GCE

Further Mathematics A

Y545/01: Additional Pure Mathematics

Advanced GCE

Mark Scheme for June 2019

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

Subject-specific Marking Instructions for A Level Further Mathematics A

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

Question			Answer	AOs	Marks	Guidance
1			$u_2 = 3$	1.1a	M1	Use of the given r.r. (with at least u_2 correct)
			$u_3 = 0.8$	1.1	M1	Repeated use of the given r.r. (with at least u_3 correct)
			$u_4 = 0.6$, $u_5 = 2$ and $u_6 = 5$	1.1	A1	u_4, u_5 & u_6 correct
			Explanation that $u_0 = u_5$ and $u_1 = u_6$ $\Rightarrow$ periodicity, period 5	2.4	E1	(since a second-order r.r.)
					[4]	
2	(a)		$f_x = 2x\sin y - 2y\sin x$ $\quad$ $f_y = x^2\cos y + 2\cos x$	1.1a 1.1	B1 B1	
			$f_{xx} = 2\sin y - 2y\cos x$ $\quad$ $f_{yy} = -x^2\sin y$	1.1 2.5	B1 B1	
			$f_{xy} = f_{yx} = 2x\cos y - 2\sin x$	1.1	B1	Both must be written down, or stated equal
					[5]	
	(b)	i	When $x = y = \frac{1}{2}\pi$, $f_x = f_y = 0$	1.1a	M1	Allow statement only
			$\Rightarrow$ stationary point	2.2a	A1	
			Visible check/calculation that $z = \left(\frac{1}{2}\pi\right)^2 \times 1 + 2\times\left(\frac{1}{2}\pi\right)\times 0 = \frac{1}{4}\pi^2$	2.1	B1	
					[3]	
		ii	$\lvert\mathbf{H}\rvert = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}$ attempted with numerical values	1.1	M1	Considering (the determinant of) the Hessian matrix, or equivalent
			$\lvert\mathbf{H}\rvert = \begin{vmatrix} 2 & -2 \\ -2 & -\frac{1}{4}\pi^2 \end{vmatrix} = -\frac{1}{2}\pi^2 - 4$ (**or** –8.935-ish)	2.1	A1	Correctly calculated value of $\lvert\mathbf{H}\rvert$
			$\Rightarrow$ Saddle-point since $\lvert\mathbf{H}\rvert < 0$	2.2a	B1 [3]	**FT** conclusion from $\lvert\mathbf{H}\rvert < 0$
3	(a)		$7x \equiv 6 \equiv 25 \equiv 44 \equiv 63 \ldots$	1.1	M1	Adding 19s to RHS to reach a multiple of 7
			$\Rightarrow$ $x \equiv 9 \pmod{19}$	1.1	A1	Must be a general result (i.e. not just $x = 9$)
			Alt.			
			$7^{-1} \pmod{19}$ is 11 and multiplying through by this		M1	
			$\Rightarrow$ $x \equiv 9 \pmod{19}$		A1	Must be a general result (i.e. not just $x = 9$)
					[2]	No requirement to justify by hcf(7, 19) = 1 e.g.
	(b)		$x \equiv 3 \pmod 4$ $\Rightarrow$ x is odd while $x \equiv 4 \pmod 6$ $\Rightarrow$ x is even	3.1a	M1	Consideration of parity (i.e. mod 2) or equivalent
			Statement that there are no solutions	2.1	A1	For valid reason
			Alt.			
			h = hcf(4, 6) = 2 and solutions exist provided $h \mid (4 - 3)$		M1	Considering h and "$b - a$"

			But $2 \nmid 1$ hence no solutions		A1	Conclusion with justification
					[2]	

