Oxford Cambridge and RSA

Thursday 7 October 2021 – Afternoon

A Level Further Mathematics A

Y541/01 Pure Core 2

Time allowed: 1 hour 30 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

Answer **all** the questions.

1 Two matrices, **A** and **B**, are given by $\mathbf{A} = \begin{pmatrix} 1 & -2 & -1 \\ 2 & -3 & 1 \\ a & 1 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -6 & 3 & -4 \\ -1 & 6 & -4 \\ 8 & -8 & -1 \end{pmatrix}$ where a is a constant.

Find the value of a for which $\mathbf{AB} = \mathbf{BA}$. **[3]**

2 **In this question you must show detailed reasoning.**

The complex numbers z_1 and z_2 are given by $z_1 = 3 - 7\text{i}$ and $z_2 = 2 + 4\text{i}$.

(a) Express each of the following as exact numbers in the form $a + b\text{i}$.

(i) $3z_1 + 4z_2$ **[1]**

(ii) $z_1 z_2$ **[2]**

(iii) $\dfrac{z_1}{z_2}$ **[2]**

(b) Write z_1 in modulus-argument form giving the modulus in exact form and the argument correct to **3** significant figures. **[3]**

3 The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 1 \\ -3 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix}$.

The plane Π has equation $\mathbf{r}.\begin{pmatrix} 2 \\ -5 \\ -3 \end{pmatrix} = 4$.

(a) Find the position vector of the point of intersection of l_1 and Π. **[3]**

(b) Find the acute angle between l_1 and Π. **[3]**

A is the point on l_1 where $\lambda = 1$.

l_2 is the line with the following properties.

- l_2 passes through A
- l_2 is perpendicular to l_1
- l_2 is parallel to Π

(c) Find, in vector form, the equation of l_2. **[3]**

4 **In this question you must show detailed reasoning.**

Determine the value of $\sum_{r=1}^{100} (2r+3)^2$. **[3]**

5 **In this question you must show detailed reasoning.**

(a) Using the definition of $\cosh x$ in terms of exponentials, show that $\cosh 2x \equiv 2\cosh^2 x - 1$. **[2]**

(b) Solve the equation $\cosh 2x = 3\cosh x + 1$, giving all your answers in exact logarithmic form. **[6]**

6 **In this question you must show detailed reasoning.**

The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$.

(a) Define the transformation represented by **A**. **[1]**

(b) Show that the area of any object shape is invariant under the transformation represented by **A**. **[1]**

The matrix **B** is given by $\mathbf{B} = \begin{pmatrix} 7 & 2 \\ 21 & 7 \end{pmatrix}$. You are given that **B** represents the transformation which is the result of applying the following **three** transformations in the given order.

- A shear which leaves the y-axis invariant and which transforms the point (1, 1) to the point (1, 4).
- The transformation represented by **A**.
- A stretch of scale factor p which leaves the x-axis invariant.

(c) Determine the value of p. **[4]**

7 **In this question you must show detailed reasoning.**

(a) Find the values of A, B and C for which $\frac{x^3+x^2+9x-1}{x^3+x^2+4x+4} \equiv A + \frac{Bx+C}{x^3+x^2+4x+4}$. **[1]**

(b) Hence express $\frac{x^3+x^2+9x-1}{x^3+x^2+4x+4}$ using partial fractions. **[5]**

(c) Using your answer to part **(b)**, determine $\int_0^2 \frac{x^3+x^2+9x-1}{x^3+x^2+4x+4}\,dx$ expressing your answer in the form $a + \ln b + c\pi$ where a is an integer, and b and c are both rational. **[4]**

8 A particle P of mass 2 kg can only move along the straight line segment OA, where OA is on a rough horizontal surface. The particle is initially at rest at O and the distance OA is 0.9 m.

When the time is t seconds the displacement of P from O is x m and the velocity of P is $v\,\mathrm{m\,s^{-1}}$. P is subject to a force of magnitude $4e^{-2t}$ N in the direction of A for any $t \geqslant 0$. The resistance to the motion of P is modelled as being proportional to v.

At the instant when $t = \ln 2$, $v = 0.5$ and the resultant force on P is 0 N.

(a) Show that, according to the model, $\frac{dv}{dt} + v = 2e^{-2t}$. **[3]**

(b) Find an expression for v in terms of t for $t \geqslant 0$. **[5]**

(c) By considering the behaviour of v as t becomes large explain why, according to the model, P's speed must reach a maximum value for some $t > 0$. **[2]**

(d) Determine the maximum speed considered in part **(c)**. **[2]**

(e) Determine the greatest value of t for which the model is valid. **[4]**

9 The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} 2 & 3 \\ 0 & 2 \end{pmatrix}$.

(a) By considering $\mathbf{A}$, $\mathbf{A}^2$, $\mathbf{A}^3$ and $\mathbf{A}^4$ make a conjecture about the form of the matrix $\mathbf{A}^n$ in terms of n for $n \geqslant 1$. **[2]**

(b) Use induction to prove the conjecture made in part **(a)**. **[4]**

10 **In this question you must show detailed reasoning.**

(a) By using an appropriate Maclaurin series prove that if $x > 0$ then $e^x > 1 + x$. **[2]**

(b) Hence, by using a suitable substitution, deduce that $e^t > et$ for $t > 1$. **[1]**

(c) Using the inequality in part **(b)**, and by making a suitable choice for t, determine which is greater, e^{π} or π^{e}. **[3]**

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