GCE

Further Mathematics A

Y532/01: Statistics

Advanced Subsidiary GCE

Mark Scheme for June 2019

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
SEEN	Acknowledgement of work not gaining credit; also replaces NR (no response)
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

Subject-specific Marking Instructions for AS Level Further Mathematics A

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to **3 s.f.** unless the question specifically asks for another level of accuracy. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

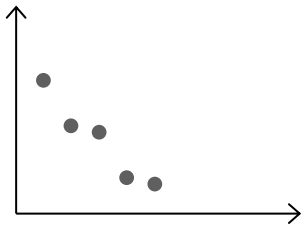
g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1	(a)		t: 2, 3, 4, 5, 6 $P(T=t)$: $\frac{1}{9}$, $\frac{2}{9}$, $\frac{3}{9}$, $\frac{2}{9}$, $\frac{1}{9}$	M1	3.1b	Attempt at distribution of T, *or* sample space (needs ÷ 9)	*NOT* from e.g. $\frac{6\times 2}{3}$
			$E(T) = \frac{1}{9}\times 2+\frac{2}{9}\times 3+\frac{3}{9}\times 4+\frac{2}{9}\times 5+\frac{1}{9}\times 6$	M1	1.1	Attempt sum of $t \times P(T=t)$	
			$= 4$ **AG**	A1	2.1	Fully correct working ("show that")	www
			Alternative				
			$E(S_1) = \frac{1}{3}\times 1+\frac{1}{3}\times 2+\frac{1}{3}\times 3$ [= 2]	M1		Probabilities for S_1 seen *as part of* attempt to use $S_1 + S_2$ (or $2S_1$)	*Or* $\frac{3+1}{2}\times 2$
			$E(T) = 2E(S_1)$	M1			$\left(\frac{1}{3}\times 1+\frac{1}{3}\times 2+\frac{1}{3}\times 3\right)^2$ M1M0A0
			$= 4$ **AG**	A1			
				[3]			
	(b)		$E(T^2) = 4\times\frac{1}{9}+9\times\frac{2}{9}+16\times\frac{3}{9}+25\times\frac{2}{9}+36\times\frac{1}{9}$ [$= \frac{156}{9}$]	M1	1.1	*Or:*	If sample space used, must ÷9
			$Var(T) = E(T^2) - [E(T)]^2 = \frac{156}{9} - 4^2$	M1	1.1	$2^2\times\frac{1}{9}+1^2\times\frac{2}{9}+0^2\times\frac{3}{9}+1^2\times\frac{2}{9}+2^2\times\frac{1}{9}$ M2	
			$= \frac{4}{3}$	A1	1.1	Allow from no working if 3/3 in (a)	
			Alternative				
			$E(S_1^2) = \frac{1}{3}\times 1+\frac{1}{3}\times 4+\frac{1}{3}\times 9 = \frac{14}{3}$	M1		Probs for S_1 *as part of* $S_1 + S_2$	*Or* $\frac{n^2-1}{12}\times 2$
			$Var(S_1) = \frac{14}{3} - 2^2$ [$= \frac{2}{3}$]	M1		*Not* $2\times\frac{14}{3} - 4^2$	
			$Var(T) = 2Var(S_1) = \frac{4}{3}$	A1			
				[3]			
	(c)	(i)	$0.3E(T) - 1.5$ [= (£)–0.3]	B1	3.3	Allow –30 or "30*p* loss" etc	
				[1]			
		(ii)	$0.3^2\times Var(T)$	M1	1.2	$0.3\times Var(T)$: M1	*Not* isw if later – 1.5
			$= (£^2)0.12$ or $\frac{3}{25}$	A1ft	1.1	FT on their $\frac{4}{3}$ provided > 0	Ignore units
				[2]			

