Mark Scheme

Summer 2023

Pearson Edexcel GCE

In Mathematics (9MA0)

Paper 32 Mechanics

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Summer 2023
Publications Code 9MA0_32_2306_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

1. The total number of marks for the paper is 50.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol √ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ✱ The answer is printed on the paper
- ⎾ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.
   If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.
6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives

answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

General Principles for Mechanics Marking

(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.

- Omission or extra g in a resolution is an accuracy error not method error.

- Omission of mass from a resolution is a method error.

- Omission of a length from a moments equation is a method error.

- Omission of units or incorrect units is not (usually) counted as an accuracy error.

- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.

- Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.

- Use of g = 9.81 should be penalised once per (complete) question.

  N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.

- Marks must be entered in the same order as they appear on the mark scheme.

- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),……then that working can only score marks for that part of the question.

- Accept column vectors in all cases.

- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft

- Mechanics Abbreviations

  M(A) Taking moments about A.

  N2L Newton's Second Law (Equation of Motion)

  NEL Newton's Experimental Law (Newton's Law of Impact)

  HL Hooke's Law

  SHM Simple harmonic motion

  PCLM Principle of conservation of linear momentum

  RHS, LHS Right hand side, left hand side

Question	Scheme	Marks	AOs
1(a)	16 ($\mathrm{m\,s^{-1}}$) seen as the answer	B1	1.1b
		(1)	
1(b)	$s=\frac{1}{2}\times3.2\times5^2$ **OR** $s=\frac{(0+16)}{2}\times5$ **OR** $s=(16\times5)-\frac{1}{2}\times3.2\times5^2$ **OR** $16^2=2\times3.2\times s$ **OR** from a *v*-*t* graph, $s=\frac{1}{2}\times5\times16$	M1	3.1b
	s = 40 (m)	A1	1.1b
		(2)	
			(3 marks)

Notes:

1a	B1	cao. Must be positive. Ignore any working.		
1b	M1	Complete method to find an equation in s only, possibly using their '16' Allow 'reversed motion': use of $s=vt-\frac{1}{2}at^2$ with v = 0 i.e. $s=-\frac{1}{2}\times3.2\times5^2$ can score M1 and s = -40 so distance is 40 (m) can score the A1		
	A1	cao. Must be positive.		
		N.B. correct answer only, in (b), can score both marks.		

Question	Scheme	Marks	AOs
2(a)	Resolve vertically, $R = 5g = 49$ (N)	B1	1.1b
		(1)	
2(b)	Equation of motion: $28 - F = 5 \times 1.4$	M1	3.1a
	$F = 21$	A1	1.1b
		(2)	
2(c)	$\mu = 0.43$ (2sf required)	B1 **ft**	3.4
		(1)	
			(4 marks)

Notes:

2a	B1	Allow either $5g$ or 49. No penalty for using $g = 9.81$ or 10. Ignore any working. Must be a positive number. B0 if m is involved. **N.B**. Could be seen on a diagram, provided it's clearly the reaction.
2b	M1	Equation with correct terms, dimensionally correct, condone sign errors.
	A1	cao but allow $\frac{15g}{7}$. Ignore units.
2c	B1**ft**	$\mu = \frac{\text{their (b)}}{\text{their (a)}}$. **Answer must be a positive number given to 2sf**. **N.B**. B0 if they use $g = 9.81$ or 10 in this part of the question. Do not allow restarts. Allow $\mu > 1$.

Question	Scheme	Marks	AOs
3(a)	7**i** – 3**j** seen or implied by Pythagoras	B1	1.1b
	Use Pythagoras: $\sqrt{7^2+(-3)^2}$	M1	3.1a
	$\sqrt{58}$, 7.6 or better ($\mathrm{m\,s^{-1}}$)	A1	1.1b
		(3)	
3(b)	$t^2-3t+7=2t^2-3$ **OR** $\dfrac{t^2-3t+7}{2t^2-3}=\dfrac{1}{1}$ = 1	M1	2.1
	t = 2 only	A1	1.1b
		(2)	
3(c)	Differentiate **v** wrt t to give a vector.	M1	3.1a
	$(2t-3)\mathbf{i}+4t\mathbf{j}$	A1	1.1b
		(2)	
3(d)	$2t-3=0$	M1	3.1a
	t = 1.5	A1	1.1b
		(2)	
			(9 marks)

Notes: Allow column vectors throughout.

