OCR
Oxford Cambridge and RSA

…day June 20XX – Morning/Afternoon

A Level Further Mathematics A

Y544 Discrete mathematics

SAMPLE MARK SCHEME

Duration: 1 hour 30 minutes

MAXIMUM MARK 75

This document consists of 16 pages

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

2. Subject-specific Marking Instructions for A Level Further Mathematics A

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

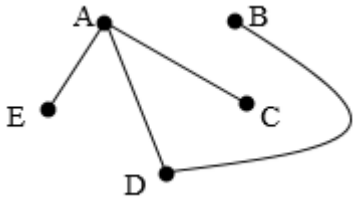
f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

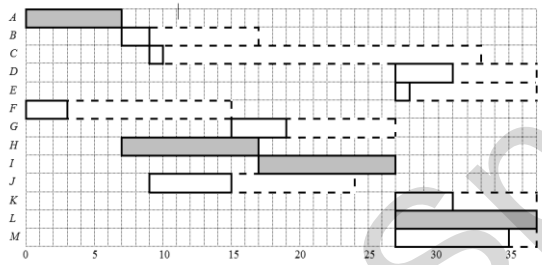
g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some papers. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1	(i)		Travelling salesperson problem	B1 [1]	1.2		
1	(ii)		<table><tr><td></td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td></tr><tr><td>A</td><td>-</td><td>12</td><td>8</td><td>6</td><td>4</td></tr><tr><td>B</td><td>12</td><td>-</td><td>10</td><td>(8)</td><td>10</td></tr><tr><td>C</td><td>(8)</td><td>10</td><td>-</td><td>13</td><td>10</td></tr><tr><td>D</td><td>(6)</td><td>8</td><td>13</td><td>-</td><td>10</td></tr><tr><td>E</td><td>(4)</td><td>10</td><td>10</td><td>10</td><td>-</td></tr></table>				
			Using Prim's algorithm starting at A	B1	1.1a	Stating that Prim has been used	(or Kruskal, if it was used)
			$AE = 4$ $AD = 6$ $AC = 8$ (or $DB = 8$)	M1	1.1	A valid order of building the tree for their starting point (arcs or vertices with arcs indicated on matrix)	
			$DB = 8$ (or $AC = 8$) Total weight $= 26$ km	A1	1.1	Valid method seen, leading to 26	
				B1 [4]	3.3	Correct (labelled) tree Weights need not be shown	
1	(iii)		Weight of MST + two least weight arcs to F $= 26 + 2(AF) + 5(EF)$ $= 33$ (km)	M1 A1FT [2]	3.4 1.1	 Their 26 (from **(ii)**) $+2+5$	
1	(iv)	(a)	F-A-E-B-D-C-F $= 2+4+10+8+13+9 = 46$ F-A-E-C-B-D-F $= 2+4+10+10+8+7 = 41$ F-A-E-D-B-C-F $= 2+4+10+8=10+9 = 43$	M1 M1 A1 [3]	3.4 1.1 1.1	At least one route F-A-E- At least one correct cycle All three correct cycles	

Question			Answer	Marks	AO	Guidance	
1	(iv)	(b)	F-A-E-B-D-C-F $= 46$ F-A-E-C-B-D-F $= 41$ F-A-E-D-B-C-F $= 43$ The length of Fiona's route is no more than 41(km)	**M1** **A1** **[2]**	**3.4** **2.2a**	Calculating the length of at least two of the paths given in part (a). 41 from cycle F-A-E-C-B-D-F	
1	(v)		E.g. The information so far says that the best route is between 26 and 41 km, but the only route that we have constructed has length 41, so we don't know.	**E1** **[1]**	**2.2b**	For an answer that makes clear the uncertainty due to the lower bound not being the length of a cycle that has been found.	
2	(i)		Critical activities: Survey, planning, extension, decoration  **(See page 15 for larger diagram)**	**M1** **A1** **B1** **M1** **A1** **M1** **A1** **[7]**	**2.2a** **1.1** **1.1** **3.3** **1.1** **1.1** **1.1**	At most one error Activities labelled, one activity per row Early start and finish times (solid boxes) correct for A, B, C,F, G, H All correct Float (dashed boxes) any two correct All correct	*A, H, I, L* and no others May have critical activities on a single row
2	(ii)		Latest start for $J = 23 - 6 = 17$ Float for $J = (23 - 9) - 6 = 8$ $17 - 15 < 6$, so float is all interfering	**B1** **M1** **E1** **[3]**	**1.1** **3.4** **1.1**	 No independent float	

