Oxford Cambridge and RSA

GCE

Further Mathematics A

Y533/01: Mechanics

Advanced Subsidiary GCE

Mark Scheme for Autumn 2021

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations and abbreviations

Annotation in RM assessor	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	
Other abbreviations in mark scheme	**Meaning**
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

Question			Answer	Marks	AO	Guidance	
1	(a)		$\omega = 2\pi / 0.84$ soi awrt 7.48 rad s^{-1}	M1 A1 [2]	1.1 1.1	Correct formula for angular velocity used $\left(\frac{50}{21}\pi\right)$	
1	(b)		$v = 2.8\times$ "7.48…" or $2\pi\times2.8 / 0.84$ awrt 20.9 m s^{-1}	M1 A1 [2]	1.1 1.1	Correct formula for speed used $\left(\frac{20}{3}\pi\right)$	FT their value for ω if used
1	(c)		$a =$ "20.9⋯"^{2} / 2.8 or $2.8\times$ "7.48⋯"^{2} or "20.9⋯" × "7.48⋯" awrt 157 (or 156) ms^{-2}	M1 A1 [2]	1.1 1.1	Any correct formula for acceleration used 156 if rounded values used. $\left(\frac{1000}{63}\pi^2\right)$	FT their value for v if used
1	(d)		…towards O	B1 [1]	1.2	Any indication that the acceleration is towards the centre of the circle	

Question			Answer	Marks	AO	Guidance	
2	(a)		$D = 15000 / 20 = 750$ $D - R = 800 \times 0.4$ $R = 750 - 320 = 430$	**B1** **M1** **A1** **[3]**	3.4 3.3 1.1	"$P = Fv$" used in the solution Use of NII with a driving force (might be incorrectly derived from power), R and correct ma term. AG	
2	(b)		Need $15000 / v_{max}$ = "430" $v_{max} = 34.9$ so max speed is 34.9 ms^{-1} (3 sf)	**M1** **A1** **[2]**	3.4 1.1	Driving force = resistive force and "$P = Fv$"	
2	(c)		$D - R - 800g \times \sin\alpha = 800 \times 0.15$ $(= 15000 / v - 60v - 1568 = 120)$ $60v^2 + 1688v - 15000 = 0$ 7.10 or –35.2 Since $v > 0$, speed is 7.10 ms^{-1} (3 sf)	**M1** **M1** **A1** **A1FT** **[4]**	3.1b 3.1a 1.1 2.3	NII with a driving force, R, a component of weight (condone incorrect component) and correct ma term. Reduction to 3 term quadratic equation (must be equation) BC (condone 7.09 from incorrect rounding for this mark) FT their quadratic, if one positive and one negative root (ie if $ac < 0$) for selecting their positive root with valid reason given.	Both roots must be seen for this mark **SC1** if **A0A0** for 7.10 ms^{-1} with no justification

Question			Answer	Marks	AO	Guidance	
3	(a)		Cons of Momentum: $0.5 \times 3.15 = 0.5v_A + 0.8 \times 2v_A$ $v_A = 0.75$ So $v_B = 2v_A = 1.5$	**M1** **A1** **A1** **[3]**	1.1 1.1 1.1		Or $0.5 \times 3.15 = 0.5 \times \frac{1}{2}v_B + 0.8 \times v_B$ $v_B = 1.5$ $v_A = \frac{1}{2}v_B = 0.75$