3a	B1	cao
	M1	Use of Pythagoras, including the square root, on a **velocity** vector at t = 0
	A1	cao. Must come from a correct **v**.
3b	M1	Equating **i** and **j** components of **v** or a ratio of 1:1 to obtain a quadratic in t only. If they use a constant, e.g. $t^2-3t+7=k$ and $2t^2-3=k$, k must be eliminated to earn this mark. **N.B.** M0 (since wrong working seen) if they write down $\mathbf{i}+\mathbf{j}=\left(t^2-3t+7\right)\mathbf{i}+\left(2t^2-3\right)\mathbf{j}$ **OR** $\begin{pmatrix}1\\1\end{pmatrix}=\begin{pmatrix}t^2-3t+7\\2t^2-3\end{pmatrix}$ **OR** $t^2-3t+7=1$ and $2t^2-3=1$

		and then $t^2 - 3t + 7 = 2t^2 - 3$
	A1	$t = 2$
		N.B. Allow M1A1 for a **correct** trial and error method where they obtain $\mathbf{v} = 5\mathbf{i} + 5\mathbf{j}$ when t = 2 but M0 if they don't get t = 2
3c	M1	At least one power decreasing by 1 in **each** component in their **v** (M0 if clearly dividing by t) **Both i** and **j** needed in their answer or a column vector Allow recovery if the **i** and **j** disappear and then reappear.
	A1	cao (must be a vector) isw e.g. if they find the magnitude or put t = 0 or differentiate again **i**'s and **j**'s do not need to be collected. **N.B**. Allow M1A0 for $2t - 3\mathbf{i} + 4t\mathbf{j}$
3d	M1	$2t - 3 = 0$ or (their **derivative** of the **i**-component of **v)** = 0 **N.B**. M0 if they equate the derivative of both components of **v** to zero.
	A1	cao **N.B.** Correct answer, with no working, can score both marks.

4(a)	$\mathbf{v}_B = (-16\mathbf{i} - 3\mathbf{j}) + 5(2.4\mathbf{i} + \mathbf{j})$	M1	3.4
	$\mathbf{v}_B = (-4\mathbf{i} + 2\mathbf{j})$	A1	1.1b
	$\sqrt{(-4)^2 + 2^2}$	M1	3.1a
	$\sqrt{20}$ = $2\sqrt{5}$, 4.5 or better ($\mathrm{m\,s^{-1}}$)	A1	1.1b
		(4)	
4(b)	Using *A* as the initial position: $\mathbf{r}_C = \mathbf{v}_A t + \frac{1}{2}\mathbf{a}t^2 + \mathbf{r}_A$ where $t = T$ $(4\mathbf{i} + c\mathbf{j}) = (-16\mathbf{i} - 3\mathbf{j})T + \frac{1}{2}(2.4\mathbf{i} + \mathbf{j})T^2 + (44\mathbf{i} - 10\mathbf{j})$ **OR** $\begin{pmatrix}4\\c\end{pmatrix} = \begin{pmatrix}-16\\-3\end{pmatrix}T + \frac{1}{2}\begin{pmatrix}2.4\\1\end{pmatrix}T^2 + \begin{pmatrix}44\\-10\end{pmatrix}$ Equating **i**-components, to give a **quadratic** equation in *T* only. Allow *t* instead of *T*. **N.B.** Allow omission of 44 for this M mark. Also allow ± 4 but M0 if 4 is not used at all i.e. $4 = -16T + \frac{1}{2} \times 2.4T^2$ scores M1A0A0	M1	3.1a
	$4 = -16T + \frac{1}{2} \times 2.4T^2 + 44$	A1	1.1b
	(T =) 10	A1	1.1b
	ALTERNATIVE using *B* as the initial position: (The position vector of *B*, $\mathbf{r}_B$, should be $-6\mathbf{i} - 12.5\mathbf{j}$ but no credit for finding this) $\mathbf{r}_C = \mathbf{v}_B t + \frac{1}{2}\mathbf{a}t^2 + \mathbf{r}_B$ using **their** $\mathbf{v}_B$ from (a) and **their** $\mathbf{r}_B$ $(4\mathbf{i} + c\mathbf{j}) = (-4\mathbf{i} + 2\mathbf{j})t + \frac{1}{2}(2.4\mathbf{i} + \mathbf{j})t^2 + (-6\mathbf{i} - 12.5\mathbf{j})$ $\begin{pmatrix}4\\c\end{pmatrix} = \begin{pmatrix}-4\\2\end{pmatrix}t + \frac{1}{2}\begin{pmatrix}2.4\\1\end{pmatrix}t^2 + \begin{pmatrix}-6\\-12.5\end{pmatrix}$ Equating **i**-components, to give a **quadratic** equation in *t* only. Allow if they have *T* instead of *t*.	M1	3.1a