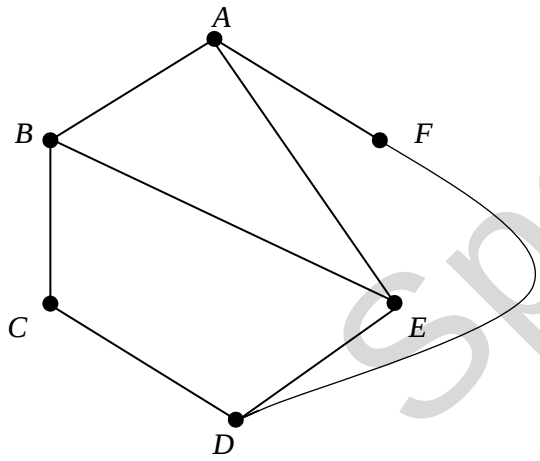
Question			Answer	Marks	AO	Guidance	
2	(iii)		When each activity starts at its earliest start time (as in the cascade chart) the only days with more than three activities are days 28 to 31	**M1**	**2.2a**	FT their cascade chart if possible Identifying problem (at day 28) (may be implied from resolution)	If their cascade chart has no day with more than three activities, and they say so ⇒ **M1A0**
			e.g. Delay repairing the roof (*E*) until after repairing the brickwork (*D*) has finished and then do the kitchens and bathrooms (*K*)	**M1**	**3.1b**	A schedule in the minimum completion time with no more than three activities each day…	
				A1	**3.4**	…in which precedence is not violated	
				[3]			

Question			Answer	Marks	AO	Guidance	
3	(i)		5! or ${}^5P_5 = 120$	B1 [1]	1.1		
3	(ii)		${}^5C_3 = 10$ ways to choose the three letters that will be put in the correct envelopes. The other two letters must be swapped over, so there is only one way to put them in the wrong envelopes. So there are $10 \times 1 = 10$ arrangements with exactly 3 letters in the correct envelopes.	M1 E1 [2]	1.1 1.1	Any valid attempt 10 from valid reasoning	A list of the ten possibilities $\Rightarrow$ **M1A0** unless supported with reasoning to show that there are no other possibilities
3	(iii)	(a)	The first symbol cannot be *A*. If it is *B* then it must be followed by *C* and then *A* (so that *C* is not last) and if it is *C* then it must be followed by *A* and then *B* (so that *B* is not second).	B1 [1]	1.1	Or list the six permutations and identify the two derangements (but not just writing down the two derangements with no explanation for why there are no others)	
3	(iii)	(b)	${}^5C_2 = 10$ ways to choose the two letters that will be put into the correct envelopes and 2 ways to arrange the other three letters so that none are in the correct envelopes. So there are $10 \times 2 = 20$ arrangements with exactly 2 letters in the correct envelopes.	E1 [1]	2.2a	20 from valid reasoning	

Question			Answer	Marks	AO	Guidance	
3	(iv)		There are $n-1$ choices for the first position. Then the symbol that was in the first position either goes in the position corresponding to the symbol that is now in the first position or not.	M1	3.1a	Or any equivalent partial argument	
			In the first case we are left with $n-2$ symbols with the corresponding $n-2$ positions, which can be done $\mathrm{D}(n-2)$ ways. In the second case we can rename the first symbol with the name of the second symbol and then we have $n-1$ symbols with $n-1$ corresponding positions, which can be done $\mathrm{D}(n-1)$ ways. Hence the result given.	E1 [2]	2.1	Achieving the given result $\mathrm{D}(n)=(n-1)\times(\mathrm{D}(n-1)+\mathrm{D}(n-2))$	
3	(v)		$\mathrm{D}(4)=3(2+1)=9$ $\mathrm{D}(5)=4(9+2)=44$ There are 44 ways in which all five letters can be in the wrong envelopes	M1 A1 [2]	3.1a 3.2a	**OR** 1 way with all five correct, 10 ways with three correct, 20 ways with two correct and $5\times 9=45$ ways with one correct $120-(45+20+10+1)=120-76=44$	**OR** **M1** use inclusion-exclusion: $5!-\frac{5!}{1!}+\frac{5!}{2!}-\frac{5!}{3!}+\frac{5!}{4!}-\frac{5!}{5!}$ $=120-120+60-20+5-1$ **A1** $=44$

