GCE

Further Mathematics A

Y541/01: Pure Core 2

Advanced GCE

Mark Scheme for June 2019

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations and abbreviations

Annotation in scoris	Meaning
✓and✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

Subject-specific Marking Instructions for ALevel Further Mathematics A

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows thatthe method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for g. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. ‘Fresh starts’ will not affect an earlier decision about a misread. Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1	(a)		**DR**				
			$(r+2)(r+1)$	**B1**	1.1a	Correct factorisation of denominator soi	
			$\frac{A}{r+1}+\frac{B}{r+2}$	**M1**	1.1	Correct general form for partial fractions soi by correct answer	Could be incorrect if recovered. eg $\frac{A}{r+1}+\frac{B}{r+2}+C$ and $C = 0$
			$A = 1, B = -1$	**A1**	1.1	Both	$\frac{1}{r+1}-\frac{1}{r+2}$
			$\Sigma = \frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\frac{1}{4}\ \ldots -\frac{1}{n+1}+\frac{1}{n+1}-\frac{1}{n+2}$	**M1**	2.1	Condone omission of $+\frac{1}{4}\ and\ -\frac{1}{n+1}$ for M1 only	
			$=\frac{1}{2}-\frac{1}{n+2}$	**A1**	1.1	AG. Cancellation must be evident	
				[5]			
	(b)		**DR**				
			$\sum^{\infty}=\frac{1}{2}$	**B1**	2.2a		
			since $\frac{1}{n+2}\to 0$ as $n\to\infty$	**B1**	2.4	Or $\lim_{n\to\infty}\left(\frac{1}{2}-\frac{1}{n+2}\right)=\frac{1}{2}-0=\frac{1}{2}$	Indication that $1/(n + 2)$ is close to zero (accept "small") when n is large
				[2]			

Question			Answer	Marks	AO	Guidance	
2	(a)		$\dfrac{\left\lvert \begin{pmatrix}4\\-5\\1\end{pmatrix}.\begin{pmatrix}3\\6\\-2\end{pmatrix}-15\right\rvert}{\left\lvert\begin{pmatrix}3\\6\\-2\end{pmatrix}\right\rvert}$	M1	1.1a	Substitution into formula. Condone missing mod signs on top	Numerator can be $\left\lvert \mathbf{q}.\begin{pmatrix}3\\6\\-2\end{pmatrix}\right\rvert$ where $\mathbf{q}$ is vector from C to point on Π Or substituting $\mathbf{r}=\begin{pmatrix}4\\-5\\1\end{pmatrix}+\lambda\begin{pmatrix}3\\6\\-2\end{pmatrix}$ into equation of Π to find a value for λ.
			$\left\lvert\begin{matrix}3\\6\\-2\end{matrix}\right\rvert=\sqrt{3^2+6^2+2^2}$ or $\begin{pmatrix}4\\-5\\1\end{pmatrix}.\begin{pmatrix}3\\6\\-2\end{pmatrix}=4\times 3+(-5)\times 6+1\times(-2)$	M1	1.1	Soi	
			5	A1 [3]	1.1		
	(b)		$\boldsymbol{r}=\begin{pmatrix}5\\2\\4\end{pmatrix}-\begin{pmatrix}4\\3\\1\end{pmatrix}=\begin{pmatrix}1\\-1\\3\end{pmatrix}$	M1		Finding the vector $\mathbf{r}$ from a point on one line to a point on the other	
			$\begin{pmatrix}1\\-1\\3\end{pmatrix}.\begin{pmatrix}1\\-2\\1\end{pmatrix}=\sqrt{11}\sqrt{6}\cos\theta$	*M1		Using dot product to find an expression for $\cos\theta$	NB Ignore attempts to use the formula for the distance between 2 skew lines.
			$cos\theta=\pm\dfrac{6}{\sqrt{66}}$	A1		can be implied by $\cos\theta=0.739$, or $\theta=0.740$ or 2.402 or 42.4° or 137.6°	
			$d=\lvert\boldsymbol{r}\rvert sin\theta=\lvert\boldsymbol{r}\rvert\sqrt{1-cos^2\theta}$	dep*M1			
			$\sqrt{5}$	A1			