Question			Answer	AOs	Marks	Guidance
4	(a)		PS is $T_n = -29$	1.1	B1	Particular Solution
			CS from $\lambda^2 + 2 = 0$, $\lambda = \pm\mathrm{i}\sqrt{2}$, is $T_n = A\left(\mathrm{i}\sqrt{2}\right)^n + B\left(-\mathrm{i}\sqrt{2}\right)^n$	1.1a 1.1	M1 A1	Complementary Solution attempted from auxiliary equation; correct
			General Solution is $T_n = A\left(\mathrm{i}\sqrt{2}\right)^n + B\left(-\mathrm{i}\sqrt{2}\right)^n - 29$	1.2	B1	**FT** GS = CS (with 2 arbitrary constants) + PS (with none)
			Use of $T_0 = -27$ and $T_1 = 27$ to create simultaneous equations in *A*, *B*	1.1a	M1	
			Two correct equations: $A + B = 2$ and $A - B = -28\mathrm{i}\sqrt{2}$	1.1	A1	Or unsimplified equivalents
			Solving attempt at two equations: $A = 1 - 14\mathrm{i}\sqrt{2}$ and $B = 1 + 14\mathrm{i}\sqrt{2}$	1.1a	M1	
			i.e. $T_n = \left(1 - 14\mathrm{i}\sqrt{2}\right)\left(\mathrm{i}\sqrt{2}\right)^n + \left(1 + 14\mathrm{i}\sqrt{2}\right)\left(-\mathrm{i}\sqrt{2}\right)^n - 29$	1.1	A1	**cao**
			Alt. version			
			PS is $T_n = -29$		B1	Particular Solution
			CS from $\lambda^2 + 2 = 0$, $\lambda = \pm\mathrm{i}\sqrt{2}$, is $T_n = \left(\sqrt{2}\right)^n\left(C\cos\frac{n\pi}{2} + D\sin\frac{n\pi}{2}\right)$		M1 A1	Complementary Solution attempted from auxiliary equation; correct
			General Solution is $\left(\sqrt{2}\right)^n\left(C\cos\frac{n\pi}{2} + D\sin\frac{n\pi}{2}\right) - 29$		B1	**FT** GS = CS (with 2 arbitrary constants) + PS (with none)
			Use of $T_0 = -27$ and $T_1 = 27$ to create simultaneous equations in *C*, *D*		M1	
			Two correct equations: $C - 29 = -27$ and $\sqrt{2}\,D - 29 = 27$		A1	Unsimplified
			Solving attempt at two equations: $C = 2$ and $D = 28\sqrt{2}$		A1	
			i.e. $T_n = \left(\sqrt{2}\right)^n\left(2\cos\frac{n\pi}{2} + 28\sqrt{2}\sin\frac{n\pi}{2}\right) - 29$		A1	**cao**
					[8]	
	(b)		Substituting $n = 20 \Rightarrow$ $T_{20} = \left(1 - 14\mathrm{i}\sqrt{2}\right) \times 1024 + \left(1 + 14\mathrm{i}\sqrt{2}\right) \times 1024 - 29$	2.1	M1	**Or** by applying the given recurrence relation (e.g. **BC**)
			$= 2048 - 29 = 2019$	1.1	A1 [2]	

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>AOs</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>5</td><td>(a)</td><td></td><td>$11^2 = 121 \equiv 41 \pmod{80}$ so $41 \in S$
$11 \times 41 = 451 \equiv 51 \pmod{80}$ so $51 \in S$
$41^2 \equiv 1 \pmod{80}$ so $1 \in S$
$$\begin{array}{c|cccc} \times_{80} & 1 & 11 & 41 & 51 \\ \hline 1 & 1 & 11 & 41 & 51 \\ 11 & 11 & 41 & 51 & 1 \\ 41 & 41 & 51 & 1 & 11 \\ 51 & 51 & 1 & 11 & 41 \end{array}$$</td><td>3.1a
2.5
2.2a

1.1a

1.1</td><td>B1
B1
B1

M1

A1

[5]</td><td>

B1 for each other element found (possibly tabulated)

4×4 table attempted (with at least R_1 and C_1 correct) using the correct 4 elements

All correct</td></tr>
<tr><td></td><td>(b)</td><td></td><td>Choice of a suitable method, such as
Listing orders of elements: 1, 4, 2, 4 in G and 1, 2, 2, 2 in H

OR Stating that G is the cyclic group of order 4 while H is the Klein-4 group

OR Stating that G has an element of order 4 while H has all (non-identity) elements of order 2</td><td>2.1