Question			Answer	Marks	AO	Guidance	
4	(i)		<table><tr><th></th><th>X</th><th>Y</th><th>Z</th><th>row min</th><th>maxi min</th></tr><tr><td>P</td><td>4</td><td>5</td><td>–4</td><td>–4</td><td></td></tr><tr><td>Q</td><td>3</td><td>–1</td><td>2</td><td>–1</td><td></td></tr><tr><td>R</td><td>4</td><td>0</td><td>2</td><td>0</td><td>*</td></tr><tr><td>col max</td><td>4</td><td>5</td><td>2</td><td></td><td></td></tr><tr><td>minimax</td><td></td><td></td><td>*</td><td></td><td></td></tr></table>				
			Play-safe for player *A* is strategy *R*	**M1**	**1.1**		
			Play-safe for player *B* is strategy *Z*	**M1**	**1.1**		
			Unstable, since $0 \neq 2$	**A1** **[3]**	**2.2a**	oe	Play-safe values not equal
4	(ii)		If player *A* uses strategy *R*, player *B* would get the best pay-off by changing from *Z* to *Y*	**B1** **[1]**	**2.2a**		
4	(iii)		Increasing entry in (R, Z) has no effect on row min and increases col max. Row maximin is still 0 and col minimax is greater than 2, they are not equal and the game is still unstable.	**E1**	**2.4**	Describing what happens if entry is increased	
			Decreasing entry has no effect on col max, as $(Q, Z) = 2$, and may or may not decrease row min. Row maximin is either still 0 or is less than 0 and col minimax is still 2, they are not equal and the game is still unstable.	**E1** **[2]**	**2.4**	Describing what happens if entry is decreased	
4	(iv)		e.g. row *R* column *Y* is increased to 2 (or more)	**M1** **A1** **[2]**	**2.1** **2.3**	Identifying a suitable cell Giving a valid new value	

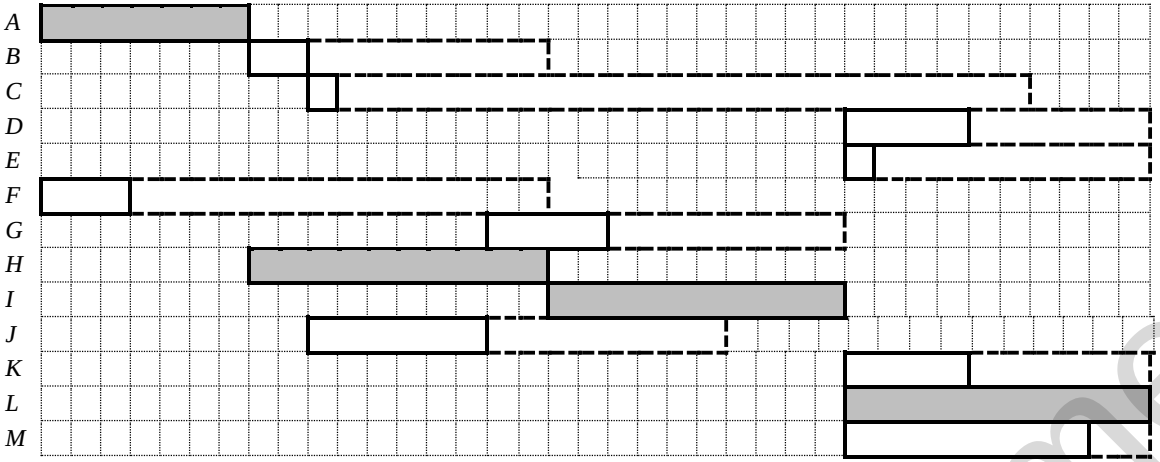
Question			Answer	Marks	AO	Guidance	
4	(v)		Player *A*: (*P*, *Y*), (*Q*, *X*), (*R*, *X*) Player *B*: (Q, *X*), (*R*, *Y*), (*P*, *Z*)	M1	1.1	Identifying cells where row maxima occur	
			When player *A* chooses row *Q* and player *B* choses column *X* it is a Nash equilibrium	A1	2.5	(*Q*, *X*) is a Nash equilibrium	
			Neither player would want to change, unless the other changed as well	E1	2.4	Explaining Nash equilibrium in context	
				[3]			
5	(i)		The number of red tulips is $25x+40y+20z$ and the number of white tulips is $25x+30y+30z$	B1	3.3	A correct expression for the number of red tulips	May be seen as an explicit part of an inequality
			$\Rightarrow 25x+40y+20z \le 50+(25x+30y+30z)$	M1	1.1	A correct inequality involving 50	May scale first
			$\Rightarrow 10y-10z \le 50 \Rightarrow y-z \le 5$	A1	1.1	Manipulating inequality and adding slack variable to get row 2	
			Add slack variable *s* to get row 2: $y-z+s=5$	[3]			
5	(ii)		Column *x* row 4	B1	1.1	Describing this cell	50 in row 4 or 50 in column *x*
				[1]			
5	(iii)		Entry for row 2 in column *x* (pivot column) was 0	E1	1.1	Value in pivot column was 0	
				[1]			
5	(iv)	(a)	$x=4.8,\ y=0,\ z=0$	B1	1.1	$x=4.8$, *y* and *z* both 0	
				[1]			
5	(iv)	(b)	Not optimal (for continuous problem) since still a negative in top row (column *y*)	B1	3.4	Optimal solution not yet achieved	Recognising that further iterations are needed
			Practical problem requires integer values, so practical solution is to buy 4 of pack A only (100 tulips of each colour)	B1	3.5a	Need integer values so 4 of pack A	Interpretation in context (packs or tulips)
				[2]			

