



Oxford Cambridge and RSA

**Thursday 16 May 2019 – Afternoon**

**AS Level Further Mathematics B (MEI)**

**Y413/01 Modelling with Algorithms**

**Time allowed: 1 hour 15 minutes**



**You must have:**

- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

**You may use:**

- a scientific or graphical calculator

**INSTRUCTIONS**

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION**

- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets [ ].
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

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## 3

Answer **all** the questions.

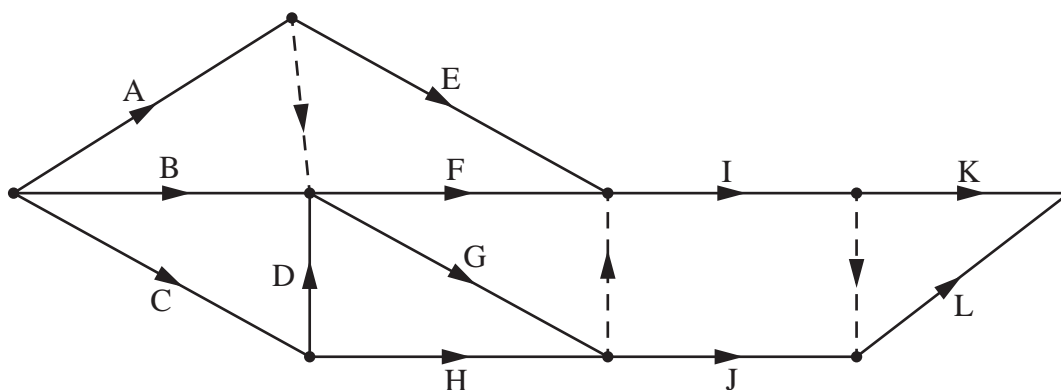
**1**                    24    17    9    25    18    11    23    19    30    15

- (a) Show the result of applying the first fit algorithm to pack items with the sizes listed above into bins that have a capacity of 50. [2]
- (b) Use the quick sort algorithm to sort the list of numbers above into descending order. You should use the first value as the pivot for each sublist. [3]
- (c) Show the result of using the first fit decreasing algorithm to pack items with the sizes listed above into bins that have a capacity of 50. [2]

The number of comparisons is used as a measure of the complexity of both the first fit and first fit decreasing packing algorithms.

- (d) Explain why, in the worst case, both packing algorithms have the same order of complexity. [1]

- 2 The network in Fig. 2.1 represents a project using activity on arc. The durations of the activities are not yet shown.



**Fig. 2.1**

- (a) Complete the table in the Printed Answer Booklet to show the immediate predecessor(s) for each activity. [2]

It is given that activity E is a critical activity. The manager of this project therefore claims that activities A, E, I and K form a critical path for the network.

- (b) Is the manager's claim correct? Explain your reasoning. [2]

Fig. 2.2 lists all the activities and their corresponding durations, in hours.

Activity	A	B	C	D	E	F	G	H	I	J	K	L
Duration, hours	7	4	6	2	10	6	3	7	5	6	5	3

**Fig. 2.2**

- (c) (i) Carry out a forward pass and a backward pass through the activity network, showing the early event time and late event time at each vertex of the network. [4]
- (ii) State the minimum project completion time. [1]

Each activity requires one person.

- (d) Use the diagram in the Printed Answer Booklet to show how three people can complete the project in the minimum time. Each column in the diagram represents 1 hour. For each person, write the letter of the activity they are doing in each box, or leave the box blank if the person is resting for that 1 hour. [2]

The manager claims the project can in fact be completed by two people in the minimum project completion time.

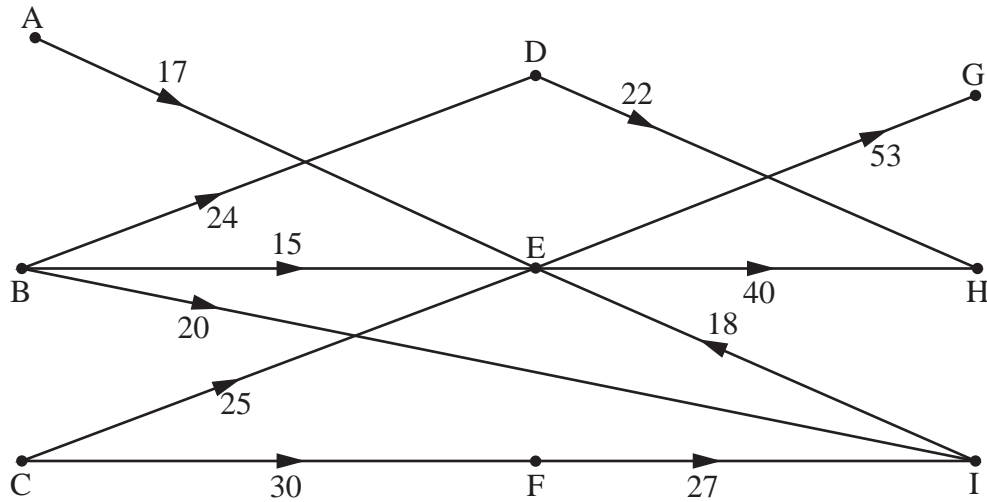
- (e) Show that the manager's claim is incorrect. [1]

