GCE

Further Mathematics A

Y535/01: Additional Pure Mathematics

Advanced Subsidiary GCE

2020 Mark Scheme (DRAFT)

This is a DRAFT mark scheme. It has not been used for marking as this paper did not receive any entries in the series it was scheduled for. It is therefore possible that not all valid approaches to a question may be captured in this version. You should give credit to such responses when marking learner's work.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Question			Answer	Marks	AO	Guidance
1	(a)		30 (mod 31) or –1 (mod 31)	**B1** **[1]**	**1.1**	**BC** No other answer to be accepted Note: $13 \times 19 = 247 = 7 \times 31 + 30 \equiv 30 \pmod{31}$
	(b)		$13x \equiv 9 \equiv 40 \equiv 71 \equiv \ldots \equiv 195$ so $x \equiv 15 \pmod{31}$ **OR** $x = 31n + 15$	**M1** **A1** **A1**	**1.1** **1.1** **2.2a**	Repeatedly adding 31s arriving at a multiple of 13 $n \in \mathbb{Z}$ need not be stated
			Alternative method $13 \times 19 \equiv -1 \Rightarrow 13 \times (19 \times 13 \times 19) \equiv 1$ so $19 \times 13 \times 19 \equiv 12$ is the reciprocal of 13 (mod 31) Then $12 \times 13x \equiv 12 \times 9$ $\Rightarrow x \equiv 15 \pmod{31}$	**M1** **M1** **A1**		Method for finding reciprocal (inverse) of 13 (mod 31) using **(a)** Multiplication by the reciprocal correct answer
				[3]		

Question			Answer	Marks	AO	Guidance
2	(a)		$xyh = 1000 \Rightarrow h = \dfrac{1000}{xy}$ $A = xy + 2xh + 2yh$ $= xy + 2000\left(\dfrac{1}{x} + \dfrac{1}{y}\right)$	B1 B1 M1 A1 [4]	3.1b 1.1 2.1 1.1	 **soi** Substitution of h expression from **(a) (i)** **AG** shown with supporting working
	(b)	(i)	$\dfrac{\partial A}{\partial x} = y + 2000\left(\dfrac{-1}{x^2}\right)$ and $\dfrac{\partial A}{\partial y} = x + 2000\left(\dfrac{-1}{y^2}\right)$ Both p.d.s set to zero and solving $x = y = 10 \times 2^{\frac{1}{3}}$	M1 A1 B1 M1 A1 [5]	1.1 1.1 1.1 2.1 1.1	Partially differentiating A w.r.t. x or y; either correct 2nd correct: **FT** 1st, with $x \leftrightarrow y$ $x^2y = xy^2 = 2000$ Both correct
		(ii)	Substg. x, y back into formula for A; $300 \times 2^{\frac{2}{3}}$	M1 A1 [2]	1.1 1.1	Any exact equivalent e.g. $150 \times 2^{\frac{5}{3}}$, $75 \times 2^{\frac{8}{3}}$ **or** awrt 476 **BC**
3	(a)		13 divides each pair of digits of N (26, 13, 26, 52)	B1 [1]	2.4	Or applying a standard divisibility test
	(b)		$4 \mid 52$ (the final two digits of N) $\Rightarrow 4 \mid N$ $9 \mid$ digit-sum of N (= 27) $\Rightarrow 9 \mid N$ Since hcf(4, 9) = 1, $4 \times 9 = 36 \mid N$	B1 B1 B1 [3]	1.1 1.1 2.4	Applying these two divisibility tests Must explain that 4, 9 are co-prime as well as state the conclusion
	(c)		By *Euclid's Lemma*, $13 \mid 36 \times 725\,907$ and hcf(13, 36) = 1 $\Rightarrow 13 \mid 725\,907$	M1 A1 [2]	2.4 2.2a	M for stating "Euclid's Lemma" (or full description of its result) Clear outline of necessary conditions

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AO</th><th>Guidance</th></tr>
<tr><td>4</td><td>(a)</td><td></td><td>
<table>
<tr><th>$\times_{14}$</th><th>2</th><th>4</th><th>6</th><th>8</th><th>10</th><th>12</th></tr>
<tr><td>2</td><td>4</td><td>8</td><td>12</td><td>2</td><td>6</td><td>10</td></tr>
<tr><td>4</td><td>8</td><td>2</td><td>10</td><td>4</td><td>12</td><td>6</td></tr>
<tr><td>6</td><td>12</td><td>10</td><td>8</td><td>6</td><td>4</td><td>2</td></tr>
<tr><td>8</td><td>2</td><td>4</td><td>6</td><td>8</td><td>10</td><td>12</td></tr>
<tr><td>10</td><td>6</td><td>12</td><td>4</td><td>10</td><td>2</td><td>8</td></tr>
<tr><td>12</td><td>10</td><td>6</td><td>2</td><td>12</td><td>8</td><td>4</td></tr>
</table>
</td><td>B1
B1
B1
B1
[4]</td><td>1.1
1.1
1.1
1.1</td><td>For any two lines (Rs or Cs) correct
For at least two Rs and two Cs correct
For LSP applying to complete table
For symmetry about main diagonal
(Must be fully correct for all 4 marks)</td></tr>
<tr><td></td><td>(b)</td><td></td><td>Closed since no other elements appear in the table
Identity is 8
Inverses: 6 is self-inverse
$2^{-1} = 4$ and $4^{-1} = 2$; $10^{-1} = 12$ and $12^{-1} = 10$
(Hence a group)</td><td>B1
B1
B1
B1
[4]</td><td>2.4
2.2a
1.2
2.5</td><td>Don’t accept “closed, from table” only
Any clear indication of inverses (not just statement they exist)
That is, (2, 4) and (10, 12) are inverse-pairs
Associativity and conclusion not required</td></tr>
<tr><td></td><td>(c)</td><td>(i)</td><td>{8, 6} {8, 2, 4}</td><td>B1 B1
[2]</td><td>2.2a 1.1</td><td>One correct; both (and no extras). Ignore {8} and G</td></tr>
<tr><td></td><td></td><td>(ii)</td><td>10, 12</td><td>B1 B1
[2]</td><td>1.1 1.1</td><td>One correct; both (and no extras)</td></tr>
<tr><td>5</td><td>(a)</td><td></td><td>Complementary Solution is $V_n = A \times 2^n$
For Particular Solution, try $V_n = an + b$
Then $V_{n+1} = 2V_n + n \Rightarrow an + (a + b) = 2an + 2b + n$
Comparing coefficients: $a = 2a + 1$ and $a + b = 2b$
$\Rightarrow a = b = -1$
General Solution is thus $V_n = A \times 2^n - n - 1$</td><td>B1
M1
A1
M1
A1
B1
[6]</td><td>1.2
1.1a
1.1
1.1
1.1
1.1</td><td>Allow $V_n = an$ for method mark
Substitution and comparing of coefficients
FT GS = CS + PS provided CS has one arbitrary constant and PS has none (and is a polynomial)</td></tr>
</table>

