OCR
Oxford Cambridge and RSA

Thursday 15 October 2020 – Afternoon

AS Level Further Mathematics A

Y533/01 Mechanics

Time allowed: 1 hour 15 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for AS Level Further Mathematics A
- a scientific or graphical calculator

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INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

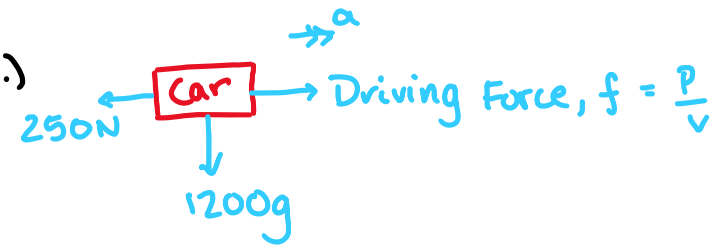
Answer **all** the questions.

1 A car of mass 1200 kg is driven on a long straight horizontal road. There is a constant force of 250 N resisting the motion of the car. The engine of the car is working at a constant power of 10 kW.

(a) The car can travel at constant speed $v\,\mathrm{m\,s^{-1}}$ along the road. Find v. **[2]**

(b) Find the acceleration of the car at an instant when its speed is $30\,\mathrm{m\,s^{-1}}$. **[3]**

(a.)



$P = fv$

$$f = \frac{P}{v} = \frac{10 \times 10^3}{v} = \frac{10000}{v}$$

As driving force is equal to resistive forces, $f = 250\,\mathrm{N}$:

$$\frac{10000}{v} = 250$$

$$\therefore v = \frac{1000\cancel{0}}{25\cancel{0}} = 40$$

$\boxed{\therefore v = 40\,\mathrm{ms^{-1}}}$

(b.) $F = ma$

$V = 30$

$f - 250 = 1200a$

$$\frac{10\,000}{30} - 250 = 1200a$$

$$\frac{250}{3} = 1200a$$

$$a = \frac{250}{3} \div 1200 = \frac{5}{72} \approx 0.069\,\mathrm{ms^{-2}}$$ (2sf)

$\boxed{\therefore a = 0.069\,\mathrm{ms^{-2}}}$

2 A particle P of mass 4.5 kg is moving in a straight line on a smooth horizontal surface at a speed of $2.4\,\text{m s}^{-1}$ when it strikes a vertical wall directly. It rebounds at a speed of $1.6\,\text{m s}^{-1}$.

(a) Find the coefficient of restitution between P and the wall. **[1]**

(b) Determine the impulse applied to P by the wall, stating its direction. **[3]**

(c) Find the loss of kinetic energy of P as a result of the collision. **[2]**

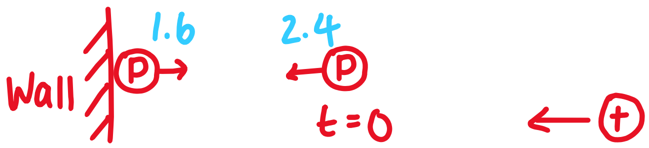
(d) State, with a reason, whether the collision is perfectly elastic. **[1]**

(a.) $v = eu$

$e = \frac{v}{u} = \frac{1.6}{2.4} = \frac{2}{3}$ $\quad \boxed{\therefore e = \frac{2}{3}}$

(b.) Impulse, $I = |m(v-u)|$

↳ This gives magnitude of impulse.



$I = 4.5(-1.6-2.4) = 4.5(-4) = -18$ Ns or kg ms^{-1}

$\boxed{\therefore I = 18 \text{ Ns in the final direction of motion of } P.}$

(c.) Loss of KE = KE start − KE end

$KE = \frac{1}{2}mv^2$

Loss of KE $= \frac{1}{2}(4.5)(2.4)^2 - \frac{1}{2}(4.5)(1.6)^2$

$= \frac{1}{2}(4.5)\left[2.4^2 - 1.6^2\right]$

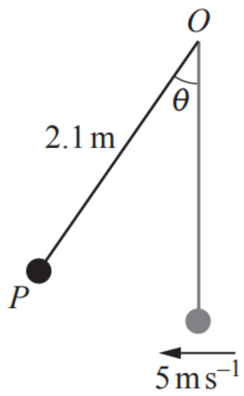
$= \frac{1}{2}(4.5)(3.2)$

$= 7.2$ J $\quad \boxed{\therefore \text{Loss of KE} = 7.2\text{ J}}$

(d.) As KE is lost, collision is not perfectly elastic.