- 3 A network has 9 vertices, A to I. The table in Fig. 3 shows the distance between each pair of vertices for which there is a connecting arc.

	A	B	C	D	E	F	G	H	I
A		5	8	16					
B	5		2			4	11		
C	8	2		8		1			
D	16		8		2				
E				2		3			5
F		4	1		3			2	12
G		11						2	
H						2	2		7
I					5	12		7	

**Fig. 3**

- (a) Apply the tabular form of Prim's algorithm to the network, starting at vertex A, to find a minimum spanning tree for the network. In your solution you should give
- the order in which the arcs are selected, and
  - the total length of the arcs in the minimum spanning tree.
- [4]
- (b) Without drawing the network, use Dijkstra's algorithm to find the shortest route from A to I.
- [5]
- (c) Determine the length of the shortest route from C to G via A.
- [1]



**Fig. 4.1**

Fig. 4.1 shows a network representing a system of pipes. Fluid flows continuously from three sources, A, B and C, to two sinks, G and H. The weights on the arcs show the capacities of the pipes in gallons per hour.

- (a) Add a supersource, S, and a supersink, T, to the network in the Printed Answer Booklet. Give appropriate weightings and directions to the connected arcs. [2]
- (b) Calculate the capacity of the cut  $X = \{S, A, B, C, E\}$ ,  $Y = \{D, F, G, H, I, T\}$ . [1]

An LP formulation is set up to find the maximum flow through the system.

Part of the LP formulation is shown below.

$$\text{Maximise } AE + BD + BE + BI + CE + FI$$

$$\begin{aligned} \text{Subject to } & BD - DH = 0 \\ & CF - FI = 0 \\ & BD \leq 24 \\ & BE \leq 15 \\ & BI \leq 20 \\ & CE \leq 25 \\ & CF \leq 30 \end{aligned}$$

- (c) Explain the purpose of each of the following lines from the LP formulation.
- (i) Maximise  $AE + BD + BE + BI + CE + FI$ . [2]
- (ii)  $BD - DH = 0$ . [1]

(d) Complete the LP formulation.

[2]

The complete LP was run in an LP solver and the output is shown in Fig. 4.2.

VARIABLE	VALUE
AE	17.00000
BD	22.00000
BE	15.00000
BI	18.00000
CE	25.00000
CF	0.00000
DH	22.00000
IE	18.00000
EH	22.00000
EG	53.00000
FI	0.00000

**Fig. 4.2**

(e) (i) State the value of the flow given by this output.

[1]

(ii) Use a suitable cut to prove that this is the maximum flow.

[2]

The capacity of pipe EH is increased by an additional 20 gallons per hour.

(f) (i) Determine the effect of this change on the maximum flow through the system.

[1]

(ii) State the effect of this change on the LP formulation.

[1]

5 An LP problem in  $x$ ,  $y$  and  $z$  is formulated as follows.

$$\text{Maximise } P = 3x + 2y + 5z$$

$$\begin{aligned} \text{subject to } & x + y + z \geq 26 \\ & 2x + y + z \leq 34 \\ & 2x - z = 10 \\ & x, y, z \geq 0 \end{aligned}$$

- (a) • Complete the initial tableau in the Printed Answer Booklet so that the two-stage simplex method may be used to solve this problem.
- Show how the constraints for the problem have been made into equations using slack variables, surplus variables and artificial variables.
  - Show how the rows for the two objective functions are formed. [9]
- (b) By re-writing the LP problem in terms of  $x$  and  $y$  only, use a 2-D graphical method to determine the maximum value of  $P$ , and the corresponding values of  $x$ ,  $y$  and  $z$ . [8]

**END OF QUESTION PAPER**

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