Question			Answer	Marks	AO	Guidance
5	(b)		$V_1 = 8 \Rightarrow A = 5$ so $V_n = 5 \times 2^n - n - 1$	M1	3.1a	soi (or **BC**)
			So $V_{20} = 5\,242\,859$	A1 [2]	1.1	accept exact value only.
6	(a)		$\mathbf{a} \times \mathbf{b} = -14\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}$	B1	1.1	A correct vector product (possibly **BC**)
			Use of formula Area $\Delta = \frac{1}{2}\|\mathbf{a} \times \mathbf{b}\|$	M1	1.1	Including an attempt at a vector product
			Area $\Delta OAB = 5\sqrt{3}$	A1 [3]	1.1	Accept alternative exact equivalents (e.g. $\sqrt{75}$)
	(b)		$(\mathbf{r} - \mathbf{a}) \times (\mathbf{b} - \mathbf{a}) = \mathbf{0}$ is the line through A and B	M1	2.2a	
			so $\mathbf{c} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a})$ or $\mathbf{c} = (1 - \lambda)\mathbf{a} + \lambda\mathbf{b}$	A1	3.1a	
			Area $\Delta OAC = \frac{1}{2}\|\mathbf{a} \times \mathbf{c}\| = \frac{1}{2}\|(1 - \lambda)\mathbf{a} \times \mathbf{a} + \lambda\,\mathbf{a} \times \mathbf{b}\|$	M1	2.1	From this point on, work may appear with numerical equivalent set-out
			$= \frac{1}{2}\|\mathbf{0} + \lambda\,\mathbf{a} \times \mathbf{b}\|$	M1	3.1a	Use of $\mathbf{a} \times \mathbf{a} = \mathbf{0}$
			Area $\Delta OAC = \frac{1}{2}$ Area $\Delta OAB \Rightarrow \lambda = \pm\frac{1}{2}$	A1	1.1	
			giving $\mathbf{c} = -\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ or $\mathbf{c} = 3\mathbf{i} + \mathbf{j} + 4\mathbf{k}$	A1	2.1	
			Alternative method			
			C is on the line AB	B1		
			Common "base" OA means that C is either the internal or the external bisector of AB	M1 A1		(For half the "height")
				M1		At least one must be attempted
			i.e. $\mathbf{c} = \frac{1}{2}(\mathbf{a} + \mathbf{b})$ or $\frac{1}{2}(3\mathbf{a} - \mathbf{b})$	A1		
			giving $\mathbf{c} = -\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ or $\mathbf{c} = 3\mathbf{i} + \mathbf{j} + 4\mathbf{k}$	A1		Both correct
				[6]		

Question			Answer	Marks	AO	Guidance
7	(a)	(i)	E.g. $T_0 = 100\,000$ is the initial population as given $T_{k+1} = (1-r)\,T_k$ because a death-rate of r means that $1-r$ of the population is left after each week. $0 \le k \le 12$ because the model given is only valid for twelve weeks.	B1 B1 B1 [3]	1.1 3.3 2.1	
		(ii)	$T_{12} = a^{12}\,T_0$ $1 - r = \sqrt[12]{0.00355} = 0.624\,96\ldots \Rightarrow r = 0.375$ to 3s.f.	M1 A1 [2]	3.1b 1.1	$a = r$ or $1 - r$ **AG**
	(b)	(i)	After 16 weeks, the number of frogs is $0.62496\ldots^{16} \times 100\,000 = 54.154\ldots$ So $54.154\ldots \times p \ge 30$ $\Rightarrow p \ge \dfrac{30}{54.154...} = 0.5539... = 0.554$ to 3 sf	B1 M1 A1 [3]	3.5c 3.1b 1.1	Allow use of 'T_{16}'. Or, starting again $0.62496\ldots^{4} \times 355$ For 'their population' $\times\, p \ge 30$
		(ii)	**E.g.** The same weekly death-rate factor continues unchanged. The females will all lay eggs. Tadpoles instantly change to frogs and lay eggs at exactly the same time.	B1 [1]	3.3	
	(c)		**E.g.** 30 surviving females would produce 75000 eggs, so the population is smaller than it was to start with, so each 'round' will result in smaller and smaller populations.	B1 [1]	3.5a	No greater detail of analysis is required beyond "they would appear to be dying out so the figure of 30 in the model is not a good one"

OCR (Oxford Cambridge and RSA Examinations)
The Triangle Building
Shaftesbury Road
Cambridge
CB2 8EA

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk