Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel Level 3 GCE

Tuesday 13 June 2023

Afternoon (Time: 2 hours)

Paper reference **9MA0/02**

Mathematics

Advanced

PAPER 2: Pure Mathematics 2

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 15 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

1.

$$f(x) = x^3 + 2x^2 - 8x + 5$$

(a) Find $f''(x)$

(2)

(b) (i) Solve $f''(x) = 0$

(ii) Hence find the range of values of x for which $f(x)$ is concave.

(2)

a) $f(x) = x^3 + 2x^2 - 8x + 5$

$f'(x) = 3x^2 + 4x - 8$

$f''(x) = 6x + 4$ 1 1

b)(i) $6x + 4 = 0$

$6x = -4$

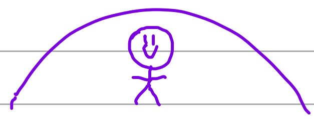
$x = -\frac{2}{3}$ 1

(ii) concave when $f''(x) < 0$

$6x + 4 < 0$

$x < -\frac{2}{3}$ 1

concave: "the rate of change of gradient is decreasing"



Question 1 continued

(Total for Question 1 is 4 marks)

2. A sequence $u_1, u_2, u_3 \ldots$ is defined by

$$u_1 = 35$$

$$u_{n+1} = u_n + 7\cos\left(\frac{n\pi}{2}\right) - 5(-1)^n$$

(a) (i) Show that $u_2 = 40$

(ii) Find the value of u_3 and the value of u_4

(3)

Given that the sequence is periodic with order 4

(b) (i) write down the value of u_5

(ii) find the value of $\sum_{r=1}^{25} u_r$

(3)

a) (i) $u_2 = 35 + 7\cos\left(\frac{\pi}{2}\right) - 5(-1)^1$

$= 40$ (1)

(ii) $u_3 = 40 + 7\cos\left(\frac{2\pi}{2}\right) - 5(-1)^2$

$= 28$ (1)

$u_4 = 28 + 7\cos\left(\frac{3\pi}{2}\right) - 5(-1)^3$

$= 33$ (1)

b) (i) $u_5 = u_1 = 35$ (1)

(ii) $\sum_{r=1}^{25} u_r = \sum_{r=1}^{24} u_r + u_{25}$

$= 6(35 + 40 + 28 + 33) + 35$ (1)

$= 851$ (1)

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Question 2 continued

(Total for Question 2 is 6 marks)

3. Given that

$$\log_2(x + 3) + \log_2(x + 10) = 2 + 2\log_2 x$$

(a) show that

$$3x^2 - 13x - 30 = 0$$

(3)

(b) (i) Write down the roots of the equation

$$3x^2 - 13x - 30 = 0$$

(ii) Hence state which of the roots in part (b)(i) is not a solution of

$$\log_2(x + 3) + \log_2(x + 10) = 2 + 2\log_2 x$$

giving a reason for your answer.

(2)

a) $\log_2(x+3) + \log_2(x+10) = 2 + 2\log_2 x$

$\log_2(x+3) + \log_2(x+10) - \log_2 x^2 = 2$

$\log_2\left(\frac{(x+3)(x+10)}{x^2}\right) = 2$ 1

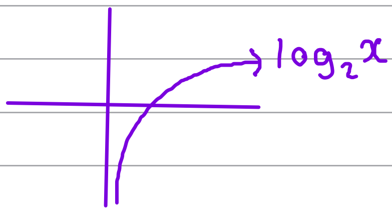
$\frac{x^2+13x+30}{x^2} = 2^2 = 4$

$x^2 + 13x + 30 = 4x^2$ 1

$3x^2 - 13x - 30 = 0$ 1

b) (i) $x = 6, \ x = -\frac{5}{3}$ 1

(ii) $x \neq -\frac{5}{3}$ because $\log_2\left(-\frac{5}{3}\right)$ is not real. 1



$\log x$ only takes positive inputs

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Question 3 continued

(Total for Question 3 is 5 marks)

4. Coffee is poured into a cup.

The temperature of the coffee, H °C, t minutes after being poured into the cup is modelled by the equation

$$H = Ae^{-Bt} + 30$$

where A and B are constants.

