OCR
Oxford Cambridge and RSA

Friday 22 May 2020 – Afternoon

AS Level Further Mathematics A

Y535/01 Additional Pure Mathematics

Time allowed: 1 hour 15 minutes

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You must have:
- the Printed Answer Booklet
- the Formulae Booklet for AS Level Further Mathematics A
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **4** pages.

ADVICE
- Read each question carefully before you start your answer.

Answer **all** the questions.

1 **(a)** Evaluate 13×19 modulo 31. **[1]**

(b) Solve the linear congruence $13x \equiv 9 \pmod{31}$. **[3]**

2 An open-topped rectangular box is to be manufactured with a fixed volume of $1000\,\text{cm}^3$. The dimensions of the base of the box are x cm by y cm. The surface area of the box is $A\,\text{cm}^2$.

(a) Show that $A = xy + 2000\left(\frac{1}{x} + \frac{1}{y}\right)$. **[4]**

(b) **(i)** Use partial differentiation to determine, in exact form, the values of x and y for which A has a stationary value. **[5]**

(ii) Find the stationary value of A. **[2]**

3 In this question, N is the number 26 132 652.

(a) Without dividing N by 13, explain why 13 is a factor of N. **[1]**

(b) Use standard divisibility tests to show that 36 is a factor of N. **[3]**

It is given that $N = 36 \times 725\,907$.

(c) Use the results of parts **(a)** and **(b)** to deduce that 13 is a factor of 725 907. **[2]**

4 **(a)** For the set $S = \{2, 4, 6, 8, 10, 12\}$, under the operation $\times_{14}$ of multiplication modulo 14, complete the Cayley table given in the Printed Answer Booklet. **[4]**

(b) Show that $(S, \times_{14})$ forms a group, G. (You may assume that $\times_{14}$ is associative.) **[4]**

(c) **(i)** Write down all the proper subgroups of G. **[2]**

(ii) Given that G is cyclic, write down all the possible generators of G. **[2]**

5 **(a)** Determine the general solution of the first-order recurrence relation $V_{n+1} = 2V_n + n$. **[6]**

(b) Given that $V_1 = 8$, find the exact value of V_{20}. **[2]**

6 The points A and B have position vectors $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = -3\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}$ respectively.

(a) Determine the area of triangle OAB, giving your answer in an exact form. **[3]**

The point C lies on the line $(\mathbf{r} - \mathbf{a}) \times (\mathbf{b} - \mathbf{a}) = \mathbf{0}$ such that the area of triangle OAC is half the area of triangle OAB.

(b) Determine the two possible position vectors of C. **[6]**

Turn over for question 7

7 In a conservation project, a batch of 100 000 tadpoles which have just hatched from eggs is introduced into an environment which has no frog population. Previous research suggests that for every 1 million tadpoles hatched only 3550 will live to maturity at 12 weeks, when they become adult frogs.

It is assumed that the steady decline in the population of tadpoles, from all causes, can be explained by a weekly death-rate factor, r, which is constant across each week of this twelve-week period.

Let T_k denote the total number of tadpoles alive at the end of k weeks after the start of this project.

(a) (i) Explain why a recurrence system for T_k is given by $T_0 = 100000$ and $T_{k+1} = (1-r)T_k$ for $0 \leqslant k \leqslant 12$. **[3]**

(ii) Show that $r = 0.375$, correct to 3 significant figures. **[2]**

The proportion of females within each batch of tadpoles is p, where $0 < p < 1$. In a simple model of the frog population the following assumptions are made.

- The death rate factor for adult frogs is also r and is the same for males and females.
- The frog population will survive provided there are at least **thirty** female frogs alive sixteen weeks after the start of this project.

(b) (i) Find the smallest value of p for which the frog population will survive according to the model. **[3]**

(ii) Write down one assumption that has been made in order to obtain this result. **[1]**

Each surviving female will then lay a batch of eggs from which 2500 tadpoles are hatched.

(c) By considering the total number of tadpoles hatched, give one criticism of the assumption that the frog population will survive provided there are at least **thirty** female frogs alive sixteen weeks after the start of this project. **[1]**

END OF QUESTION PAPER