Question			Answer	Marks	AO	Guidance	
2	(a)		$1 - P(\leq 27)$ $= 0.0525$ **BC**	**M1** **A1** **[2]**	3.4 1.1	Allow M1 for $1 - 0.9657 = 0.0343$ In range [0.0524, 0.0525] **BC**	
	(b)		Po(40) $1 - P(\leq 55)$ $= 0.00968$	**M1*** **depM1** **A1** **[3]**	3.1b 1.1a 1.1	Po(2×their 20) stated or implied Allow M1 for $1 - P(\leq 56) = 0.00658$ or $1 - 0.990 = 0.01$ or 0.0097 Awrt 0.00968	
	(c)		Orders on one day are independent of orders on the other	**B1** **[1]**	3.2b	Use "orders independent", clearly referred to the two different days, needs context [*not* "events"], and nothing else	*Not* anything affecting given separate Poissons, such as "orders must be independent" or "constant average rate".
3	(a)		Minimum possible non-zero value of Σd^2 … … is 2 $1 - \frac{6 \times 2}{9 \times 80}$ $\frac{59}{60}$ or 0.983(33…)	**M1** **A1** **M1** **A1** **[4]**	3.1a 2.2a 1.1a 1.1	Find minimum Σd^2 2 stated or used Use correct formula Answer, exact or 0.983 or better	Allow for $\Sigma d^2 = 1$ Allow $1 - \frac{6\Sigma d^2}{9 \times 80}$ used
	(b)		e.g. 	**M1** **A1** **[2]**	1.2 1.2	Points strictly decreasing Not in straight line, no errors SC: Curve, no points: M1	Ignore lines drawn < 5 points: M1A0. > 5 points: M1A1 Needn't be in first quadrant

4	**(a)**		$7!\times 4!$ $\div 10!$ $= \frac{120960}{3628800} = \frac{1}{30}$	**M1** **M1** **A1**	2.1 1.1a 1.1	Allow for $6!\times 4!$ or $6!\times 4!\times 2$ Divide by 10!, needs at least one factorial in numerator Answer, exact or awrt 0.0333	3/3 for $\frac{1}{30}$ www
			Alternative: $7\times\frac{6}{10}\times\frac{4}{9}\times\frac{5}{8}\times\frac{3}{7}\times\frac{4}{6}\times\frac{2}{5}\times\frac{3}{4}\times\frac{1}{3}\times\frac{2}{2}\times\frac{1}{1}$	**M1** **M1** **A1**		no 7, one other error only one error correct answer	
				[3]			
	(b)		Women placed in 6! ways, men in 4! [= 720×24]	**B1**	2.1	$6!\times 4!$ anywhere, or $6!\times$attempt at ${}^7P_4$	Or $6!\times 7\times 6\times 5\times 4$ B2
			4 slots *m* in *m*W*m*W*m*W*m*W*m*W*m*W*m* = ${}^7C_4$	**M1**	3.1b	Or ${}^7P_4$. Allow for *m* and *W* reversed	${}^7P_4\times 6!$: B1M1
			${}^7C_4\times\frac{6!\times 4!}{10!}$ $= \frac{1}{6}$	**M1** **A1**	1.1a 1.1	Needs attempt at both terms Or 0.167 or 0.1667 etc	$4\times(6!\times 4!)/10! = 2/105$: B1M1
			Alternative: PIE $(10! - 12\times 9! + (3\times 4\times 8! + 12\times 2\times 8!) - 24\times 7!)/10!$ [= (3628800 – 4354560 + 1451520 – 120960)/10!]	**M2** **A1** **A1**		Signs alternating, at least one term √ Allow one term omitted or wrong Correct answer	
				[3]			
Three together: $7\times 6\times\frac{6!4!}{10!} = \frac{1}{5}$			Two pairs: $\frac{7\times 6}{2}\times\frac{6!4!}{10!} = \frac{1}{10}$			One pair: $7\times\frac{6\times 5}{2}\times\frac{6!4!}{10!} = \frac{1}{2}$	
5	**(a)**		H_0: $\rho = 0$, H_1: $\rho \neq 0$, where ρ is population pmcc	**B1**	2.5	Must use symbols. Allow no definition of letter if ρ used	*Not* "H_0 : there is no assoc'n, H_1: there is association"
			0.701 > 0.4973	**B1**	1.1a	Correct CV stated, allow 0.497	
			Reject H_0. There is significant evidence of association between the marks on the two papers	**M1ft** **A1** **[4]**	1.1 2.2b	FT on wrong CV *Not* FT. Needs context, and not too definite.	*Not* <u>There is</u> association …"
	(b)	**(i)**	–0.534	**B2** **[2]**	1.1a 1.1	SC: if B0, give B1 for two of 1440, 2066, –921 and $S_{xy}/\sqrt{(S_{xx}S_{yy})}$	–0.53: B1

