giving a reason for your answer. [1]

5 In this question you must show detailed reasoning.

The matrix \mathbf{A} is given by $\begin{pmatrix} \frac{1}{3} & -\frac{2}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \\ -\frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \end{pmatrix}$ and the vector \mathbf{e} is given by $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$. You are given that \mathbf{e} is an

eigenvector of \mathbf{A} with an associated eigenvalue of -1 .

\mathbf{f} is any vector which is perpendicular to \mathbf{e} .

(a) Show that \mathbf{f} is also an eigenvector of \mathbf{A} . [4]

(b) State the eigenvalue associated with \mathbf{f} . [1]

You are now given that \mathbf{A} represents a reflection in 3-D space.

(c) Explain the significance of \mathbf{e} and \mathbf{f} in relation to the transformation that \mathbf{A} represents. [2]

(d) State the cartesian equation of the plane of reflection of the transformation represented by \mathbf{A} . [1]

6 A surface S is defined by $z = f(x, y) = 4x^4 + 4y^4 - 17x^2y^2$.

(a) (i) Show that there is only one stationary point on S . [5]

The value of z at the stationary point is denoted by s .

(ii) State the value of s . [1]

(iii) By factorising $f(x, y)$, sketch the contour lines of the surface for $z = s$. [3]

(iv) Hence explain whether the stationary point is a maximum point, a minimum point or a saddle point. [1]

C is a point on S with coordinates $(a, a, f(a, a))$ where a is a constant and $a \neq 0$. Π is the tangent plane to S at C .

(b) (i) Find the equation of Π in the form $\mathbf{r} \cdot \mathbf{n} = p$. [3]

(ii) The shortest distance from the origin to Π is denoted by d . Show that $\frac{d}{a} \rightarrow \frac{3\sqrt{2}}{4}$ as $a \rightarrow \infty$. [3]

(iii) Explain whether the origin lies above or below Π . [1]

END OF QUESTION PAPER

OCR

Oxford Cambridge and RSA

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact The OCR Copyright Team, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.