OCR
Oxford Cambridge and RSA

# Thursday 16 May 2019 – Afternoon

## AS Level Further Mathematics A

**Y534/01** Discrete Mathematics

**Time allowed: 1 hour 15 minutes**

*7771494489*

**You must have:**
- Printed Answer Booklet
- Formulae AS Further Mathematics A

**You may use:**
- a scientific or graphical calculator

**INSTRUCTIONS**
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

**INFORMATION**
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.


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Answer **all** the questions.

**1** Alfie has a set of 15 cards numbered consecutively from 1 to 15.
He chooses two of the cards.

**(a)** How many different sets of two cards are possible? **[2]**

Alfie places the two cards side by side to form a number with 2, 3 or 4 digits.

**(b)** Explain why there are fewer than ${}^{15}P_2 = 210$ possible numbers that can be made. **[1]**

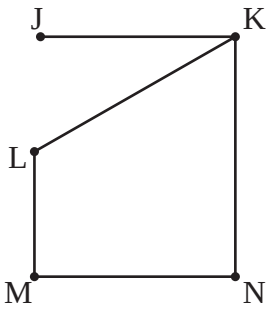
**(c)** Explain why, with these cards, 1 is the lead digit more often than any other digit. **[1]**

Alfie makes the number 113, which is a 3-digit prime number.

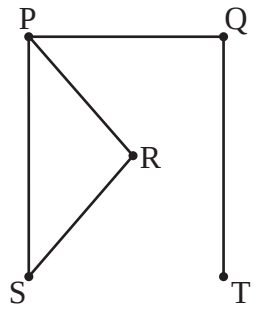
Alfie says that the problem of working out how many 3-digit prime numbers can be made using two of the cards is a construction problem, because he is trying to find all of them.

**(d)** Explain why Alfie is wrong to say this is a construction problem. **[1]**

**2** Two graphs are shown below.




Graph G1




Graph G2

**(a)** List the vertex degrees for each graph. **[3]**

**(b)** Prove that the graphs are non-isomorphic. **[2]**

The two graphs are joined together by adding an arc connecting J and T.

**(c)** **(i)** Explain how you know that the resulting graph is not Eulerian. **[1]**

**(ii)** Describe how the graph can be made Eulerian by adding one more arc. **[1]**

The vertices of the graph $K_3$ are connected to the vertices of the graph $K_4$ to form the graph $K_7$.

**(d)** Explain why 12 arcs are needed connecting $K_3$ to $K_4$. **[3]**

**3** **(a)** Give an example of a standard sorting algorithm that can be used when some of the values are not known until after the sorting has been started. **[1]**

Becky needs to sort a list of numbers into increasing order.
She uses the following algorithm:

STEP 1: Let *L* be the first value in the input list.
Write this as the first value in the output list and delete it from the input list.

STEP 2: If the input list is empty go to STEP 7.
Otherwise let *N* be the new first value in the input list and delete this value from the input list.

STEP 3: Compare *N* with *L*.

STEP 4: If *N* is less than or equal to *L*

- write the value of *N* immediately before *L* in the output list,
- replace *L* with the first value in the new output list,
- then go to STEP 2.

STEP 5: If *N* is greater than *L*

- if *L* is the value of the last number in the output list, go to STEP 6;
- otherwise, replace *L* with the next value in the output list and then go to STEP 3.

STEP 6: Write the value of *N* immediately after *L* in the output list.
Let *L* be the first value in the new output list and then go to STEP 2.

STEP 7: Print the output list and STOP.

**(b)** Trace through Becky's algorithm when the input list is

6 9 5 7 6 4

Complete the table in the Printed Answer Booklet, starting a new row each time that STEP 3 or STEP 7 is used.
You should not need all the lines in the Answer Booklet. **[5]**

Becky measures the efficiency of her sort by counting using the number of times that STEP 3 is used.

**(c)** **(i)** How many times did Becky use STEP 3 in sorting the list from part **(b)**? **[1]**

**(ii)** What is the greatest number of times that STEP 3 could be used in sorting a list of 6 values? **[1]**

A computer takes 15 seconds to sort a list of 60 numbers using Becky's algorithm.

**(d)** Approximately how long would you expect it to take the computer to sort a list of 300 numbers using the algorithm? **[3]**

**4** The table shows the activities involved in a project, their durations in hours and their immediate predecessors. The activities can be represented as an activity network.

| Activity | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| Duration | 2 | 4 | 5 | 4 | 3 | 3 | 2 | 4 |
| Immediate predecessors | - | A | - | A, C | B, C | B, D | D, E | F, G |

**(a)** Use standard algorithms to find the activities that form

- the longest path(s)
- the shortest path(s)

through the activity network.

You must show working to demonstrate the use of the algorithms. **[8]**

Only one of the paths from part **(a)** has a practical interpretation.

**(b)** What is the practical interpretation of the total weight of that path? **[1]**

The duration of activity E can be changed. No other durations change.

**(c)** What is the smallest increase to the duration of E that will make activity E become part of a longest path through the network? **[1]**

**5** Corey is training for a race that starts in 18 hours time. He splits his training between gym work, running and swimming.

- At most 8 hours can be spent on gym work.
- At least 4 hours must be spent running.
- The total time spent on gym work and swimming must not exceed the time spent running.

Corey thinks that time spent on gym work is worth 3 times the same time spent running or 2 times the same time spent swimming. Corey wants to maximise the worth of the training using this model.

**(a)** Formulate a linear programming problem to represent Corey's problem.
Your formulation must include defining the variables that you are using. **[5]**

Suppose that Corey spends the maximum of 8 hours on gym work.

**(b)** **(i)** Use a graphical method to determine how long Corey should spend running and how long he should spend swimming. **[5]**

**(ii)** Describe why this solution is not practical. **[1]**

**(iii)** Describe how Corey could refine the LP model to make the solution more realistic. **[1]**

**6** Drew and Emma play a game in which they each choose a strategy and then use the tables below to determine the pay-off that each receives.

Drew's pay-off

| | | Emma | | |
|---|---|---|---|---|
| | | X | Y | Z |
| Drew | P | 3 | 14 | 11 |
| | Q | 12 | 4 | 7 |
| | R | 11 | 4 | 6 |

Emma's pay-off

| | | Emma | | |
|---|---|---|---|---|
| | | X | Y | Z |
| Drew | P | 13 | 2 | 5 |
| | Q | 4 | 12 | 9 |
| | R | 5 | 12 | 10 |

**(a)** Convert the game into a zero-sum game, giving the pay-off matrix for Drew. **[2]**

**(b)** Determine the optimal mixed strategy for Drew. **[5]**

**(c)** Determine the optimal mixed strategy for Emma. **[5]**

**END OF QUESTION PAPER**

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