



# AS Level Further Mathematics A Y531/01 Pure Core

# Monday 14 May 2018 – Afternoon

Time allowed: 1 hour 15 minutes

#### You must have:

- Printed Answer Booklet
- Formulae AS Level Further Mathematics A

#### You may use:

• a scientific or graphical calculator

## **INSTRUCTIONS**

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer all the questions.
- Write your answer to each question in the space provided in the Printed Answer Booklet. If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- Do not write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by  $gm s^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use g = 9.8.

#### **INFORMATION**

- The total mark for this paper is 60.
- The marks for each question are shown in brackets [ ].
- You are reminded of the need for clear presentation in your answers.
- The Printed Answer Booklet consists of 12 pages. The Question Paper consists of 4 pages.

[3]

Answer all the questions.

- 1 (i) Find a vector which is perpendicular to both  $\begin{pmatrix} 1\\3\\-2 \end{pmatrix}$  and  $\begin{pmatrix} -3\\-6\\4 \end{pmatrix}$ . [2]
  - (ii) The cartesian equation of a line is  $\frac{x}{2} = y 3 = 2z + 4$ .

Express the equation of this line in vector form.

2 In this question you must show detailed reasoning.

The cubic equation  $2x^3 + 3x^2 - 5x + 4 = 0$  has roots  $\alpha$ ,  $\beta$  and  $\gamma$ . By making an appropriate substitution, or otherwise, find a cubic equation with integer coefficients whose roots are  $\frac{1}{\alpha}$ ,  $\frac{1}{\beta}$  and  $\frac{1}{\gamma}$ .

3 In this question you must show detailed reasoning.

The complex numbers  $z_1$  and  $z_2$  are given by  $z_1 = 2 - 3i$  and  $z_2 = a + 4i$  where a is a real number.

- (i) Express  $z_1$  in modulus-argument form, giving the modulus in exact form and the argument correct to 3 significant figures. [3]
- (ii) Find  $z_1 z_2$  in terms of a, writing your answer in the form c + id. [2]
- (iii) The real and imaginary parts of a complex number on an Argand diagram are x and y respectively. Given that the point representing  $z_1 z_2$  lies on the line y = x, find the value of a. [2]
- (iv) Given instead that  $z_1 z_2 = (z_1 z_2)^*$  find the value of a. [2]
- **4** The matrix **A** is given by  $\mathbf{A} = \begin{pmatrix} 2 & 1 & 2 \\ 1 & -1 & 1 \\ 2 & 2 & a \end{pmatrix}$ .
  - (i) Show that  $\det \mathbf{A} = 6 3a$ . [2]
  - (ii) State the value of a for which A is singular. [1]
  - (iii) Given that **A** is non-singular find  $\mathbf{A}^{-1}$  in terms of a. [4]

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- 5 In this question you must show detailed reasoning.
  - (i) Express  $(2+3i)^3$  in the form a+ib. [3]
  - (ii) Hence verify that 2+3i is a root of the equation  $3z^3 8z^2 + 23z + 52 = 0$ . [3]
  - (iii) Express  $3z^3 8z^2 + 23z + 52$  as the product of a linear factor and a quadratic factor with real coefficients. [4]
- 6 The matrices **A** and **B** are given by  $\mathbf{A} = \begin{pmatrix} t & 6 \\ t & -2 \end{pmatrix}$  and  $\mathbf{B} = \begin{pmatrix} 2t & 4 \\ t & -2 \end{pmatrix}$  where t is a constant.
  - (i) Show that  $|\mathbf{A}| = |\mathbf{B}|$ . [2]
  - (ii) Verify that  $|\mathbf{A}\mathbf{B}| = |\mathbf{A}||\mathbf{B}|$ . [3]
  - (iii) Given that  $|\mathbf{AB}| = -1$  explain what this means about the constant t. [2]
- 7 Prove by induction that  $2^{n+1} + 5 \times 9^n$  is divisible by 7 for all integers  $n \ge 1$ . [6]
- 8 The  $2 \times 2$  matrix **A** represents a transformation T which has the following properties.
  - The image of the point (0,1) is the point (3,4).
  - An object shape whose area is 7 is transformed to an image shape whose area is 35.
  - T has a line of invariant points.
  - (i) Find a possible matrix for **A**. [8]

The transformation S is represented by the matrix **B** where  $\mathbf{B} = \begin{pmatrix} 3 & 1 \\ 2 & 2 \end{pmatrix}$ .

- (ii) Find the equation of the line of invariant points of S. [2]
- (iii) Show that any line of the form y = x + c is an invariant line of S. [3]

#### **END OF QUESTION PAPER**

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4



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