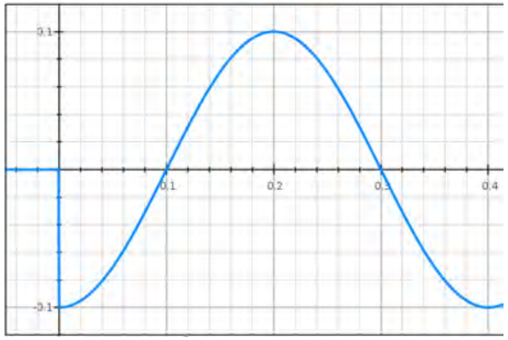
Question			Answer	Marks	AO	Guidance	
			Alternate method 1				
			$\mathbf{r} = \begin{pmatrix}5\\2\\4\end{pmatrix} + \mu\begin{pmatrix}1\\-2\\1\end{pmatrix} - \begin{pmatrix}4\\3\\1\end{pmatrix}$	**M1**	3.1a	Finding the vector from a particular point on one line to a general point on the other	
			$\left(\begin{pmatrix}5\\2\\4\end{pmatrix} + \mu\begin{pmatrix}1\\-2\\1\end{pmatrix} - \begin{pmatrix}4\\3\\1\end{pmatrix}\right) . \begin{pmatrix}1\\-2\\1\end{pmatrix} = 0$	***M1**	1.1a	Dotting the vector found with one of the direction vectors and setting to 0	
			($\mu = -1$ so) $\begin{pmatrix}4\\4\\3\end{pmatrix}$	**A1**	1.1	Solving to find (a parameter value and hence) position vector or coordinates of equivalent point on other line	or ($\lambda = -\frac{1}{2}$ and) $\begin{pmatrix}5\\1\\2\end{pmatrix}$
			$d = \sqrt{(4-4)^2 + (3-4)^2 + (1-3)^2}$	**dep*M1**	1.1	Distance formula for their 2 points	
			$\sqrt{5}$	**A1**	1.1		
			Alternate method 2				
			$\mathbf{r} = \begin{pmatrix}5\\2\\4\end{pmatrix} + \mu\begin{pmatrix}1\\-2\\1\end{pmatrix} - \begin{pmatrix}4\\3\\1\end{pmatrix}$	**M1**		Finding the vector from a particular point on one line to a general point on the other	As above
			$\lvert \boldsymbol{r} \rvert = \sqrt{6\mu^2 + 12\mu + 11}$	***M1** **A1**			
			$\mu = -1$	**dep*M1**		Attempts to find parameter (Calculus or completes square)	
			$d = \sqrt{5}$	**A1**			
			Alternate method 3				
			$\boldsymbol{r} = \begin{pmatrix}5\\2\\4\end{pmatrix} - \begin{pmatrix}4\\3\\1\end{pmatrix} = \begin{pmatrix}1\\-1\\3\end{pmatrix}$	**M1**		Finding the vector **r** from a point on one line to a point on the other	

Question			Answer	Marks	AO	Guidance	
			$\left\|\begin{pmatrix}1\\-1\\3\end{pmatrix}\times\begin{pmatrix}1\\-2\\1\end{pmatrix}\right\| = \left\|\begin{matrix}5\\2\\-1\end{matrix}\right\|$	***M1**		Calculates cross product of 'their ***r***' and direction vector.	Allow if full method seen or if 2 terms correct
			$\ldots = \sqrt{5^2+2^2+1^2} = \sqrt{30}$	**A1**			
			$d = \dfrac{\sqrt{30}}{\left\|\begin{pmatrix}1\\-2\\1\end{pmatrix}\right\|} = \sqrt{5}$	**dep*M1** **A1**			
				[5]			
3			**DR**				
			$\int (x-1)^{-\frac{3}{2}}\,dx = -2(x-1)^{-\frac{1}{2}}(+c)$	**B1**	1.1a		
			$\int_5^N (x-1)^{-\frac{3}{2}}\,dx = \left[-2(x-1)^{-\frac{1}{2}}\right]_5^N$	**M1**	2.1	Consideration of a finite upper limit	
			$-\dfrac{2}{\sqrt{N-1}}+\dfrac{2}{\sqrt{5-1}}$	**A1**	1.1		
			$\lim\limits_{N\to\infty}\dfrac{1}{\sqrt{N-1}} = 0$ oe	**B1**	2.1	Not *just* eg $\dfrac{1}{\infty}=0$	Can be seen as part of limit of both terms, but must be explicitly shown as zero
			$\int_5^\infty (x-1)^{-\frac{3}{2}}\,dx = \lim\limits_{N\to\infty}\left\{-\dfrac{2}{\sqrt{N-1}}+\dfrac{2}{\sqrt{5-1}}\right\} = 1$	**A1** **[5]**	2.2a	AG. Convincing argument equating improper integral to solution	