2.4</td><td>M1

A1

[2]</td><td>MUST be for two groups of order 4

Using knowledge of small-order groups: M1 for clear indication that there are only two groups of order 4, A1 for the details</td></tr>
<tr><td></td><td>(c)</td><td>i</td><td>G has proper subgroup $\{1, 41\}$ only
H has proper subgroups $\{1, 9\}$, $\{1, 31\}$ and $\{1, 39\}$</td><td>3.1a
1.2
2.2a</td><td>B1
M1
A1
[3]</td><td>Ignore statement of $\{1\}$ and group throughout
Listing any two correct subgroups of order 2
All three and no extras</td></tr>
<tr><td></td><td></td><td>ii</td><td>G and H must have different structures since they have differing numbers of (proper) subgroups</td><td>2.4</td><td>B1

[1]</td><td>(Must have different numbers of subgroups in (c)(i))</td></tr>
</table>

Question			Answer	AOs	Marks	Guidance
6	(a)		$\mathbf{p} \cdot \mathbf{q} \times \mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -17 \\ -14 \end{pmatrix}$ or $\begin{vmatrix} 1 & 3 & 2 \\ 2 & 1 & -4 \\ 3 & -1 & 5 \end{vmatrix} = -75$	1.1a 1.1	M1 A1	One vector product and a scalar product Or via the determinant method
			$\mathbf{p} \times (\mathbf{q} \times \mathbf{r}) = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \times \begin{pmatrix} 1 \\ -17 \\ -14 \end{pmatrix} = \begin{pmatrix} 23 \\ 17 \\ -19 \end{pmatrix}$	1.1a 1.1	M1 A1	Two vector products, in correct order
			$(\mathbf{p} \times \mathbf{q}) \times \mathbf{r} = \begin{pmatrix} -5 \\ 10 \\ -5 \end{pmatrix} \times \begin{pmatrix} 2 \\ -4 \\ 5 \end{pmatrix} = \begin{pmatrix} 30 \\ 15 \\ 0 \end{pmatrix}$	1.1a 1.1	M1 A1 [6]	Two vector products, in correct order
	(b)		No, since $\mathbf{p} \times (\mathbf{q} \times \mathbf{r}) \neq (\mathbf{p} \times \mathbf{q}) \times \mathbf{r}$	2.2a	B1 [1]	Correct answer with reason
	(c)		$\mathbf{b} \times \mathbf{c} = \mathbf{n}$ is normal/perpendicular to the plane containing **b**, **c** (or B, C) Then $\mathbf{a} \times \mathbf{n}$ is perpendicular to this normal and hence in the plane of **b**, **c** so that $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$ is of the form $\lambda\mathbf{b} + \mu\mathbf{c}$ for scalar constants λ and μ.	3.1a 3.2a	M1 A1 [2]	Consideration of fact that the vector product gives a vector perpendicular to the two original vectors Fully explained ("planes" not essential; directions suffice)
	(d)		$(\mathbf{a} \times \mathbf{b}) \times \mathbf{c} = -\mathbf{c} \times (\mathbf{a} \times \mathbf{b})$ $= -\{(\mathbf{c} \cdot \mathbf{b})\,\mathbf{a} - (\mathbf{c} \cdot \mathbf{a})\,\mathbf{b}\}$ $= (\mathbf{c} \cdot \mathbf{a})\,\mathbf{b} - (\mathbf{c} \cdot \mathbf{b})\,\mathbf{a}$	3.1a 2.1 2.2a	M1 M1 A1	Anti-commutative property used Attempted use of given form, with $\mathbf{a} \to \mathbf{c}$, $\mathbf{b} \to \mathbf{a}$, $\mathbf{c} \to \mathbf{b}$ Correct, in required form (order of dot products may be reversed, of course)
			Alt. NB $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c} = -\mathbf{c} \times (\mathbf{a} \times \mathbf{b}) = -\mathbf{c} \times -(\mathbf{b} \times \mathbf{a}) = \mathbf{c} \times (\mathbf{b} \times \mathbf{a})$ $= (\mathbf{c} \cdot \mathbf{a})\,\mathbf{b} - (\mathbf{c} \cdot \mathbf{b})\,\mathbf{a}$		M1 A1	
					[3]	