3 A particle P of mass 5.6 kg is attached to one end of a light rod of length 2.1 m. The other end of the rod is freely hinged to a fixed point O.

The particle is initially at rest directly below O. It is then projected horizontally with speed $5\,\text{m s}^{-1}$. In the subsequent motion, the angle between the rod and the downward vertical at O is denoted by θ radians, as shown in the diagram.



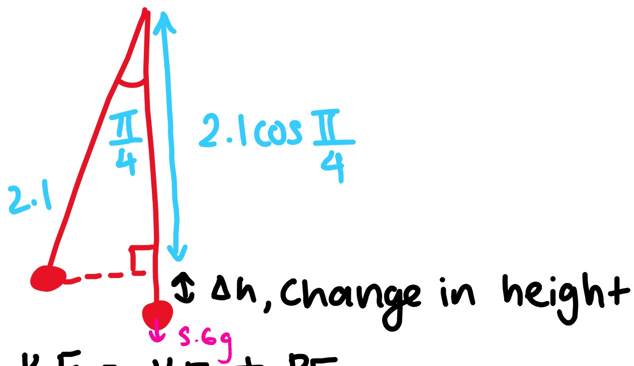
(a) Find the speed of P when $\theta = \frac{1}{4}\pi$. **[4]**

(b) Find the value of θ when P first comes to instantaneous rest. **[2]**

(a.) Conservation of Energy: Energy Start = Energy End

$KE = \frac{1}{2}mv^2 \rightarrow$ Initial Energy $= \frac{1}{2}(5.6)(5)^2 = 70$

$PE = mgh \rightarrow PE = 5.6g\left(2.1 - 2.1\cos\frac{\pi}{4}\right) = 33.755\ldots$



$\therefore KE = KE + PE$

$70 = \frac{1}{2}(5.6)v^2 + 33.755\ldots$

$36.24\ldots = 2.8v^2$

$v = \sqrt{\frac{36.24\ldots}{2.8}} = 3.597\ldots \approx 3.6\,\text{ms}^{-1}$ (2sf)

$\boxed{\therefore v = 3.6\,\text{ms}^{-1}}$

(b.) When $v = 0$, $PE = 70$.

$$5.6g(2.1 - 2.1\cos\theta) = 70$$

$$2.1 - 2.1\cos\theta = \frac{70}{5.6g}$$

$$2.1\cos\theta = 2.1 - \frac{70}{5.6g}$$

$$2.1\cos\theta = \frac{202}{245}$$

$$\cos\theta = \frac{202}{245} \div 2.1$$

$$\cos\theta = \frac{404}{1029}$$

$$\theta = \cos^{-1}\left(\frac{404}{1029}\right) = 1.1673\ldots \approx 1.17 \text{ rads (3sf)}$$

$$\boxed{\therefore \theta = 1.17 \text{ rads}}$$

4 A particle P of mass 2.4 kg is moving in a straight line OA on a horizontal plane. P is acted on by a force of magnitude 30 N in the direction of motion. The distance OA is 10 m.

(a) Find the work done by this force as P moves from O to A. **[2]**

The motion of P is resisted by a constant force of magnitude R N. The velocity of P increases from $12\,\text{m s}^{-1}$ at O to $18\,\text{m s}^{-1}$ at A.