Question			Answer	Marks	AO	Guidance	
4	(a)		$\mathbf{A}=\begin{pmatrix}1&0\\k&1\end{pmatrix}$	B1	1.2	Correct form of **A** seen	
			$\mathbf{A}=\begin{pmatrix}1&0\\k&1\end{pmatrix}\begin{pmatrix}2\\1\end{pmatrix}=\begin{pmatrix}2\\2k+1\end{pmatrix}=\begin{pmatrix}2\\9\end{pmatrix}$ $\begin{pmatrix}1&0\\k&1\end{pmatrix}\begin{pmatrix}2\\1\end{pmatrix}=\begin{pmatrix}2\\2k+1\end{pmatrix}=\begin{pmatrix}2\\9\end{pmatrix}$	M1	1.1	Correctly multiplying object vector into their **A** to find image and equating to given image	their **A** must have at least 1 unknown element
			$k = 4$ so $\mathbf{A}=\begin{pmatrix}1&0\\4&1\end{pmatrix}$	A1 **[3]**	1.1		
	(b)		det**A** = 1 – 0 = 1 The determinant is the area scale factor so a determinant of 1 leaves the area unchanged	B1 B1 **[2]**	1.1 2.4	Correctly finding determinant Convincing explanation. Must include both ideas.	

Question			Answer	Marks	AO	Guidance	
5	(a)		$F = ma \Rightarrow \frac{1}{2}t - v = 2\frac{dv}{dt} \Rightarrow \frac{dv}{dt} + \frac{1}{2}v = \frac{1}{4}t$	B1 [1]	3.3	AG. Intermediate step as shown or both sides halved	
	(b)		IF = $e^{\int\frac{1}{2}dt} = e^{\frac{1}{2}t}$	B1	1.1	Or $e^{\frac{1}{2}t+c}$ or $Ae^{\frac{1}{2}t}$	
			$e^{\frac{1}{2}t}\frac{dv}{dt} + \frac{1}{2}e^{\frac{1}{2}t}v = \frac{d}{dt}\left(ve^{\frac{1}{2}t}\right) = \frac{1}{4}te^{\frac{1}{2}t}$	M1	1.1	Multiplying both sides by IF and writing new LHS as an exact derivative	
			$ve^{\frac{1}{2}t} = \frac{1}{2}te^{\frac{1}{2}t} - \frac{1}{2}\int e^{\frac{1}{2}t}dt = \frac{1}{2}te^{\frac{1}{2}t} - e^{\frac{1}{2}t}(+c)$	A1	1.1		
			$0 = -1 + c \Rightarrow c = 1$	M1	3.4	Use initial conditions to determine c	Allow if using a solution of correct form with wrong (non-zero) coefficients
			$v = \frac{1}{2}t - 1 + e^{-\frac{1}{2}t}$	A1	1.1		
			Alternate method AE: $\lambda + \frac{1}{2} = 0, \lambda = -\frac{1}{2}$ CF: $(v =)Ae^{-\frac{1}{2}t}$	B1			
			Trial function: $v = at + b,\ \frac{dv}{dt} = a$ $a + \frac{1}{2}(at + b) = \frac{1}{4}t$ $a = \frac{1}{2}, b = -1$	M1A1			
			GS: $(v =)Ae^{-\frac{1}{2}t} + \frac{1}{2}t - 1$				
			$v = 0, t = 0$ gives $A = 1$ $v = e^{-\frac{1}{2}t} + \frac{1}{2}t - 1$	M1		Substitutes boundary condition into general solution to find A	Allow if using a solution of correct form with wrong (non-zero) coefficients