Question			Answer	AOs	Marks	Guidance
7	(a)		$\frac{dy}{dx} = x^2 - \frac{1}{4x^2} \Rightarrow 1 + \left(\frac{dy}{dx}\right)^2 = 1 + \left(x^4 - \frac{1}{2} + \frac{1}{16x^4}\right)$ **or** $\left(x^2 + \frac{1}{4x^2}\right)^2$	1.1	M1 A1	Not necessarily as a perfect square
			$A = 2\pi\int\left(\frac{x^3}{3} + \frac{1}{4x}\right)\left(x^2 + \frac{1}{4x^2}\right)dx$ **or** $2\pi\int\left(\frac{x^5}{3} + \frac{x}{3} + \frac{1}{16x^3}\right)dx$	1.1a 1.1	A1	Correct integrand **soi** at any stage (any correct form, including involving a square-root)
			$= 2\pi\left[\frac{x^6}{18} + \frac{x^2}{6} - \frac{1}{32x^2}\right]_{\frac{1}{2}}^{\frac{3}{2}} = 2\pi\left(\frac{1145}{576} - \frac{-95}{576}\right) = \frac{155}{72}\pi$	1.1	A1 **[4]**	**BC** or from correct integration
	(b)		Line is $y = \frac{3}{4}x + \frac{1}{6}$ and cuts the x-axis at $(-\frac{2}{9}, 0)$	3.3	B1	Cutting-point on x-axis **BC**
			DR Distance $(-\frac{2}{9}, 0)$ to $\left(\frac{3}{2}, \frac{31}{24}\right)$ is $\sqrt{\left(\frac{31}{18}\right)^2 + \left(\frac{31}{24}\right)^2} = \frac{155}{72}$	3.4	M1	Attempt at either of the two slant heights (or use of calculus)
			Distance $(-\frac{2}{9}, 0)$ to $\left(\frac{1}{2}, \frac{13}{24}\right)$ is $\sqrt{\left(\frac{13}{18}\right)^2 + \left(\frac{13}{24}\right)^2} = \frac{65}{72}$	1.1	A1	Both correct
			Curved SA required is $\pi RL - \pi rl = \pi\left(\frac{31}{24}.\frac{155}{72} - \frac{13}{24}.\frac{65}{72}\right) = \frac{55}{24}\pi$	1.1	A1	**Cao** (possibly **BC**)
			Alt. Line is $y = \frac{3}{4}x + \frac{1}{6}$		B1	
			$SA = 2\pi\int\left(\frac{3}{4}x + \frac{1}{6}\right)\left(1 + \left(\frac{3}{4}\right)^2\right)dx$		M1 A1	Attempt at surface area integral with y and "ds" Correct integrand in any form (ignore limits for now)
			$= \frac{55}{24}\pi$		A1	**Cao** (possibly **BC**)
					[4]	
	(c)		Mean value of y is $M = \frac{1}{\frac{3}{2} - \frac{1}{2}}\int_{0.5}^{1.5}\left(\frac{x^3}{3} + \frac{1}{4x}\right)dx$	3.3	M1	Attempt to find the mean value of y over this interval
			$= \left[\frac{x^4}{12} + \frac{1}{4}\ln x\right]_{0.5}^{1.5} = \frac{5}{12} + \frac{1}{4}\ln 3$ **or** 0.691 …	1.1	A1	**BC**
			Curved SA cylinder = $2\pi rh$ (with $r = y_m$, $h = 1$) $= 2\pi\left(\frac{5}{12} + \frac{1}{4}\ln 3\right) = 4.3437$ to 4 d.p.	3.4 1.1	M1 A1 **[4]**	**Or** by integration Allow exact answer and condone **awrt** 4.344 (i.e. to 4 s.f.)

