Oxford Cambridge and RSA

Accredited

A Level Further Mathematics A
Y541 Pure Core 2
Sample Question Paper

Date – Morning/Afternoon
Time allowed: 1 hour 30 minutes

OCR supplied materials:
- Printed Answer Booklet
- Formulae A Level Further Mathematics A

You must have:
- Printed Answer Booklet
- Formulae A Level Further Mathematics A
- Scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.**
- Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total number of marks for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

1 Find $\sum_{r=1}^{n}(r+1)(r+5)$. Give your answer in a fully factorised form. **[4]**

2 **In this question you must show detailed reasoning.**

The finite region R is enclosed by the curve with equation $y=\frac{8}{\sqrt{16+x^2}}$, the x-axis and the lines $x=0$ and $x=4$. Region R is rotated through 360° about the x-axis. Find the exact value of the volume generated. **[4]**

3 **(i)** Find $\sum_{r=1}^{n}\left(\frac{1}{r}-\frac{1}{r+2}\right)$. **[3]**

(ii) What does the sum in part **(i)** tend to as $n\to\infty$? Justify your answer. **[1]**

4 It is given that $\frac{5x^2+x+12}{x^3+kx}\equiv\frac{A}{x}+\frac{Bx+C}{x^2+k}$ where k, A, B and C are positive integers.

Determine the set of possible values of k. **[5]**

5 **In this question you must show detailed reasoning.**

Evaluate $\int_0^\infty 2x\mathrm{e}^{-x}\,\mathrm{d}x$.

[You may use the result $\lim_{x\to\infty} x\mathrm{e}^{-x}=0$.] **[4]**

6 The equation of a plane Π is $x-2y-z=30$.

(i) Find the acute angle between the line $\mathbf{r}=\begin{pmatrix}3\\2\\-5\end{pmatrix}+\lambda\begin{pmatrix}-5\\3\\2\end{pmatrix}$ and Π. **[4]**

(ii) Determine the geometrical relationship between the line $\mathbf{r}=\begin{pmatrix}1\\4\\2\end{pmatrix}+\mu\begin{pmatrix}3\\-1\\5\end{pmatrix}$ and Π. **[4]**

7 **(i)** Use the Maclaurin series for $\sin x$ to work out the series expansion of $\sin x\sin 2x\sin 4x$ up to and including the term in x^3. **[4]**

(ii) Hence find, in exact surd form, an approximation to the least positive root of the equation $2\sin x\sin 2x\sin 4x=x$. **[3]**

8 The equation of a curve is $y = \cosh^2 x - 3\sinh x$. Show that $\left(\ln\left(\dfrac{3+\sqrt{13}}{2}\right), -\dfrac{5}{4}\right)$ is the only stationary point on the curve. **[8]**

9 A curve has equation $x^4 + y^4 = x^2 + y^2$, where x and y are not both zero.

(i) Show that the equation of the curve in polar coordinates is $r^2 = \dfrac{2}{2-\sin^2 2\theta}$. **[4]**

(ii) Deduce that no point on the curve $x^4 + y^4 = x^2 + y^2$ is further than $\sqrt{2}$ from the origin. **[2]**

10 Let $C = \displaystyle\sum_{r=0}^{20} \binom{20}{r} \cos r\theta$. Show that $C = 2^{20} \cos^{20}\left(\tfrac{1}{2}\theta\right) \cos 10\theta$. **[8]**

11 During an industrial process substance X is converted into substance Z. Some of the substance X goes through an intermediate phase, and is converted to substance Y, before being converted to substance Z. The situation is modelled by

$$\frac{dy}{dt} = 0.3x - 0.2y \quad \text{and} \quad \frac{dz}{dt} = 0.2y + 0.1x$$

where x, y and z are the amounts in kg of X, Y and Z at time t hours after the process starts.

Initially there is 10 kg of substance X and nothing of substances Y and Z. The amount of substance X decreases exponentially. The initial rate of decrease is 4 kg per hour.

(i) Show that $x = Ae^{-0.4t}$, stating the value of A. **[3]**

(ii) (a) Show that $\dfrac{dx}{dt} + \dfrac{dy}{dt} + \dfrac{dz}{dt} = 0$. **[2]**

(b) Comment on this result in the context of the industrial process. **[2]**

(iii) Express y in terms of t. **[5]**

(iv) Determine the maximum amount of substance Y present during the process. **[3]**

(v) How long does it take to produce 9 kg of substance Z? **[2]**

END OF QUESTION PAPER