Question			Answer	Marks	AO	Guidance	
				A1			
				[5]			
	(c)		$t = 2 \Rightarrow v = e^{-1}$ (so velocity is e^{-1} ms^{-1})	B1 [1]	3.4	or awrt 0.368	
	(d)		change the RHS of DE in part (a) from $\frac{1}{4}t$ to $\frac{1}{4}$. oe	B1 [1]	3.5c		
	(e)		$\frac{d}{dt}\left(ve^{\frac{1}{2}t}\right) = \frac{1}{4}e^{\frac{1}{2}t} \Rightarrow ve^{\frac{1}{2}t} = \int \frac{1}{4}e^{\frac{1}{2}t}dt$ $ve^{\frac{1}{2}t} = \frac{1}{2}e^{\frac{1}{2}t} + c$. oe	B1	2.2a	**SC1** $ve^{\frac{1}{2}t} = \frac{N}{2}e^{\frac{1}{2}t} + c$ oe	Or $\frac{dv}{dt} + \frac{1}{2}v = \frac{1}{4}$ rearranged to $\int \frac{2dv}{1-2v} = \frac{1}{2}\int dt = \frac{1}{2}t$ giving $-\ln(1-2v) + c' = \frac{1}{2}t$
			$e^{-1}e^{1} = \frac{1}{2}e^{1} + c \Rightarrow c = 1 - \frac{1}{2}e$	M1	3.3	substitute their boundary condition from (c) into correct GS to find c	$c' = 1 + \ln(1 - 2e^{-1})$
			$v = \frac{1}{2} + \left(1 - \frac{1}{2}e\right)e^{-\frac{1}{2}t}$	A1 [3]	1.1		

Question		Answer	Marks	AO	Guidance	
6	(a)		B1 B1 B1 B1 [4]	1.2 3.4 3.4 3.1b	At least one cycle of $A\cos \omega t$ graph Amplitude 0.1 Period 0.4 Intersect with the t-axis at 0.1 and 0.3 (values must be indicated or implied unambiguously (eg by tick-marks and a single value))	graph must instantaneously horizontal at top/bottom, continuous and not vertical at any point. Ignore any graph outside [0, 0.4] Non-inverted cos graph can still get 4/4
	(b)	So $x = \pm 0.1\cos(5\pi t)$ or $x = 0.1\sin(5\pi t \pm \frac{1}{2}\pi)$ when t = 0.75 or 0.35 $x = -\frac{\sqrt{2}}{20}$ (= –0.0707 to 3 sf)	M1 A1 [2]	3.1b 3.4	or by argument from sketch	Condone amplitude of 0.2 for M1
7	(a)	(2 + 3i) – (1 – i) (= ±(1 + 4i)) soi $\lvert 1 + 4i \rvert = \sqrt{1^2 + 4^2}$ 17	B1 M1 A1 [3]	1.1 2.2a 1.1	Either way round Can be implied by vector Or finding the square of their side	
	(b)	(±i)×(±(1 + 4i)) (2 + 3i) ± i(1 + 4i) and (1 – i) ± i(1 + 4i) So vertices at –3 and –2 + 4i	*M1 dep*M1 A1	3.1a 2.2a 3.2a	Method to find a complex number representing perpendicular side. Can be implied by ±(4 – i) Method to find both pairs of numbers	Or vector form if take geometric approach

Question			Answer	Marks	AO	Guidance	
			Or at 5 – 2i and 6 + 2i	A1 [4]	3.2a	Both clearly paired and in complex number form for final A1	If **M1M0A0A0** then add **SC1** for any two correct vertices