	N.B. Allow omission of their -6 or if they use 44 for this M mark. Also allow ± 4 but M0 if 4 is not used at all. e.g. $4 = -4t + \frac{1}{2} \times 2.4t^2$ scores M1A0A0		
	$4 = -4t + \frac{1}{2} \times 2.4t^2 - 6$	A1	1.1b
	t = 5 so (T =) 10	A1	1.1b
		(3)	
4(c)	Equating **j**-components, with their value of T or t substituted, to give an equation, which must have a square term, in c only. **N.B.** Allow $\pm c$ in their equation. **(N.B.** Allow omission of -10 or their -12.5 for this M mark i.e. if using A as initial position $c = (-3 \times 10) + \frac{1}{2} \times 1 \times 10^2$ scores M1M0A0 **OR** if using B as initial position $c = (2 \times 5) + \frac{1}{2} \times 1 \times 5^2$ scores M1M0A0)	M1	2.1
	if using A as initial position $c = (-3 \times 10) + \frac{1}{2} \times 1 \times 10^2 + (-10)$ **N.B.** Allow $\pm c$ and/or $\pm(-10)$ in their equation **OR** if using B as initial position $c = (2 \times 5) + \frac{1}{2} \times 1 \times 5^2 + (-12.5)$ **N.B.** Allow $\pm c$ and/or $\pm(-12.5)$ in their equation	M1	1.1b
	c = 10	A1	1.1b
		(3)	
			(10 marks)

Notes: Accept column vectors throughout

4a	M1	Use of **v** = **u** + **a***t* with t = 5 to give an unsimplified $\mathbf{v}_B$ M0 if **u** = **0**

		N.B. If using integration, they must get to the same stage i.e. have found the constant and put $t = 5$ M0 if they omit the constant altogether
	A1	Correct $\mathbf{v}_B$ with **i**'s and **j**'s collected
	M1	Use of Pythagoras on *their* $\mathbf{v}_B$ to give a magnitude (need the root)
	A1	Must be positive
4b	M1	Equating components of **i** to give an equation in T or t only. **N.B.** (they could use integration to get to the same stage) for this M mark, they **only** need to be equating the **i**-components, and receive no credit until they do so. M0 if **u** = **0**
	A1	A correct equation in T or t only (could be in $(T - 5)$ if using B as initial position)
	A1	$T = 10$
4c	M1	Equating components of **j** to give an equation in c only but allow omission of their initial position
	M1	With their value of T or t and **must include t = 0 position** (should be -10 if using A **OR** their -12.5 if using B)
	A1	cao

