Oxford Cambridge and RSA

# Monday 20 May 2019 – Afternoon

## AS Level Further Mathematics A

### **Y533/01** Mechanics

**Time allowed: 1 hour 15 minutes**

**You must have:**
- Printed Answer Booklet
- Formulae AS Level Further Mathematics A

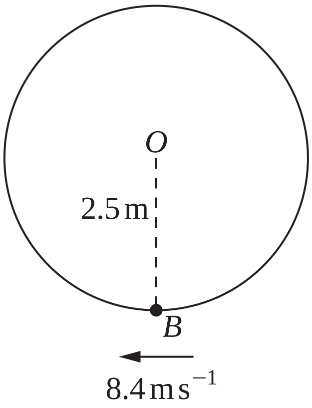
**You may use:**
- a scientific or graphical calculator

## INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

## INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

Answer **all** the questions.

**1**




A smooth wire is shaped into a circle of radius 2.5 m which is fixed in a vertical plane with its centre at a point *O*. A small bead *B* is threaded onto the wire. *B* is held with *OB* vertical and is then projected horizontally with an initial speed of $8.4\,\mathrm{m\,s^{-1}}$ (see diagram).

**(a)** Find the speed of *B* at the instant when *OB* makes an angle of 0.8 radians with the downward vertical through *O*. **[3]**

**(b)** Determine whether *B* has sufficient energy to reach the point on the wire vertically above *O*. **[3]**

**2** A particle *A* of mass 3.6 kg is attached by a light inextensible string to a particle *B* of mass 2.4 kg.

*A* and *B* are initially at rest, with the string slack, on a smooth horizontal surface. *A* is projected directly away from *B* with a speed of $7.2\,\mathrm{m\,s^{-1}}$.

**(a)** Calculate the speed of *A* after the string becomes taut. **[3]**

**(b)** Find the impulse exerted on *A* at the instant that the string becomes taut. **[2]**

**(c)** Find the loss in kinetic energy as a result of the string becoming taut. **[2]**

**3** A car of mass 1500 kg has an engine with maximum power 60 kW. When the car is travelling at $10\,\mathrm{m\,s^{-1}}$ along a straight horizontal road using maximum power, its acceleration is $3.3\,\mathrm{m\,s^{-2}}$.

In an initial model of the motion of the car it is assumed that the resistance to motion is constant.

**(a)** Using this initial model, find the greatest possible steady speed of the car along the road. **[4]**

In a refined model the resistance to motion is assumed to be proportional to the speed of the car.

**(b)** Using this refined model, find the greatest possible steady speed of the car along the road. **[5]**

The greatest possible steady speed of the car on the road is measured and found to be $21.6\,\mathrm{m\,s^{-1}}$.

**(c)** Explain what this value means about the models used in parts **(a)** and **(b)**. **[2]**

**4** A student is studying the speed of sound, $u$, in a gas under different conditions.

He assumes that $u$ depends on the pressure, $p$, of the gas, the density, $\rho$, of the gas and the wavelength, $\lambda$, of the sound in the relationship $u = kp^{\alpha}\rho^{\beta}\lambda^{\gamma}$, where $k$ is a dimensionless constant. (The wavelength of a sound is the distance between successive peaks in the sound wave.)

**(a)** Use the fact that density is mass per unit volume to find $[\rho]$. **[1]**

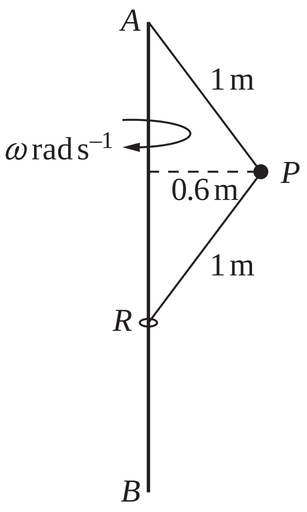
**(b)** Given that the units of $p$ are $\mathrm{N\,m^{-2}}$, determine the values of $\alpha$, $\beta$ and $\gamma$. **[7]**

**(c)** Comment on what the value of $\gamma$ means about how fast sounds of different wavelengths travel through the gas. **[1]**

The student carries out two experiments, $A$ and $B$, to measure $u$. Only the density of the gas varies between the experiments, all other conditions being unchanged. He finds that the value of $u$ in experiment $B$ is double the value in experiment $A$.

**(d)** By what factor has the density of the gas in experiment $A$ been multiplied to give the density of the gas in experiment $B$? **[2]**

**5**




As shown in the diagram, $AB$ is a long thin rod which is fixed vertically with $A$ above $B$. One end of a light inextensible string of length 1 m is attached to $A$ and the other end is attached to a particle $P$ of mass $m_1$ kg. One end of another light inextensible string of length 1 m is also attached to $P$. Its other end is attached to a small smooth ring $R$, of mass $m_2$ kg, which is free to move on $AB$.

Initially, $P$ moves in a horizontal circle of radius 0.6 m with constant angular velocity $\omega$ rad s$^{-1}$. The magnitude of the tension in string $AP$ is denoted by $T_1$ N while that in string $PR$ is denoted by $T_2$ N.

**(a)** By considering forces on $R$, express $T_2$ in terms of $m_2$. **[2]**

**(b)** Show that

**(i)** $T_1 = \frac{49}{4}(m_1 + m_2)$, **[2]**

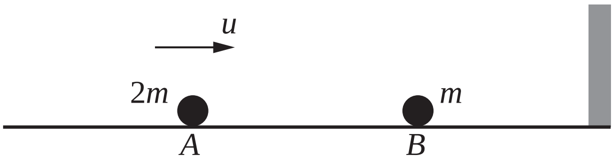
**(ii)** $\omega^2 = \frac{49(m_1 + 2m_2)}{4m_1}$. **[3]**

**(c)** Deduce that, in the case where $m_1$ is much bigger than $m_2$, $\omega \approx 3.5$. **[2]**

In a different case, where $m_1 = 2.5$ and $m_2 = 2.8$, $P$ slows down. Eventually the system comes to rest with $P$ and $R$ hanging in equilibrium.

**(d)** Find the total energy lost by $P$ and $R$ as the angular velocity of $P$ changes from the initial value of $\omega$ rad s$^{-1}$ to zero. **[5]**

**6** Particles *A* of mass $2m$ and *B* of mass $m$ are on a smooth horizontal floor. *A* is moving with speed $u$ directly towards a vertical wall, and *B* is at rest between *A* and the wall (see diagram).




*A* collides directly with *B*. The coefficient of restitution in this collision is $\frac{1}{2}$.

*B* then collides with the wall, rebounds, and collides with *A* for a second time.

**(a)** Show that the speed of *B* after its second collision with *A* is $\frac{1}{2}u$. **[6]**

The first collision between *A* and *B* occurs at a distance $d$ from the wall. The second collision between *A* and *B* occurs at a distance $\frac{1}{5}d$ from the wall.

**(b)** Find the coefficient of restitution for the collision between *B* and the wall. **[5]**

**END OF QUESTION PAPER**

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