Question			Answer	Marks	AO	Guidance	
8	(a)		**DR** $\sin\theta = \dfrac{e^{i\theta} - e^{-i\theta}}{2i}$	*B1	1.1a		Condone $2i\sin\theta = e^{i\theta} - e^{-i\theta}$
			$\sin^6\theta = \left(\dfrac{e^{i\theta} - e^{-i\theta}}{2i}\right)^6 = -\dfrac{1}{64}\left(e^{i\theta} - e^{-i\theta}\right)^6$	M1	2.1	Raising expression for sin θ to the power 6 and $(2i)^6 = -64$.	Allow use of $sin\theta = \frac{e^{i\theta}+e^{-i\theta}}{2i}$ for 1st two M marks only
			$(e^{i\theta} - e^{-i\theta})^6 =$ $e^{6i\theta} - 6e^{4i\theta} + 15e^{2i\theta} - 20 + 15e^{-2i\theta} - 6e^{-4i\theta} + e^{-6i\theta}$	M1	1.1	Genuine attempt to use binomial expansion with correct evaluated binomial coefficients. Condone sign errors	If i omitted from denominator their expression for sin θ then only this M mark can still be awarded
			$e^{6i\theta} + e^{-6i\theta} - 6\left(e^{4i\theta} + e^{-4i\theta}\right) + 15\left(e^{2i\theta} + e^{-2i\theta}\right) - 20$ $= 2\cos 6\theta - 6\times 2\cos 4\theta + 15\times 2\cos 2\theta - 20$	M1	2.1	Collecting terms and <u>using</u> $e^{i\phi} + e^{-i\phi} = 2\cos\phi$ at least once.	
			$\therefore sin^6\,\theta =$ $-\dfrac{1}{64}\left(2\cos 6\theta - 12\cos 4\theta + 30\cos 2\theta - 20\right)$ $= \dfrac{1}{32}\left(10 - 15\cos 2\theta + 6\cos 4\theta - \cos 6\theta\right)$	dep*A1 [5]	1.1	AG. Fully correct argument	
	(b)		**DR** $\theta = \dfrac{\pi}{8}$ and eg $\cos 2\theta = \dfrac{\sqrt{2}}{2}$	*M1	2.1	Choice of θ soi and calculation of at least one cos term.	

Question			Answer	Marks	AO	Guidance	
			$\sin^6\frac{\pi}{8}=\frac{1}{32}\left(10-15\times\frac{\sqrt{2}}{2}-\frac{-\sqrt{2}}{2}(+6(0))\right)$	**dep*M1**	1.1	Substitution and calculation of all cos terms	Terms must be shown distinct either in this line or in the form of $\cos n\frac{\pi}{8}$
			$\sin\frac{\pi}{8}=\sqrt[6]{\frac{1}{64}\left(20-15\sqrt{2}+\sqrt{2}\right)}$ $=\frac{1}{2}\sqrt[6]{20-14\sqrt{2}}$	**A1** **[3]**	2.2a	AG Some intermediate working must be seen	

Question			Answer	Marks	AO	Guidance	
9	**(a)**		**DR**				
			$\frac{1}{2}\int\left(\sqrt{sin\theta}e^{\frac{1}{3}cos\theta}\right)^2\,\mathrm{d}\theta$	**M1**	3.1a	Correct form, in terms of θ,	M1 can be implied by 1.0757... BC
			$\mathrm{A}=\frac{1}{2}\int_0^{\pi}\sin\theta\mathrm{e}^{\frac{2}{3}\cos\theta}\,\mathrm{d}\theta$	***A1**	2.1	Integrand has been squared out. Must include limits (can be seen later)	
			$=\frac{1}{2}\times-\frac{3}{2}\left[\mathrm{e}^{\frac{2}{3}\cos\theta}\right]_0^{\pi}$	**dep*M1**	1.1a	Might be as result of substitution Allow coefficient error for **M1**	eg $\frac{3}{4}\left[\mathrm{e}^u\right]_{-\frac{2}{3}}^{\frac{2}{3}}$ or $\frac{3}{4}\left[\mathrm{e}^{\frac{2}{3}u}\right]_{-1}^{1}$ oe
			$\frac{3}{4}\left(\mathrm{e}^{\frac{2}{3}}-\mathrm{e}^{-\frac{2}{3}}\right)$	**A1** **[4]**	1.1	isw	
	(b)		**DR** $\frac{\mathrm{d}r}{\mathrm{d}\theta}=\frac{1}{2}\cos\theta(\sin\theta)^{-\frac{1}{2}}e^{\frac{1}{3}\cos\theta}+$	***M1** **A1**	3.1a 1.1	Attempt to differentiate using product and chain rules.	Must be in the form $uv'+u'v$ with at most one of u, v, u' or v' incorrect or omitted