Question			Answer	Marks	AO	Guidance	
5	(v)		u becomes a non-basic variable $x = 4.8 - 0.96y - 1.06z - 0.02u$ $P - x - y - z = 0$ $\Rightarrow P - (4.8 - 0.96y - 1.06z - 0.02u) - y - z = 0$ $\Rightarrow P - 0.04y + 0.06z + 0.02u = 4.8$	**B1** **B1** **B1** **[3]**	**1.2** **1.1** **1.1**	cao Working leading to correct expression corresponding to row 1 of iterated tableau	
5	(vi)		$\Rightarrow P - 0.04y + 0.06z + 0.02u = 4.8$ When y, z and $u = 0$, $P = 4.8$ $x = 4.8 - 0.96y - 1.06z - 0.02u$ When y, z and $u = 0$, $x = 4.8$	**B1** **B1** **[2]**	**2.2a** **2.2a**		Must refer to equations not just find value(s) or use tableau to read off value(s)
6	(i)			**B1** **[1]**	**1.1**	Arcs may be drawn crossing	

Question			Answer	Marks	AO	Guidance	
6	(ii)		$V + R = E + 2 \Rightarrow 6 + R = 8 + 2 \Rightarrow R = 4$	**M1**	**1.1a**	Showing substitution of $V = 6$ and $E = 8$ to get $R = 4$	
			$\{A, B, E\}$, $\{B, C, D, E\}$, $\{A, D, E, F\}$	**M1**	**1.1**	Any three of these sets	
			$\{A, B, C, D, F\}$	**A1**	**1.1**	All four sets and no others	
				[3]			
6	(iii)		If *AB* is part of a Hamiltonian cycle then (to include *C*) must have *A* – *B* – *C* – *D* but then either have *D* – *E* – *A* or *D* – *F* – *A*, but cannot visit both *E* and *F*	**M1**	**1.1**	Partially correct attempt at reasoning	Assuming that *AB* is included and reaching a contradiction
				E1	**2.4**	Explaining that if *AB* is used the cycle can visit at most two of *C*, *E* and *F*	
			Ignoring arc *AB* means that part of the cycle must be *E* – *B* – *C* – *D*	**M1**	**3.1a**	Identifying cycle *E* – *B* – *C* – *D* – *F* – *A* – *E*	
			To include *A* and *F*, cycle must continue *D* – *F* – *A* – *E*. So there is only one Hamiltonian cycle.	**E1**	**3.2a**	Showing that there are no other Hamiltonian cycles	
				[4]			
6	(iv)		STEP 1: E = colour 1 STEP 2: *A*, *B* and D = colour 2 STEP 3: *C* and F = colour 1 STEP 2: Stop	**B1**	**1.1**	*C*, *E* and *F* are colour 1 *A*, *B* and *D* are colour 2	Need not show STEP numbers Could draw solution, but if so it must be in the answer space for this part
			The two colours correspond to the two sets in a bipartite graph, but *A* and *B* are both colour 2 (so in the same set) yet they are connected by the arc *AB*	**E1FT**	**2.2a**	Arc *AB* has both ends the same colour	*A* and *B* are the same colour and are connected by an arc **SC1** If algorithm not followed, but valid argument given e.g. that *ABE* is an odd length cycle
				[2]			