3	(b)		$e = (\pm)\dfrac{\text{"1.5"} - \text{"0.75"}}{3.15 - 0}$	M1	1.1	Speed of separation over speed of approach. Using their values from 3(a) provided c.o.m. used (and in subsequent questions)	
			$\frac{5}{21}$ or awrt 0.238	A1 [2]	1.1		
3	(c)		Because e is the ratio of two speeds (in ms^{-1}) (the units cancel and so) it is a dimensionless quantity.	B1 [1]	2.4	oe	
3	(d)		Initial KE = ½ × 0.5 × 3.15^2	M1	1.1	$\frac{3969}{1600}$ = 2.48… Correct KE calc	Or change/gain of KE of B = 0.8 × "1.5"^{2}
			Final KE = ½ × 0.5 × "0.75"^{2} + ½ × 0.8 × "1.5"^{2}	M1	1.1	$\frac{333}{320}$ = 1.04… KE calculation with correct m and their u and $2u$	Change/loss of KE of A = ±½ × 0.5 × "0.75"^{2} ∓ ½ × 0.5 × 3.15^2
			KE Loss = 2.48… – 1.04... = 1.44 J	A1 [3]	1.1	FT their speeds if positive. $\frac{36}{25}$ = 1.44	2.34 – 0.9 = 1.44J NB Must be positive value for the amount lost
3	(e)		Not perfectly elastic since KE is lost oe	B1 [1]	2.4	eg $e \neq 1$ oe (but just e = 0.238… is insufficient)	
3	(f)		Change in B's momentum = 0.8 × "1.5"	M1	1.1	Using impulse = change in momentum (condone sign error)	Or by finding the change in A's momentum: 0.5 × 0.75 – 0.5 × 3.15
			(±)1.2 Ns or kgms^{-1}	A1	1.1	Impulse on B (Hence impulse B exerts on A is (±)1.2 Ns)	= (±)1.2 Ns
			in the opposite direction to A's original direction of motion	A1 [3]	1.1	This statement oe needed for full marks	in the opposite direction to A's original motion

Question			Answer	Marks	AO	Guidance	
4	(a)	(i)	Gain in KE = ½ × 4.2 × 4.5^2 (J) Work done by force = 35 × 2.4 (J) Energy lost = 84.0 – 42.5 = awrt 41.5 J	**M1** **M1** **A1** **[3]**	1.1 1.1 1.1	Correct formula for KE used. Can be implied by awrt 42.5 Correct formula for WD by force used. Can be implied by awrt 84.0	Do not allow the assumption that the resistance is constant, e.g. by use of suvat, also in part (ii) SC2 if using suvat to find correct average resistance and hence total energy lost.
4	(a)	(ii)	R = 41.5 / 2.4 So average resistive force is awrt 17.3 N	**M1** **A1** **[2]**	3.1b 1.1	Their energy loss divided by 2.4	SC1 only for 17.3N, if using suvat/N2L
4	(b)	(i)	Other resistive forces (eg air resistance) can be ignored.	**B1** **[1]**	3.3		"No friction" is not a valid answer here
4	(b)	(ii)	Need ½ × 4.2 × 4.5^2 = 4.2gh h = 1.033… Distance = 1.033 / sin20° = awrt 3.02 m	**M1** **A1** **A1**	2.2b 1.1 1.1	Equating KE with PE (4.2 may be missing on both sides).	If "resistive force" term included then **M0** unless recovered.
			Alternative method: $a = -g\sin20°$ $0^2 = 4.5^2 + 2\times -g\sin20°\times s$ Distance = awrt 3.02 m	 **M1** **M1** **A1**		Correctly deducing the acceleration up the slope. Using a suvat equation, or equations, which lead(s) to s from a and u given with v = 0 and consistent signs	
				[3]			

Question			Answer	Marks	AO	Guidance	
5	(a)		$[r]$ = L, $[m]$ = M and $[U]$ = LT^{-1}	B1	2.1	Correct dimensions for other parameters (U, r and m) soi (no need for them to be used for this mark to be awarded).	
			$[G] = \left[\frac{U^2 r}{m}\right]$	M1	1.1	Comparing dimensions, realising that 2 is dimensionless and rearranging	Could be done by dimensional analysis e.g. $[G] = L^{\alpha}M^{\beta}T^{\gamma}$ and equate indices using $U = \sqrt{\frac{2Gm}{r}}$ oe
			$\therefore [G] = (LT^{-1})^2LM^{-1} = L^3M^{-1}T^{-2}$	A1 **[3]**	2.2a	AG	
5	(b)		$[P] = (MLT^{-2}L)/T = ML^2T^{-3}$	B1	3.3	Using P = WD/t oe	
			Need $LT^{-1} = M^{\alpha}L^{2\alpha}T^{-3\alpha}M^{\beta}L^{\beta}T^{-2\beta}T^{\gamma}$	B1	3.3	Realising condition for equation to be dimensionally correct and substituting in dimensions.	ft errors in [P] and/or [W] here and in subsequent method marks provided M, L and T appear at least twice on the RHS
			M: $\alpha + \beta = 0$, L: $1 = 2\alpha + \beta$	M1	3.4	Comparing to obtain equations in α and β	
			$\alpha = 1$, $\beta = -1$	A1	1.1		
			T: $-1 = -3\alpha - 2\beta + \gamma$	M1	3.4	Comparing to obtain equation in γ	
			$\gamma = 0$	A1 **[6]**	1.1		
5	(c)		Because $\gamma = 0$, the modelled minimum launch speed V does not depend on the time t for which the engines operate…	B1ft **[1]**	3.5a	ie the modified model predicts that V does not vary when t varies	Or appropriate comment from their result, e.g. if $\gamma = -1$, then V is inversely proportional to t