			$(\sin\theta)^{\frac{1}{2}}(-\frac{1}{3}\sin\theta)e^{\frac{1}{3}\cos\theta}$ $\frac{dr}{d\theta}=\frac{1}{6}(\sin\theta)^{-\frac{1}{2}}e^{\frac{1}{3}\cos\theta}(3\cos\theta-2\sin^2\theta)$ $\frac{dr}{d\theta}=0\Rightarrow 3\cos\theta-2\sin^2\theta=0$	dep*M1	2.2a	Setting r' to zero and factorising/cancelling to produce a quadratic equation in cos and/or sin	
			$2\cos^2\theta+3\cos\theta-2=0$	M1	2.1	Use of $\cos^2+\sin^2=1$ to find 3 term quadratic equation in $\cos\theta$.	Or could be in $\sin^2\theta$; $4\sin^4\theta+9\sin^2\theta-9=0$
			$\cos\theta$ = ½, –2	*A1	1.1	Solving quadratic correctly	$\sin^2\theta$ = ¾, –3
			$\cos\theta\neq-2$	dep*A1	2.3	Explicitly rejecting root	Rejects $\sin^2\theta$ = –3 and $\sin\theta=-\frac{\sqrt{3}}{2}$
			$\Rightarrow\sin\theta=\frac{\sqrt{3}}{2}\Rightarrow r=\sqrt{\frac{\sqrt{3}}{2}e^{\frac{1}{3}\times\frac{1}{2}}}=\sqrt{\frac{\sqrt{3}}{2}e^{\frac{1}{6}}}$	A1 [7]	2.2a	AG. At least one intermediate step must be seen.	Can be awarded even if rejection of root(s) was implicit.

Question			Answer	Marks	AO	Guidance	
10	(a)		$f(0)=\ln(\frac{1}{2}+\cos 0)=\ln(\frac{3}{2})$	B1	1.1	soi	
			$\frac{d}{dx}\left(\ln(\frac{1}{2}+\cos x)\right)=\frac{-\sin x}{\frac{1}{2}+\cos x}\Rightarrow f'(0)=0$	M1	3.1a	Differentiating using chain rule (or rule for ln(f(x)) and evaluating when x = 0	Allow sign error in numerator
			$\frac{d^2\ln\left(\frac{1}{2}+\cos x\right)}{dx^2}=\frac{-\cos x\left(\frac{1}{2}+\cos x\right)+\sin x(-\sin x)}{\left(\frac{1}{2}+\cos x\right)^2}$	A1	1.1	Differentiating again using quotient (or product/chain) rule.	NB Simplifies to $-\frac{\frac{1}{2}\cos x+1}{\left(\frac{1}{2}+\cos x\right)^2}$
			$\ldots\Rightarrow f''(0)=-\frac{2}{3}$	A1	1.1	www	

			$\ln\left(\frac{1}{2}+\cos x\right)=\ln\left(\frac{3}{2}\right)-\frac{x^2}{3}+\ldots$	[4]			If zero scored then **SC1** for correct expansion
	(b)		$\ln\left(\frac{1}{2}+\cos x\right)=0\Rightarrow x=\frac{\pi}{3}$ (or $-\frac{\pi}{3}$)	**B1**	1.1	Finding either $\pm\pi/3$ as a root. Allow 60° for B1. Ignore other roots	
			$\therefore \ln\left(\frac{3}{2}\right)-\frac{\left(\frac{\pi}{3}\right)^2}{3}\approx 0$	**M1**	3.1a	Substituting their root, in radians, into their Maclaurin series and equating (approximately) to 0.	Or equating their expression (approximately) to 0 and rearranging for *x*: $\ln\left(\frac{3}{2}\right)-\frac{x^2}{3}\approx 0\Rightarrow x\approx\sqrt{3\ln\left(\frac{3}{2}\right)}$
			$\ln\left(\frac{3}{2}\right)-\frac{\pi^2}{27}\approx 0\Rightarrow \pi\approx\sqrt{27\ln\left(\frac{3}{2}\right)}=3\sqrt{3\ln\left(\frac{3}{2}\right)}$	**A1** [3]	3.2a	Could see ± but must be removed by final conclusion. Must use approximately equals symbol (not just equals symbol)	$\frac{\pi}{3}\approx\sqrt{3\ln\left(\frac{3}{2}\right)}\Rightarrow\pi\approx 3\sqrt{3\ln\left(\frac{3}{2}\right)}$

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