Initially, the temperature of the coffee was 85 °C.

(a) State the value of A.

(1)

$\frac{dH}{dt}$ is negative

Initially, the coffee was cooling at a rate of 7.5 °C per minute.

(b) Find a complete equation linking H and t, giving the value of B to 3 decimal places.

(3)

a) when $t=0$, $H=85$

$85 = Ae^0 + 30$

$A = 55$ 1

b) when $t=0$, $\frac{dH}{dt} = -7.5$

$H = 55e^{-Bt} + 30$

$\frac{dH}{dt} = -55Be^{-Bt}$ 1

$-7.5 = -55Be^0$

$B = \frac{3}{22} = 0.136$ (3sf) 1

$H = 55e^{-0.136t} + 30$ 1

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Question 4 continued

(Total for Question 4 is 4 marks)

5. The curve C has equation $y = f(x)$

The curve

- passes through the point $P(3, -10)$
- has a turning point at P

Given that

$$\frac{dy}{dx} = 2x^3 - 9x^2 + 5x + k$$

where k is a constant,

(a) show that $k = 12$

(2)

(b) Hence find the coordinates of the point where C crosses the y-axis.

(3)

a) $\left.\frac{dy}{dx}\right|_{x=3} = 0$ because P is a turning point

$2(3)^3 - 9(3)^2 + 5(3) + k = 0$ (1)

$54 - 81 + 15 + k = 0 \Rightarrow k = 12$ (1)

b) $y = \int \frac{dy}{dx}\,dx = \int (2x^3 - 9x^2 + 5x + 12)\,dx$

$y = \frac{1}{2}x^4 - 3x^3 + \frac{5}{2}x^2 + 12x + c$ (1)

sub in $x = 3, y = 10$

$-10 = \frac{1}{2}(3)^4 - 3(3)^3 + \frac{5}{2}(3)^2 + 12(3) + c$ (1)

$c = -28$

$y = \frac{1}{2}x^4 - 3x^3 + \frac{5}{2}x^2 + 12x - 28$

C crosses y-axis when $x = 0 \therefore y = -28$

$(0, -28)$ (1)

Question 5 continued

(Total for Question 5 is 5 marks)

6. Relative to a fixed origin O,

- A is the point with position vector $12\mathbf{i}$
- B is the point with position vector $16\mathbf{j}$
- C is the point with position vector $(50\mathbf{i} + 136\mathbf{j})$
- D is the point with position vector $(22\mathbf{i} + 24\mathbf{j})$

(a) Show that AD is parallel to BC.

(2)

Points A, B, C and D are used to model the vertices of a running track in the shape of a quadrilateral.

Runners complete one lap by running along all four sides of the track.

The lengths of the sides are measured in metres.

Given that a particular runner takes exactly 5 minutes to complete 2 laps,

(b) calculate the average speed of this runner, giving the answer in kilometres per hour.

(4)

a) Method: find $\overrightarrow{AD}$ and $\overrightarrow{BC}$ and show that one is a multiple of the other

$\overrightarrow{AD} = \overrightarrow{AO} + \overrightarrow{OD} = -12\underline{i} + (22\underline{i} + 24\underline{j}) = 10\underline{i} + 24\underline{j}$

$\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC} = -16\underline{j} + (50\underline{i} + 136\underline{j}) = 50\underline{i} + 120\underline{j}$ (1)

$= 5\overrightarrow{AD}$ (1)

$\therefore$ AD is parallel to BC

b) average speed $= \dfrac{\text{total distance}}{\text{total time}}$

total time = 5 minutes $= \dfrac{5}{60}$ hours

total distance $= 2(|AB| + |BC| + |CD| + |DA|)$

$AB = \overrightarrow{AO} + \overrightarrow{OB} = -12\underline{i} + 16\underline{j}$

$CD = \overrightarrow{CO} + \overrightarrow{OD} = -(50\underline{i} + 136\underline{j}) + (22\underline{i} + 24\underline{j}) = -28\underline{i} - 112\underline{j}$

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Question 6 continued

$|AB| = \sqrt{12^2+16^2} = 20\ m$

$|BC| = \sqrt{50^2+120^2} = 130\ m$ (1)

