



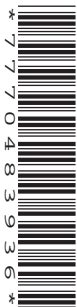
Oxford Cambridge and RSA

Monday 24 June 2019 – Morning

A Level Further Mathematics B (MEI)

Y435/01 Extra Pure

Time allowed: 1 hour 15 minutes



You must have:

- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You may use:

- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [].
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

1 The matrix \mathbf{A} is $\begin{pmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{pmatrix}$.

(a) Given that \mathbf{A} represents a reflection, write down the eigenvalues of \mathbf{A} . [1]

(b) Hence find the eigenvectors of \mathbf{A} . [3]

(c) Write down the equation of the mirror line of the reflection represented by \mathbf{A} . [1]

2 A surface S is defined by $z = 4x^2 + 4y^2 - 4x + 8y + 11$.

(a) Show that the point $P(0.5, -1, 6)$ is the only stationary point on S . [2]

(b) (i) On the axes in the Printed Answer Booklet, draw a sketch of the contour of the surface corresponding to $z = 42$. [2]

(ii) By using the sketch in part (b)(i), deduce that P must be a minimum point on S . [3]

(c) In the section of S corresponding to $y = c$, the minimum value of z occurs at the point where $x = a$ and $z = 22$.

Find all possible values of a and c . [4]

3 The matrix \mathbf{A} is $\begin{pmatrix} -1 & 2 & 4 \\ 0 & -1 & -25 \\ -3 & 5 & -1 \end{pmatrix}$.

Use the Cayley-Hamilton theorem to find \mathbf{A}^{-1} . [8]

4 T is the set $\{1, 2, 3, 4\}$. A binary operation \bullet is defined on T such that $a \bullet a = 2$ for all $a \in T$. It is given that (T, \bullet) is a group.

(a) Deduce the identity element in T , giving a reason for your answer. [2]

(b) Find the value of $1 \bullet 3$, showing how the result is obtained. [3]

(c) (i) Complete a group table for (T, \bullet) . [2]

(ii) State with a reason whether the group is abelian. [1]

- 5 A financial institution models the repayment of a loan to a client in the following way.
- An amount, $\pounds C$, is loaned to the client at the start of the repayment period.
 - The amount owed n years after the start of the repayment period is $\pounds L_n$, so that $L_0 = C$.
 - At the end of each year, interest of $\alpha\%$ ($\alpha > 0$) of the amount owed at the start of that year is added to the amount owed.
 - Immediately after interest has been added to the amount owed a repayment of $\pounds R$ is made by the client.
 - Once L_n becomes negative the repayment is finished and the overpayment is refunded to the client.
- (a) Show that during the repayment period, $L_{n+1} = aL_n + b$, giving a and b in terms of α and R . [2]
- (b) Find the solution of the recurrence relation $L_{n+1} = aL_n + b$ with $L_0 = C$, giving your solution in terms of a , b , C and n . [5]
- (c) Deduce from parts (a) and (b) that, for the repayment scheme to terminate, $R > \frac{\alpha C}{100}$. [2]

A client takes out a $\pounds 30\,000$ loan at 8% interest and agrees to repay $\pounds 3000$ at the end of each year.

- (d) (i) Use an algebraic method to find the number of years it will take for the loan to be repaid. [3]
- (ii) Taking into account the refund of overpayment, find the total amount that the client repays over the lifetime of the loan. [3]
- 6 (a) Given that $\sqrt{7}$ is an irrational number, prove that $a^2 - 7b^2 \neq 0$ for all $a, b \in \mathbb{Q}$ where a and b are not both 0. [2]
- (b) A set G is defined by $G = \{a + b\sqrt{7} : a, b \in \mathbb{Q}, a \text{ and } b \text{ not both } 0\}$.
Prove that G is a group under multiplication. (You may assume that multiplication is associative.) [7]
- (c) A subset H of G is defined by $H = \{1 + c\sqrt{7} : c \in \mathbb{Q}\}$.
Determine whether or not H is a subgroup of (G, \times) . [2]
- (d) Using (G, \times) , prove by counter-example that the statement ‘An infinite group cannot have a non-trivial subgroup of finite order’ is false. [2]

END OF QUESTION PAPER

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