		(ii)	6 candidates did very well or very badly on both papers; middle 10 tended to do badly on one paper and well on the other	**B1** **[1]**	2.4	Correct inference about scores oe, *not* "correlation/association/value of *r*". *Not* "outliers" or "anomalies".	Allow inference for one group only, provided it *is* clearly for only one group & any ref to other group is not wrong
6	**(a)**		$10p(1-p)$	**B1** **[1]**	1.2	Allow $10pq$ oe, e.g. $10p - 10p^2$	Not just $np(1-p)$
	(b)	**(i)**	0.7^4 $= 0.240(1)$	**M1** **A1** **[2]**	1.1a 1.1	$0.7^5 = 0.168$ or $0.7^6 = 0.118$: M1 Allow 0.24	*Or* $1 - 0.3(1+0.7+0.7^2+0.7^3)$ Allow M1 if also 0.3×0.7^4 [0.15 is from binomial]
		(ii)	$q/p^2 = \frac{70}{9}$ or 7.777…	**B1** **[1]**	1.1	Allow 7.78, 7.778, etc	Allow 8 only if evidence, e.g. $(1 - 0.3)/0.3^2$
	(c)		$(1-p)^2 p = \frac{4}{25} p$ $p = 0$ or $(1-p)^2 = \frac{4}{25}$ $\quad (p \neq 0)$ $(1-p) = \pm\frac{2}{5}$ $p \neq \frac{7}{5}$ $p = \frac{3}{5}$	**B1** **M1** **M1** **B1ft** **A1** **[5]**	1.1 1.1a 1.1 2.3 2.1	Correct equation Reduce to quadratic/cubic and solve Obtain two non-zero solutions Explicitly discard one solution, *either* here *or* in line 2 (not enough to give 2 answers and then only 1) Exact final answer exact (0.6) no others left, allow from ± omitted	 e.g. $p(p^2 - 2p + \frac{21}{25}) = 0$ ± omitted: M0B0A1 Allow " $p = 0, \frac{3}{5}, \frac{7}{5}$ but $p \leq 1$" SC binomial: B0 then $75p^2 = (1-p)^2$ & solve M1 0.104 [0.1035] A1 Explicitly reject 0 or –0.13 B1 SC Poisson: 0

7	**(a)**		H_0: population frequencies in ratio 9:3:3:1 H_1: population frequencies not in ratio 9:3:3:1	**B1** **[1]**	1.1	"Population" can be implied, but *not* "observed frequencies are in ratio .."	Allow "data consistent with model"
	(b)		Expected frequencies 67.5, 22.5, 22.5, 7.5 $\sum\frac{(O-E)^2}{E} = 4.033 + 4.9 + 2.5 + 0.3$ $= 11.73$ $11.73 > 11.34$ Reject H_0. There is significant evidence that the results are not consistent with theory	**B1** **M1** **A1** **A1** **M1ft** **A1ft** **[6]**	3.3 3.4 1.1 1.1 1.1 2.2b	At least 2 correct Correct method for X^2 awrt 11.7 **BC** (working not needed) Compare X^2 value to CV 11.34 ft on TS only. Needs valid TS Conclusion, contextualised, acknowledge uncertainty. *Not* FT on hypotheses reversed	Allow rounded if seen correct Must see (any) value of X^2 *Not* "results are not consistent". FT on "do not reject" needs double negative
	(c)		*b* as it makes the largest contribution to X^2	**B1** **[1]**	2.4	Must refer to test statistic, not "difference"	Or "33" or other clear identification
	(d)		Increase *b* (from 3) and *either* decrease *a* (from 9) *or* increase *c* (from 3), e.g. 7:5:3:1 or 9:5:5:1	**B1** **[1]**	3.5c	E.g. 7:4:3:1, 8:4:3:1 or 9:4:4:1 Allow description if fully correct	Inconsistent to increase *c* but not *b*
8			Probability of this or better = $\frac{1+{}^nC_2}{n!}$ $\frac{1+{}^nC_2}{n!} < 0.01$ $n = 6, p = 0.0222\ldots \quad > 0.01$ $n = 7, p = 0.004365\ldots \quad < 0.01$ The smallest value of *n* is 7	**M1** **A1** **A1** **M1** **A1** **A1** **A1** **[7]**	3.1b 1.1 1.1 1.1 2.1 1.1 2.2a	${}^nC_2$ or ${}^nP_2 = n(n-1)$ or ${}^nC_1 = n$ seen +1 $\div n!$ < 0.01 and one relevant number substituted *or* attempt to simplify factorials One correct *p*, compared with 0.01 Both of these Correct conclusion stated, allow if only one probability seen, www	2 omitted, or $(1 + {}^nC_1)/n!$: M1A1A1M1 A0A0A0 = 4/7 Needs attempt at non-1 term *OR* $100 + 50n(n - 1) < n!$ $n(n-1)[(n-2)! - 50] > 100$ SR: Spearman: ($n = 6$, 0.9905 > 0.9429): SC B2

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