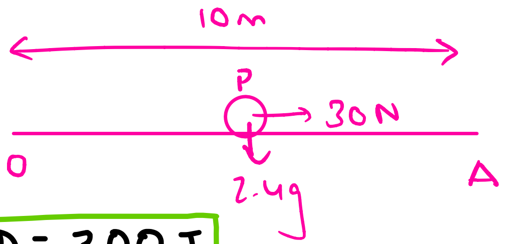
(b) Find the value of R. **[3]**

(c) Find the average power used in overcoming the resistance force on P as it moves from O to A. **[3]**

When P reaches A it collides directly with a particle Q of mass 1.6 kg which was at rest at A before the collision. The impulse exerted on Q by P as a result of the collision is 17.28 N s.

(d) (i) Find the speed of Q after the collision. **[2]**

(ii) Hence show that the collision is inelastic. **[2]**



(a.) Work Done = Fd

$WD = 30 \times 10 = 300\text{J}$ $\quad \boxed{\therefore WD = 300\text{J}}$

(b.) Energy Start + WD by Df = Energy End + WD by Resistance

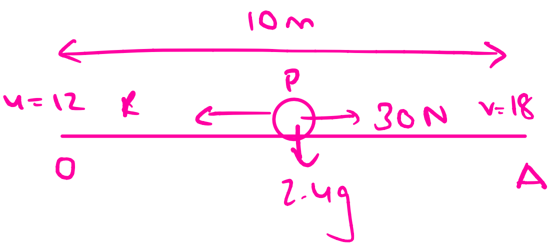
KE + WD = KE + WD by R

$\frac{1}{2}(2.4)(12)^2 + 300 = \frac{1}{2}(2.4)(18)^2 + 10R$

$172.8 + 300 = 388.8 + 10R$

$10R = 172.8 + 300 - 388.8 = 84$

$R = \frac{84}{10} = 8.4\text{ N}$ $\quad \boxed{\therefore R = 8.4\text{N}}$



(c.) $P = \frac{WD}{t}$

$S = 10$ $\quad s = \frac{1}{2}(u+v)t$

$U = 12$

$V = 18$ $\quad 10 = \frac{1}{2}(12+18)t \rightarrow t = \frac{10 \times 2}{12+18} = \frac{2}{3}$

$A = x$

$T = ?$

$P = \frac{10R}{\left(\frac{2}{3}\right)} = \frac{84}{\left(\frac{2}{3}\right)} = 126\ W$ $\qquad \boxed{\therefore P = 126\ W}$

(d.) (i) B: $\rightarrow 18$

P $\qquad$ Q

2.4 kg $\qquad$ 1.6 kg

A: $\rightarrow v_P$ $\qquad \rightarrow v_Q$

$I = m(v-u)$

$17.28 = 1.6(v_Q - 0)$

$v_Q = \frac{17.28}{1.6} = 10.8\ ms^{-1}$

$\boxed{\therefore v_Q = 10.8\ ms^{-1}}$

(ii) Conservation of Momentum : p Before = p After

$2.4(18) = 2.4v_P + 1.6(10.8)$

$v_P = \frac{2.4(18) - 1.6(10.8)}{2.4} = 10.8$ $\qquad \boxed{\therefore v_P = 10.8\ ms^{-1}}$

$v_P = v_Q$, so the particles coalese.

$\therefore$ Collision is inelastic.

5 A particle of mass m moves in a straight line with constant acceleration a. Its initial and final velocities are u and v respectively and its final displacement from its starting position is s. In order to model the motion of the particle it is suggested that the velocity is given by the equation

$$v^2 = pu^\alpha + qa^\beta s^\gamma$$

where p and q are dimensionless constants.

(a) Explain why α must equal 2 for the equation to be dimensionally consistent. **[2]**

(b) By using dimensional analysis, determine the values of β and γ. **[4]**

(c) By considering the case where $s = 0$, determine the value of p. **[1]**

(d) By multiplying both sides of the equation by $\frac{1}{2}m$, and using the numerical values of α, β and γ, determine the value of q. **[2]**

(a.) $v^2 = pu^\alpha$

$v = u$ & p is dimensionless therefore $\alpha = 2$.