$|CD| = \sqrt{28^2+112^2} = 28\sqrt{17}\ m$

$|DA| = \sqrt{10^2+24^2} = 26\ m$ (1)

distance of 2 laps $= 2(176 + 28\sqrt{17})\ m$

$= \dfrac{2(176+28\sqrt{17})}{1000}\ km$

speed $= \dfrac{[2(176+28\sqrt{17})/1000]\ km}{[5/60]\ h}$ (1) $= 6.99\ kmh^{-1}$ (3sf) (1)

Question 6 continued

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Question 6 continued

(Total for Question 6 is 6 marks)

7. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

A curve has equation

$$x^3 + 2xy + 3y^2 = 47$$

(a) Find $\frac{dy}{dx}$ in terms of x and y

(4)

The point $P(-2, 5)$ lies on the curve.

(b) Find the equation of the normal to the curve at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(3)

a) $x^3 + 2xy + 3y^2 = 47$

$3x^2 + 2y + 2x\frac{dy}{dx} + 6y\frac{dy}{dx} = 0$ ① ①

$\frac{dy}{dx}(2x + 6y) = -3x^2 - 2y$ ①

$\frac{dy}{dx} = -\frac{3x^2 + 2y}{2x + 6y}$ ①

b) $P(-2, 5)$

$\left.\frac{dy}{dx}\right|_{\substack{x=-2\\y=5}} = -\frac{3(-2)^2 + 2(5)}{2(-2) + 6(5)} = -\frac{11}{13}$ ①

$\therefore\ m_{normal} = \frac{13}{11}$

$y - 5 = \frac{13}{11}(x + 2)$ ①

$11y - 55 = 13x + 26$

$13x - 11y + 81 = 0$ ①

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Question 7 continued

(Total for Question 7 is 7 marks)

8. (a) Express $2\cos\theta + 8\sin\theta$ in the form $R\cos(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

The first three terms of an arithmetic sequence are

$$\cos x \qquad \cos x + \sin x \qquad \cos x + 2\sin x \qquad x \neq n\pi$$

Given that S_9 represents the sum of the first 9 terms of this sequence as x varies,

(b) (i) find the exact maximum value of S_9

(ii) deduce the smallest positive value of x at which this maximum value of S_9 occurs.

(3)

a) $2\cos\theta + 8\sin\theta = R\cos(\theta - \alpha)$

$= R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

$R\cos\alpha = 2$ $\Rightarrow R = \sqrt{2^2 + 8^2} = 2\sqrt{17}$ (1)

$R\sin\alpha = 8$

$\tan\alpha = \frac{8}{2}$ (1) $\Rightarrow \alpha = 1.326$ rad (3dp) (1)

b) (i) $S_9 = \frac{9}{2}(2a + (9-1)d)$ $\quad a = \cos x$, $d = \sin x$

$= 4.5(2\cos x + 8\sin x)$

$= 4.5 \times 2\sqrt{17}\cos(\theta - 1.326)$

$\therefore$ max S_9 is when $\cos(\theta - 1.326) = 1$, and $S_9 = 4.5 \times 2\sqrt{17}$ (1)

max $S_9 = 9\sqrt{17}$ (1)

(ii) $\cos(\theta - 1.326) = 1$

$\theta - 1.326 = 0$

$\theta = 1.326$ (1)

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Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 6 marks)

9. The curve C has parametric equations

$$x = t^2 + 6t - 16 \qquad y = 6\ln(t+3) \qquad t > -3$$

(a) Show that a Cartesian equation for C is

$$y = A\ln(x+B) \qquad x > -B$$

where A and B are integers to be found.

(3)

The curve C cuts the y-axis at the point P

(b) Show that the equation of the tangent to C at P can be written in the form

$$ax + by = c\ln 5$$

where a, b and c are integers to be found.