Question	Scheme	Marks	AOs
	N.B. In this question, allow misread of α for a.		
5(a)	Use horizontal motion to give an equation in T and α only: $28\cos\alpha \times T = 40$	M1	3.4
	$T = \dfrac{10}{7\cos\alpha}$ *	A1*	1.1b
		(2)	
5(b)	Use vertical motion to give an equation in T and α only	M1	3.3
	$20 = (28\sin\alpha)T - \dfrac{1}{2}gT^2$	A1	1.1b
	Eliminate T to give an unsimplified equation in α only: $20 = (28\sin\alpha)\times\dfrac{10}{7\cos\alpha} - \dfrac{1}{2}g\left(\dfrac{10}{7\cos\alpha}\right)^2$	M1	1.1b
	Use $\sec^2\alpha = 1+\tan^2\alpha$ oe to give an unsimplified equation in **tan α only**: $20 = 40\tan\alpha - \dfrac{1}{2}g\times\dfrac{100}{49}(1+\tan^2\alpha)$	M1	3.1b
	$\tan^2\alpha - 4\tan\alpha + 3 = 0$ * (allow $0 = \tan^2\alpha - 4\tan\alpha + 3$)	A1*	2.2a
		(5)	
5(c)	Solve and use of $\tan\alpha = 3$ or $\sin\alpha = \dfrac{3}{\sqrt{10}}$ or $\alpha = 71.565..^o$ to find an equation in H only.	M1	3.1b
	$0 = (28\sin\alpha)^2 - 2gH$ **where** $\tan\alpha = 3$ $(\alpha = 71.565..^o)$	M1	3.4
	H = 36 or 36.0 (m)	A1	1.1b
		(3)	
5(d)	e.g. spin of the ball, the wind, the dimensions or shape of the ball, ball is modelled as a particle, uses an inaccurate value of g, motion takes place in 3D not in 2D, g could be variable. B0 if mass or weight are mentioned. B0 for ground may not be horizontal.	B1	3.5b
		(1)	

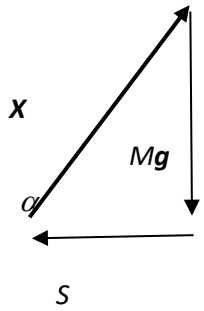
		(11 marks)
Notes:		
5a	M1	Correct no. of terms, dim correct, condone sin/cos confusion and sign errors
	A1*	Correct printed answer correctly obtained. Allow $\frac{10}{7\cos\alpha} = T$ OR $T = \frac{40}{28\cos\alpha} = \frac{10}{7\cos\alpha}$ OR $\frac{40}{28\cos\alpha} = \frac{10}{7\cos\alpha} = T$ OR *t* instead of *T*
5b	M1	Correct no. of terms, dim correct, condone sin/cos confusion and sign errors
	A1	Correct equation
	M1	Eliminate *T*, using either the given answer in (a) or their own *T* expression, from their equation to give an unsimplified equation in α only
	M1	Use $\sec^2\alpha = 1 + \tan^2\alpha$ to produce an equation in $\tan\alpha$ only
	A1*	Given answer correctly obtained. **N.B.** Must be α (or *a*) in the final answer but allow a different angle in the working.
5c	M1	Solve given equation and select larger value of $\tan\alpha$ and use it to try to obtain an equation in *H* only.
	M1	Complete method to give an equation in ***H* only**, using **<u>larger</u>** value of α , correct no. of terms, dim correct, condone sin/cos confusion and sign errors.
	A1	cao. Must be positive, (allow a negative value, changed to a positive answer). **N.B.** This answer comes from use of *g* = 9.8, so must be rounded to 2 or 3 sf.
5d	B1	B0 if any incorrect extras

