



Oxford Cambridge and RSA

Friday 22 October 2021 – Afternoon

A Level Further Mathematics A

Y545/01 Additional Pure Mathematics

Time allowed: 1 hour 30 minutes



You must have:

- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep in the centre or recycle it.

INFORMATION

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets [].
- This document has **4** pages.

ADVICE

- Read each question carefully before you start your answer.

Answer **all** the questions.

1 In this question you must show detailed reasoning.

Express the number 41723_{10} in hexadecimal (base 16). [3]

2 The following Cayley table is for G , a group of order 6. The identity element is e and the group is generated by the elements a and b .

G	e	a	a^2	b	ab	a^2b
e	e	a	a^2	b	ab	a^2b
a	a	a^2	e	ab	a^2b	b
a^2	a^2	e	a	a^2b	b	ab
b	b	a^2b	ab	e	a^2	a
ab	ab	b	a^2b	a	e	a^2
a^2b	a^2b	ab	b	a^2	a	e

(a) List all the proper subgroups of G . [4]

(b) State another group of order 6 to which G is isomorphic. [1]

3 The points P , Q and R have position vectors $\mathbf{p} = 2\mathbf{i} + \mathbf{j} + 5\mathbf{k}$, $\mathbf{q} = \mathbf{i} - \mathbf{j} + \mathbf{k}$ and $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + t\mathbf{k}$ respectively, relative to the origin O .

Determine the value(s) of t in each of the following cases.

(a) The line OR is parallel to $\mathbf{p} \times \mathbf{q}$. [2]

(b) The volume of tetrahedron $OPQR$ is 13. [4]

4 Solve the simultaneous linear congruences $x \equiv 1 \pmod{3}$, $x \equiv 5 \pmod{11}$, $2x \equiv 5 \pmod{17}$. [6]

5 The surface S has equation $x^2 + y^2 + z^2 = xyz - 1$.

(a) Show that $(2z - xy)\left(x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}\right) = 2(1 + z^2)$. [6]

(b) Deduce that S has no stationary point. [2]

6 The binary operation \diamond is defined on the set \mathbb{C} of complex numbers by

$$(a + ib) \diamond (c + id) = ac + i(b + ad)$$

where a, b, c and d are real numbers.

(a) Is \mathbb{C} closed under \diamond ? Justify your answer. [1]

(b) Prove that \diamond is associative on \mathbb{C} . [4]

(c) Determine the identity element of \mathbb{C} under \diamond . [2]

(d) Determine the largest subset S of \mathbb{C} such that (S, \diamond) is a group. [3]

7 Let $I_n = \int_0^{\frac{1}{2}\pi} \cos^n x \, dx$ for integers $n \geq 0$.

(a) Show that, for $n \geq 2$, $nI_n = (n-1)I_{n-2}$. [4]

(b) Use this reduction formula to deduce the exact value of I_8 . [2]

(c) Use the results of parts (a) and (b) to determine the exact value of $\int_0^{\frac{1}{2}\pi} \cos^6 x \sin^2 x \, dx$. [2]

8 (a) Solve the second-order recurrence system $H_{n+2} = 5H_{n+1} - 4H_n$ with $H_0 = 3$, $H_1 = 7$ for $n \geq 0$. [5]

(b) (i) Write down the quadratic residues modulo 10. [1]

(ii) By considering the sequence $\{H_n\}$ modulo 10, prove that H_n is never a perfect square. [6]

9 For each value of k the sequence of real numbers $\{u_n\}$ is given by $u_1 = 2$ and $u_{n+1} = \frac{k}{6+u_n}$.

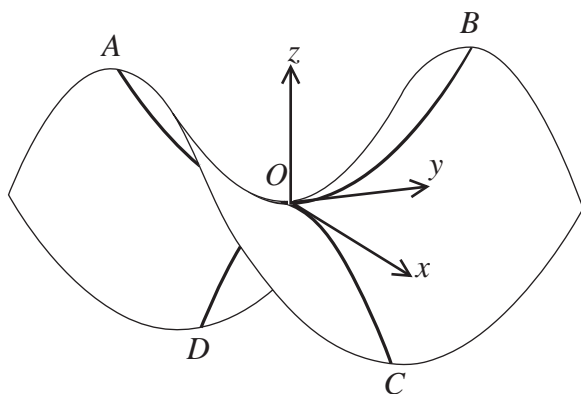
For each of the following cases, either determine a value of k or prove that one does not exist.

(a) $\{u_n\}$ is constant. [2]

(b) $\{u_n\}$ is periodic, with period 2. [3]

(c) $\{u_n\}$ is periodic, with period 4. [5]

10



A student wishes to model the saddle of a horse. They use a surface described by a function of the form $z = f(x, y)$ with a saddle point at the origin O . The z -axis is vertically upwards. The x - and y -axes lie in a horizontal plane, with the x -axis across the horse and the y -axis along the length of the horse (see diagram).

The arc AOB is part of a parabola which lies in the yz -plane. The arc COD is part of a parabola which lies in the xz -plane. The saddle is symmetric in both the xz -plane and yz -plane.

The length of the saddle, the distance AB , is to be 0.6 m with both A and B at a height of 0.27 m above O . The width of the saddle, the distance CD , is to be 0.5 m with both C and D at a depth of 0.4 m below O .

- (a) On separate diagrams, sketch the sections $x = 0$ and $y = 0$. [2]
- (b) Determine a function f that describes the saddle. [You do not need to state the domain of function f .] [5]

END OF QUESTION PAPER

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