Question		Answer	AOs	Marks	Guidance
8	(a)	**DR** $2(p-2)^{p-2} \equiv 2(-2)^{p-2} \pmod{p}$ $= -(-2)^{p-1}$	2.1 1.2	M1 A1	
		$\equiv -1 \pmod{p}$ by *FLT*	2.4	A1	
		since hcf((–) 2, p) = 1	2.2a	E1	
		Alt. I DR Since p and $(p-2)$ are consecutive *odds*, hcf($p-2$, p) = 1		E1	
		and so $(p-2)^{p-1} \equiv 1 \pmod{p}$ by *FLT*		B1	
		$\Rightarrow (p-2)(p-2)^{p-2} \equiv 1 \pmod{p}$		M1	
		$\Rightarrow (-2)(p-2)^{p-2} \equiv 1 \pmod{p} \Rightarrow 2(p-2)^{p-2} \equiv -1 \pmod{p}$		A1	**AG** correctly reasoned
		Alt. II DR $(p-2)^{p-2} = p^{p-2} + (p-2)\left(p^{p-3}\right)(-2) + \binom{p-2}{2}\left(p^{p-4}\right)(-2)^2 + \ldots$		M1	By the *Binomial Theorem*
		$\ldots + \binom{p-2}{p-4}\left(p^2\right)(-2)^{p-4} + (p-2)p(-2)^{p-3} + (-2)^{p-2} \equiv (-2)^{p-2}$ (mod p)		A1	(since each term apart from the last has a factor of p) Statement only is fine
		Then $2(p-2)^{p-2} \equiv -(-2)^{p-1} \pmod{p}$ and $(-2)^{p-1} \equiv 1 \pmod{p}$ by *FLT*		A1	
		$\Rightarrow$ result since hcf(– 2, p) = 1		E1	Can also argue via $(p-2)$ but there's more to justify
				[4]	
	(b)	**DR** mod 3, $N \equiv 2\times(1)^{34} - 2^{15}$	3.1a	M1	Working mod 3
		$\equiv 2\times1-2=0$ since $2^{\text{odd}} \equiv 2 \pmod{3}$ and $2^{\text{even}} \equiv 1 \pmod{3}$	2.4	A1	Fully justified that $3 \mid N$
		$2\times34^{34} - 2^{15} = 2^{35}\times17^{34} - 2^{15} = 2^{15}\left(2^{20}\times17^{34}-1\right)$	3.1a	B1	Indices work to factor out the 2^{15}
		$\left(2^{20}\times17^{34}-1\right) = \left(2^{10}\times17^{17}-1\right)\left(2^{10}\times17^{17}+1\right)$	2.5	M1	Use of the *difference-of-two-squares* factorisation
		Using **(a)**, $2\times17^{17} \equiv -1 \pmod{19}$	3.1a	M1	Using **(a)**'s result
		so first bracket $\equiv -2^9 - 1 = -513 = -19\times27 \equiv 0 \pmod{19}$	2.4	M1	Working mod 19 in one or both brackets
		and N is a multiple of 19	3.2a	A1	Valid conclusion

			Answer		Marks	Guidance
			Alt. (for factor 19)			
			$2\times 34^{34} - 2^{15} = 2^{35}\times 17^{34} - 2^{15} = 2^{15}\left(2^{20}\times 17^{34} - 1\right) = 2^{15}B$		**B1**	Indices work to factor out the 2^{15}
			mod 19, $2^{18} \equiv 1$ by *FLT*		**M1**	(since 19 is prime and hcf(2, 19) = 1)
			$B \equiv 2^2 \times 17^{34} - 1 = (2\times 17^{17} - 1)(2\times 17^{17} + 1)$		**M1**	Use of the *difference-of-two-squares* factorisation
			Using **(a)**, second bracket is divisible by 19		**M1**	Using **(a)**'s result
			and N is a multiple of 19		**A1**	Valid conclusion
					[7]	

Question			Answer		Marks	Guidance
8	**(b)**		**Alt.** (for factor 19)			
			$2\times 34^{34} - 2^{15} = 2^{35}\times 17^{34} - 2^{15} = 2^{33}\left(2\times 17^{17}\right)^2 - 2^{15}$		**B1**	Indices work to obtain a part **(a)** expression
			$\equiv 2^{33}(-1)^2 - 2^{15} \pmod{19}$		**M1**	Using **(a)**'s result
			$2^{33} - 2^{15} = 2^{15}(2^{18} - 1)$		**M1**	Working mod 19 with remaining numerical term(s)
			$2^{18} - 1 \equiv 0 \pmod{19}$ by *FLT* since hcf(2, 19) = 1		**M1**	Use of *FLT* **or** calculator
			and N is a multiple of 19		**A1**	Valid conclusion (*FLT* justified **or** correctly demonstrated numerical work) **Note** that $2^{18} - 1 = 262\,143 = 19 \times 13\,797$ and $2^{33} - 2^{15} = 8\,589\,901\,824 = 19 \times 452\,100\,096$
			NB $N = 2^{15} \times 3 \times 19 \times 389 \times \ldots$		**[7]**	

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