Question	Scheme	Marks	AOs
6(a)	The normal reaction at *B* is acting to the left so it must act to the right, right as it needs to balance (oppose, counter) the force at *B*, right as it prevents the rod from sliding (slipping, falling), right as the weight (mass) of the rod will mean the rod tends to slip left, mass or weight will be pushing the rod to the left so friction will oppose that. **N.B.** You may see an arrow on the diagram at *A*, instead of 'right'. B0 if they say the rod is moving oe Accept towards the wall instead of to the right.	B1	2.4
		(1)	
6(b)	Take moments about *A*	M1	3.4
	$S \times 2a\sin\theta = Mga\cos\theta$	A1	1.1b
	$S = \frac{1}{2}Mg\cot\theta$ *	A1*	2.2a
		(3)	
6(c)	Resolve vertically, $R = Mg$	B1	3.3
	Resolve horizontally, $F = S$	B1	3.3
	Other possible equations: Resolve along the rod, $F\cos\theta + R\sin\theta = S\cos\theta + Mg\sin\theta$ Resolve perp to the rod, $R\cos\theta + S\sin\theta = F\sin\theta + Mg\cos\theta$ M(*B*), $R \times 2a\cos\theta = F \times 2a\sin\theta + Mga\cos\theta$ M(*G*), $Ra\cos\theta = Fa\sin\theta + Sa\sin\theta$ **N.B.** When entering these two B marks on ePEN**,** **First** B1 is for a **vertical** resolution, **second** B1 is for a **horizontal** resolution, and if either is replaced by a different equation, enter appropriately. If both are replaced by other equations, enter in the order in which they appear in their working.		
	$F = \mu R$	B1	1.2
	$\frac{1}{2}Mg \times \frac{4}{3} = \mu Mg$	**dM**1	2.1
	$\mu = \frac{2}{3}$ oe Accept 0.67 or better	A1	2.2a
	S.C. For F ,, μR, B0 $\frac{1}{2}Mg \times \frac{4}{3}$,, μMg M1		

	$\frac{2}{3}$,, μ A0 **N.B**. If $\mu = \frac{2}{3}$ follows this, they could score all the marks.		
		(5)	
6(d)	$\sqrt{F^2 + R^2}$	M1	3.1a
	$\sqrt{\left(\frac{2}{3}Mg\right)^2 + (Mg)^2}$	M1	1.1b
	$\frac{1}{3}Mg\sqrt{13}$ or 1.2*Mg* or better	A1	2.2a
		(3)	
6(e)	New value of *S* would be **larger** as the **moment** of the **weight** about ***A*** would be larger	B1	3.5a
		(1)	
			(13 marks)

Notes:

6a	B1	Any equivalent appropriate statement.
6b	M1	Correct no. of terms, dimensionally correct, condone sin/cos confusion and sign errors. **N.B.** If *a*'s never appear, M0
	A1	Correct equation
	A1*	Correct given answer correctly obtained, **with no wrong working seen.** Allow $\frac{1}{2}Mg\cot\theta = S$ or $S = \frac{Mg\cot\theta}{2}$ or $\frac{Mg\cot\theta}{2} = S$ or $S = \frac{Mg}{2}\cot\theta$ or similar but NOT $S = \frac{1}{2}\cot\theta\ \mathrm{Mg}$ or similar **N.B.** Allow *m* instead of *M* Must be θ in final answer but allow a different angle in the working.
6c	B1	cao
	B1	cao
	B1	Seen anywhere, e.g. on the diagram
	dM1	Using $F = \mu R$, their two equations and substitute for trig (not necessarily correctly) to produce an equation in μ only. This mark is **dependent** on the 3 previous B marks.
	A1	Accept 0.67 or better

6d	M1	Use of Pythagoras with square root to find the required magnitude, but *F* and *R* do not need to be substituted
	M1	Substitute for their *F* and their *R* in terms of *Mg* and take square root to obtain magnitude in terms of *M* and *g* only. **N.B.** Must be using Pythagoras
		ALTERNATIVE: Using trig on triangle of forces  M1: $X = \frac{Mg}{\sin\alpha}$ or $\frac{S}{\cos\alpha}$ M1: substitute for $\sin\alpha$ or $\cos\alpha$ and *S*, where $\tan\alpha = \frac{Mg}{S}$ $(=\frac{3}{2})$, to obtain *X* in terms of *M* and *g* only.
	A1	Any equivalent surd form or 1.2*Mg* or better Must be in terms of *M* and *g*
6e	B1	Correct answer and any equivalent appropriate statement.

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