



Oxford Cambridge and RSA

**Monday 24 June 2019 – Morning**

**A Level Further Mathematics A**

**Y545/01** Additional Pure Mathematics

**Time allowed: 1 hour 30 minutes**



**You must have:**

- Printed Answer Booklet
- Formulae A Level Further Mathematics A

**You may use:**

- a scientific or graphical calculator

**INSTRUCTIONS**

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

**INFORMATION**

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets [ ].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

- 1 The sequence  $\{u_n\}$  is defined by  $u_0 = 2$ ,  $u_1 = 5$  and  $u_n = \frac{1 + u_{n-1}}{u_{n-2}}$  for  $n \geq 2$ .  
Prove that the sequence is periodic with period 5. [4]
- 2 A surface has equation  $z = f(x, y)$  where  $f(x, y) = x^2 \sin y + 2y \cos x$ .
- (a) Determine  $f_x$ ,  $f_y$ ,  $f_{xx}$ ,  $f_{yy}$ ,  $f_{xy}$  and  $f_{yx}$ . [5]
- (b) (i) Verify that  $z$  has a stationary point at  $(\frac{1}{2}\pi, \frac{1}{2}\pi, \frac{1}{4}\pi^2)$ . [3]  
(ii) Determine the nature of this stationary point. [3]
- 3 (a) Solve  $7x \equiv 6 \pmod{19}$ . [2]
- (b) Show that the following simultaneous linear congruences have no solution.  
 $x \equiv 3 \pmod{4}$ ,  $x \equiv 4 \pmod{6}$ . [2]
- 4 (a) Solve the second-order recurrence relation  $T_{n+2} + 2T_n = -87$  given that  $T_0 = -27$  and  $T_1 = 27$ . [8]
- (b) Determine the value of  $T_{20}$ . [2]

- 5 The group  $G$  consists of a set  $S$  together with  $\times_{80}$ , the operation of multiplication modulo 80. It is given that  $S$  is the smallest set which contains the element 11.

(a) By constructing the Cayley table for  $G$ , determine all the elements of  $S$ . [5]

The Cayley table for a second group,  $H$ , also with the operation  $\times_{80}$ , is shown below.

$\times_{80}$	1	9	31	39
1	1	9	31	39
9	9	1	39	31
31	31	39	1	9
39	39	31	9	1

(b) Use the two Cayley tables to explain why  $G$  and  $H$  are not isomorphic. [2]

(c) (i) List

- all the proper subgroups of  $G$ ,
- all the proper subgroups of  $H$ . [3]

(ii) Use your answers to (c) (i) to give another reason why  $G$  and  $H$  are not isomorphic. [1]

- 6 (a) For the vectors  $\mathbf{p} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ ,  $\mathbf{q} = \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$  and  $\mathbf{r} = \begin{pmatrix} 2 \\ -4 \\ 5 \end{pmatrix}$ , calculate

- $\mathbf{p} \cdot \mathbf{q} \times \mathbf{r}$ ,
- $\mathbf{p} \times (\mathbf{q} \times \mathbf{r})$ ,
- $(\mathbf{p} \times \mathbf{q}) \times \mathbf{r}$ . [6]

(b) State whether the vector product is associative for three-dimensional column vectors with real components. Justify your answer. [1]

It is given that  $\mathbf{a}$ ,  $\mathbf{b}$  and  $\mathbf{c}$  are three-dimensional column vectors with real components.

(c) Explain geometrically why the vector  $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$  must be expressible in the form  $\lambda\mathbf{b} + \mu\mathbf{c}$ , where  $\lambda$  and  $\mu$  are scalar constants. [2]

It is given that the following relationship holds for  $\mathbf{a}$ ,  $\mathbf{b}$  and  $\mathbf{c}$ .

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c}) \mathbf{b} - (\mathbf{a} \cdot \mathbf{b}) \mathbf{c} \quad (*)$$

(d) Find an expression for  $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}$  in the form of (\*). [3]

- 7 The points  $P\left(\frac{1}{2}, \frac{13}{24}\right)$  and  $Q\left(\frac{3}{2}, \frac{31}{24}\right)$  lie on the curve  $y = \frac{1}{3}x^3 + \frac{1}{4x}$ .

The area of the surface generated when arc  $PQ$  is rotated completely about the  $x$ -axis is denoted by  $A$ .

- (a) Find the exact value of  $A$ . Give your answer as a rational multiple of  $\pi$ . [4]

Student X finds an approximation to  $A$  by modelling the arc  $PQ$  as the straight line segment  $PQ$ , then rotating this line segment completely about the  $x$ -axis to form a surface.

- (b) Find the approximation to  $A$  obtained by student X. Give your answer as a rational multiple of  $\pi$ . [4]

Student Y finds a second approximation to  $A$  by modelling the original curve as the line  $y = M$ , where  $M$  is the mean value of the function  $f(x) = \frac{1}{3}x^3 + \frac{1}{4x}$ , then rotating this line completely about the  $x$ -axis to form a surface.

- (c) Find the approximation to  $A$  obtained by student Y. Give your answer correct to four decimal places. [4]

**8 In this question you must show detailed reasoning.**

- (a) Prove that  $2(p-2)^{p-2} \equiv -1 \pmod{p}$ , where  $p$  is an odd prime. [4]

- (b) Find two odd prime factors of the number  $N = 2 \times 34^{34} - 2^{15}$ . [7]

**END OF QUESTION PAPER**

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