Question			Answer	Marks	AO	Guidance	
6	(v)		Remove arc *AB* to give a bipartite graph with sets $\{A, B, D\}$ and $\{C, E, F\}$	**B1** **B1** **[2]**	**1.1** **1.1**	*AB* $\{A, B, D\}$ and $\{C, E, F\}$	
6	(vi)		Graph *G* is planar so arcs of graph *G* can be drawn on one plane surface with no arcs crossing, then new arcs can be drawn on a second plane surface. Hence thickness is at most 2.	**E1**	**3.1a**	Showing that new graph can be drawn using 2 layers	
			Kuratowski's theorem says that if a graph includes a subdivision of either $K_{3,3}$ or K_5 as a subgraph then it is not planar.	**B1**	**1.2**	Knowing what Kuratowski's theorem says and that the new graph does not contain a subgraph which is a subdivision of K_5 or $K_{3,3}$	
			The new graph does not contain either of these as subgraphs, however: If arc *AF* is contracted then the new graph contains $K_{3,3} = \{X, C, E\}, \{(AF), B, D\}$ as a subgraph (or similarly using arc *FD*)	**M1**	**2.1**	Attempting to create either K_5 or $K_{3,3}$ as a subset by combining arcs	**OR M1** Contract arc *BC* (or arc *CD*) and arc *AF* (or arc *FD*) so graph *G* becomes K_4 and adding the new arcs gives $K_5 = \{X, (AF), (BC), D, E\}$ as a subgraph (or similarly).
			Hence, new graph is not planar so thickness $\neq 1$ Hence thickness is at least 2 Therefore the thickness must be 2	**E1** **[4]**	**3.2a**	Deducing that new graph is non-planar and hence thickness is not 1	

Enlarged diagram for Q2(ii)



	Earliest start	Earliest finish	Latest finish
A	0	7	
B	7	9	17
C	9	10	33
D	27	31	37
E	27	28	37
F	0	3	17
G	15	19	27
H	7	17	
I	17	27	
J	9	15	23
K	27	31	37
L	27	37	
M	27	35	37

Assessment Objectives (AO) Grid

Question	AO1	AO2	AO3(PS)	AO3(M)	Total
1(i)	1	0	0	0	1
1(ii)	3	0	0	1	4
1(iii)	1	0	0	1	2
1(iv)(a)	2	0	0	1	3
1(iv)(b)	0	1	0	1	2
1(v)	0	1	0	0	1
2(i)	5	1	0	1	7
2(ii)	2	0	0	1	3
2(iii)	0	1	1	1	3
3(i)	1	0	0	0	1
3(ii)	2	0	0	0	2
3(iii)(a)	1	0	0	0	1
3(iii)(b)	0	1	0	0	1
3(iv)	0	1	1	0	2
3(v)	0	0	2	0	2
4(i)	2	1	0	0	3
4(ii)	0	1	0	0	1
4(iii)	0	2	0	0	2
4(iv)	0	2	0	0	2
4(v)	1	2	0	0	3
5(i)	2	0	0	1	3
5(ii)	1	0	0	0	1
5(iii)	1	0	0	0	1
5(iv)(a)	1	0	0	0	1
5(iv)(b)	0	0	0	2	2
5(v)	3	0	0	0	3
5(vi)	0	2	0	0	2
6(i)	1	0	0	0	1
6(ii)	3	0	0	0	3
6(iii)	1	1	2	0	4
6(iv)	1	1	0	0	2
6(v)	2	0	0	0	2
6(vi)	1	1	2	0	4
Totals	38	19	8	10	75

PS = Problem Solving
M = Modelling