(b.) $L^2T^{-2} = (LT^{-2})^\beta L^\gamma$

$-2 = -2\beta \rightarrow \therefore \beta = 1$

$\beta + \gamma = 2 \rightarrow \therefore \gamma = 2 - \beta = 2 - 1 = 1$

$\boxed{\therefore \beta = 1, \gamma = 1}$

(c.) If $s = 0$ then $v = u$, so $u^2 = pu^2 + 0$.

$\boxed{\therefore p = 1}$

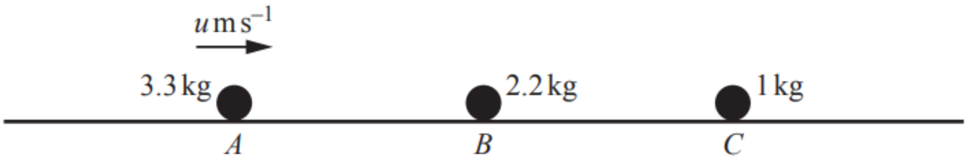
(d.) $\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \frac{1}{2}qmas = \frac{1}{2}qFs = \frac{1}{2}qW$

↳ This equation is a statement of work-energy principle.

so $\frac{1}{2}q = 1. \rightarrow \boxed{\therefore q = 2}$

6 Three particles A, B and C are free to move in the same straight line on a large smooth horizontal surface. Their masses are 3.3 kg, 2.2 kg and 1 kg respectively. The coefficient of restitution in collisions between any two of them is e.

Initially, B and C are at rest and A is moving towards B with speed $u\,\mathrm{m\,s^{-1}}$ (see diagram). A collides directly with B and B then goes on to collide directly with C.



(a) The velocities of A and B immediately after the first collision are denoted by $v_A\,\mathrm{m\,s^{-1}}$ and $v_B\,\mathrm{m\,s^{-1}}$ respectively.

- Show that $v_A = \dfrac{u(3-2e)}{5}$.
- Find an expression for v_B in terms of u and e. **[4]**

(b) Find an expression in terms of u and e for the velocity of B immediately after its collision with C. **[4]**

After the collision between B and C there is a further collision between A and B.

(c) Determine the range of possible values of e. **[4]**

(a.) B: $\rightarrow u$

A B

3.3kg 2.2kg

A: $\rightarrow v_A$ $\rightarrow v_B$

$p = mv$

$e(u_A - u_B) = v_B - v_A$

① $3.3u = 3.3v_A + 2.2v_B \xrightarrow{\times 10} 3u = 3v_A + 2\cancel{v}_B$

② $eu = -v_A + v_B \xrightarrow[\times 2]{} 2eu = -2v_A + 2\cancel{v}_B$

$3u - 2eu = 5v_A$

$\boxed{\therefore v_A = \dfrac{u(3-2e)}{5}}$

$v_B = eu + v_A$

$= eu + \dfrac{3u - 2eu}{5}$

$= \dfrac{5eu + 3u - 2eu}{5}$

$= \dfrac{3eu + 3u}{5}$

$= \dfrac{3u(1+e)}{5}$

$\boxed{\therefore v_B = \dfrac{3u(1+e)}{5}}$

(b.) B: $\rightarrow \frac{3u(1+e)}{5}$

B   C

2.2kg   1kg

A: $\rightarrow v_B$ $\rightarrow v_C$

① $2.2\left(\frac{3u(1+e)}{5}\right) = 2.2v_B + v_c \rightarrow \frac{33u(1+e)}{25} = 2.2v_B + \cancel{v_c}$

② $e\left(\frac{3u(1+e)}{5}\right) = -v_B + v_c \longrightarrow -\frac{3ue(1+e)}{5} = -v_B + \cancel{v_c}$