Question			Answer	Marks	AO	Guidance	
6	(a)		$I = mu \Rightarrow u = I/m$	B1	3.1b	Use of Impulse = change of momentum	
			Init PE $= mgr - mgr\cos\frac{\pi}{3}$	M1	1.1	(= ½mgr). Attempt to use 'mgh' to find initial PE.	Could use eg edge as zero PE level (so init PE = –½mgr) but must be clear and signs consistent
			$\frac{1}{2}mu^2 + \frac{1}{2}mgr = \frac{1}{2}mv^2 + mgr$	M1	1.1	Conservation of energy; KE & PE considered on both sides	
			$v^2 = u^2 - gr \Rightarrow v = \sqrt{\frac{I^2}{m^2} - gr}$	A1	1.1	oe e.g. $v = \frac{\sqrt{I^2 - m^2gr}}{m}$	
			Alternative method				
			$u = I/m$	B1			
			$\Delta PE = mgr\cos\frac{\pi}{3} \quad (= \frac{1}{2}mgr)$	M1			
			$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - \frac{1}{2}mgr$	M1		Subtract gain in PE	
			$v^2 = u^2 - gr \Rightarrow v = \sqrt{\frac{I^2}{m^2} - gr}$	A1			
				[4]			
	(b)		$\frac{1}{2}mv^2 = mgh \Rightarrow h = \frac{1}{2g}\left(\frac{I^2}{m^2} - gr\right) = \frac{I^2}{2gm^2} - \frac{r}{2}$	B1 [1]	1.1	oe e.g. $h = \frac{I^2 - m^2gr}{2m^2g}$	
	(c)		Consider the case where $h \to 0$	M1	3.1b	e.g. $\frac{I^2}{m^2} = gr$	
			maximum possible value of m is $\frac{I}{\sqrt{gr}}$	A1 [2]	3.5b		
	(d)		Work done against $R = r\left(\frac{\pi}{2} + \frac{\pi}{3}\right)R$	M1	3.4		
			$\frac{1}{2}mu^2 + \frac{1}{2}mgr = \frac{1}{2}mv^2 + mgr + r\left(\frac{\pi}{2} + \frac{\pi}{3}\right)R$ or $\frac{I^2}{2m} + \frac{1}{2}mgr = \frac{1}{2}mv^2 + mgr + r\left(\frac{\pi}{2} + \frac{\pi}{3}\right)R$ Or $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - \frac{1}{2}mgr - r\left(\frac{\pi}{2} + \frac{\pi}{3}\right)R$	M1	3.4	Revising the energy equation (condone incorrect initial energy from **(a)**) to include an energy loss term (work done against R). Could already be in terms of I rather than u.	Could be expressed as an inequality at this stage: eg $\frac{1}{2}mu^2 + \frac{1}{2}mgr > mgr + r\left(\frac{\pi}{2} + \frac{\pi}{3}\right)R$
			Need $v > 0$ so $I > \sqrt{m^2gr + \frac{5\pi mrR}{3}}$	A1 [3]	1.1	**AG**	

	(e)		$[I]=MLT^{-1}$ And $[RHS]=(M^2LT^{-2}L+MLMLT^{-2})^{1/2}$ Hence $[RHS]=MLT^{-1}=[I]$ so the inequality is dimensionally consistent	**M1** **A1** **[2]**	1.1 2.2a	Attempt dimensional analysis on both sides.	

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