Oxford Cambridge and RSA

Monday 24 June 2019 – Morning

A Level Further Mathematics B (MEI)

Y435/01 Extra Pure

Time allowed: 1 hour 15 minutes

* 7 7 7 0 4 8 3 9 3 6 *

You must have:
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

1 The matrix **A** is $\begin{pmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{pmatrix}$.

(a) Given that **A** represents a reflection, write down the eigenvalues of **A**. **[1]**

(b) Hence find the eigenvectors of **A**. **[3]**

(c) Write down the equation of the mirror line of the reflection represented by **A**. **[1]**

2 A surface S is defined by $z = 4x^2 + 4y^2 - 4x + 8y + 11$.

(a) Show that the point P $(0.5, -1, 6)$ is the only stationary point on S. **[2]**

(b) **(i)** On the axes in the Printed Answer Booklet, draw a sketch of the contour of the surface corresponding to $z = 42$. **[2]**

(ii) By using the sketch in part **(b)(i)**, deduce that P must be a minimum point on S. **[3]**

(c) In the section of S corresponding to $y = c$, the minimum value of z occurs at the point where $x = a$ and $z = 22$.

Find all possible values of a and c. **[4]**

3 The matrix **A** is $\begin{pmatrix} -1 & 2 & 4 \\ 0 & -1 & -25 \\ -3 & 5 & -1 \end{pmatrix}$.

Use the Cayley-Hamilton theorem to find $\mathbf{A}^{-1}$. **[8]**

4 T is the set $\{1, 2, 3, 4\}$. A binary operation $\bullet$ is defined on T such that $a \bullet a = 2$ for all $a \in T$. It is given that $(T, \bullet)$ is a group.

(a) Deduce the identity element in T, giving a reason for your answer. **[2]**

(b) Find the value of $1 \bullet 3$, showing how the result is obtained. **[3]**

(c) **(i)** Complete a group table for $(T, \bullet)$. **[2]**

(ii) State with a reason whether the group is abelian. **[1]**

5 A financial institution models the repayment of a loan to a client in the following way.

- An amount, £C, is loaned to the client at the start of the repayment period.
- The amount owed n years after the start of the repayment period is £L_n, so that $L_0 = C$.
- At the end of each year, interest of $\alpha\%$ ($\alpha > 0$) of the amount owed at the start of that year is added to the amount owed.
- Immediately after interest has been added to the amount owed a repayment of £R is made by the client.
- Once L_n becomes negative the repayment is finished and the overpayment is refunded to the client.

(a) Show that during the repayment period, $L_{n+1} = aL_n + b$, giving a and b in terms of α and R. **[2]**

(b) Find the solution of the recurrence relation $L_{n+1} = aL_n + b$ with $L_0 = C$, giving your solution in terms of a, b, C and n. **[5]**

(c) Deduce from parts **(a)** and **(b)** that, for the repayment scheme to terminate, $R > \dfrac{\alpha C}{100}$. **[2]**

A client takes out a £30 000 loan at 8% interest and agrees to repay £3000 at the end of each year.

(d) **(i)** Use an algebraic method to find the number of years it will take for the loan to be repaid. **[3]**

(ii) Taking into account the refund of overpayment, find the total amount that the client repays over the lifetime of the loan. **[3]**

6 **(a)** Given that $\sqrt{7}$ is an irrational number, prove that $a^2 - 7b^2 \neq 0$ for all $a, b \in \mathbb{Q}$ where a and b are not both 0. **[2]**

(b) A set G is defined by $G = \{a + b\sqrt{7} : a, b \in \mathbb{Q}, a \text{ and } b \text{ not both } 0\}$.
Prove that G is a group under multiplication. (You may assume that multiplication is associative.) **[7]**

(c) A subset H of G is defined by $H = \{1 + c\sqrt{7} : c \in \mathbb{Q}\}$.
Determine whether or not H is a subgroup of $(G, \times)$. **[2]**

(d) Using $(G, \times)$, prove by counter-example that the statement ‘An infinite group cannot have a non-trivial subgroup of finite order’ is false. **[2]**

END OF QUESTION PAPER