$\frac{u}{25}(e+1)(5e-11) = 3.2v_B$

$\boxed{\therefore v_B = \frac{3u(e+1)(-5e+11)}{80}}$

$\frac{33u(1+e)}{25} - \frac{3ue(1+e)}{5}$

$= \frac{33u}{25} + \frac{33ue}{25} - \frac{3ue}{5} - \frac{3ue^2}{5}$

$= u\left[\frac{33}{25} + \frac{18}{25}e - \frac{3e^2}{5}\right]$

$= \frac{u}{25}(33 + 18e - 15e^2)$

$= \frac{3u}{25}(e+1)(-5e+11)$

(c.) For another collision between A & B:

$\frac{\cancel{u}(3-2e)}{5} > \frac{3\cancel{u}(e+1)(-5e-11)}{\cancel{8}0}$

$\times 80$ $\times \cancel{80}$

$48 - 32e > 3(-5e^2 + 6e + 11)$

$\cancel{48} - 3\cancel{2e} > -15e^2 + 18e + 33$

$-\cancel{48}$ $+\cancel{32e}$ $+32e$ -48

$-15e^2 + 50e - 15 < 0 \rightarrow 3e^2 - 10e + 3 > 0$

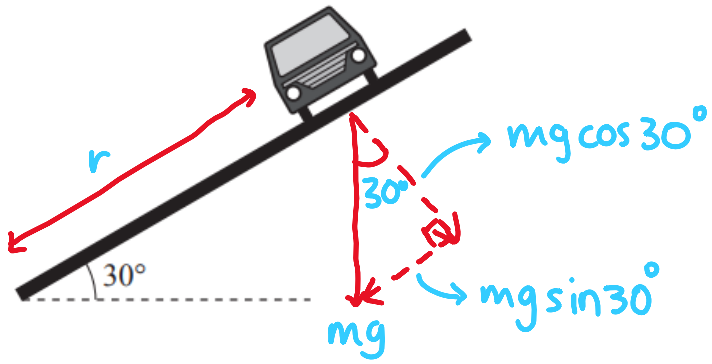
$\div -5$

$(3e-1)(e-3) > 0$

$e = \frac{1}{3}$ $e = 3$

$\boxed{\text{As } 0 < e \leq 1,\ 0 < e < \frac{1}{3}.}$

7 It is required to model the motion of a car of mass m kg travelling at a constant speed v m s^{-1} around a circular portion of banked track. The track is banked at 30° (see diagram).



In a model, the following modelling assumptions are made.

- The track is smooth.
- The car is a particle.
- The car follows a horizontal circular path with radius r m.

(a) Show that, according to the model, $\sqrt{3}v^2 = gr$. **[4]**

For a particular portion of banked track, $r = 24$.

(b) Find the value of v as predicted by the model. **[2]**

A car is being driven on this portion of the track at the constant speed calculated in part **(b)**. The driver finds that in fact he can drive a little slower or a little faster than this while still moving in the same horizontal circle.

(c) Explain

- how this contrasts with what the model predicts,
- how to improve the model to account for this. **[3]**

(a.) $F_c = \frac{mv^2}{r} \rightarrow C\sin 30° = \frac{mv^2}{r}$ ①

C is the normal contact force between the car & track.

$C\cos 30° = mg$ ②

$\frac{①}{②} = \frac{\cancel{C}\sin 30°}{\cancel{C}\cos 30°} = \frac{\cancel{m}v^2}{r} \div \cancel{m}g \rightarrow \tan 30° = \frac{v^2}{rg}$

$\frac{1}{\sqrt{3}} = \frac{v^2}{rg} \rightarrow rg = \sqrt{3}v^2$ $\boxed{\therefore \sqrt{3}v^2 = rg}$

(b.) $\sqrt{3}\, v^2 = 24g$

$$v = \sqrt{\frac{24 \times 9.8}{\sqrt{3}}} = 11.65\ldots \approx 11.7 \text{ ms}^{-1}$$

$$\boxed{\therefore v = 11.7 \text{ ms}^{-1}}$$

(c.) Model implies that only a single value for the speed is possible for a given radius, so any changes in speed could cause the car to move in a different circle.

To improve, the track should be modelled as resisting sideways motion → (ie. friction)