(4)

a) Method: rearrange x to find $t+3$, then substitute into y.

$$x = t^2 + 6t - 16$$
$$x = (t+3)^2 - 9 - 16$$
$$x = (t+3)^2 - 25 \quad ①$$
$$(t+3)^2 = x + 25$$
$$(t+3) = (x+25)^{1/2} \quad ①$$

$$y = 6\ln(t+3) = 6\ln(x+25)^{1/2}$$

$$y = 3\ln(x+25) \quad ①$$

b) when $x = 0$, $y = 3\ln 25 = 6\ln 5$ ① $\quad 3\ln 25 = 3\ln 5^2 = 6\ln 5$

$$\frac{dy}{dx} = \frac{3}{x+25} = \frac{3}{0+25} \quad ①$$

$$y - 6\ln 5 = \frac{3}{25}(x - 0) \quad ①$$

$$25y - 150\ln 5 = 3x$$

$$25y - 3x = 150\ln 5 \quad ①$$

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Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 7 marks)

10. $$f(x) = \frac{3kx - 18}{(x+4)(x-2)} \qquad \text{where } k \text{ is a positive constant}$$

(a) Express $f(x)$ in partial fractions in terms of k.

(3)

(b) Hence find the exact value of k for which

$$\int_{-3}^{1} f(x)\,dx = 21$$

(4)

a) $f(x) = \dfrac{3kx-18}{(x+4)(x-2)} = \dfrac{A}{x+4} + \dfrac{B}{x-2}$

$3kx - 18 = A(x-2) + B(x+4)$ 1
$= Ax - 2A + Bx + 4B$
$= (A+B)x + (4B - 2A)$

x: $3k = A + B \Rightarrow B = 3k - A$ ①
constant: $-18 = -2A + 4B$ ②

sub ① into ②: $-18 = -2A + 4(3k - A)$ 1
$-18 = -2A + 12k - 4A$
$6A = 12k + 18$
$A = 2k + 3$

$B = 3k - (2k+3) = k - 3$

$f(x) = \dfrac{2k+3}{x+4} + \dfrac{k-3}{x-2}$ 1

b) $\displaystyle\int_{-3}^{1} f(x)\,dx = 21 \quad \therefore \int_{-3}^{1} \frac{2k+3}{x+4} + \frac{k-3}{x-2}\,dx = 21$

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Question 10 continued

$$\Big[(2k+3)\ln|x+4| + (k-3)\ln|x-2|\Big]_{-3}^{1} = 21$$

$$(2k+3)\ln 5 + \cancel{(k-3)\ln 1}^{\,0} - \cancel{(2k+3)\ln 1}^{\,0} - (k-3)\ln 5 = 21$$

$\ln 1 = 0$

$$(2k+3-k+3)\ln 5 = 21$$

$$(k+6)\ln 5 = 21$$

$$k+6 = \frac{21}{\ln 5}$$

$$k = \frac{21}{\ln 5} - 6$$

Question 10 continued

Question 10 continued

(Total for Question 10 is 7 marks)

11.

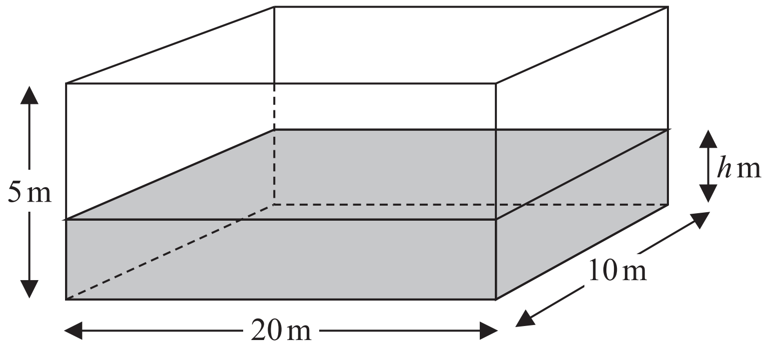

Figure 1

A tank in the shape of a cuboid is being filled with water.

The base of the tank measures 20 m by 10 m and the height of the tank is 5 m, as shown in Figure 1.

At time t minutes after water started flowing into the tank the height of the water was h m and the volume of water in the tank was V m^3

In a model of this situation

- the sides of the tank have negligible thickness
- the rate of change of V is inversely proportional to the square root of h

(a) Show that

$$\frac{dh}{dt} = \frac{\lambda}{\sqrt{h}}$$

where λ is a constant.

(3)

Given that

- initially the height of the water in the tank was 1.44 m
- exactly 8 minutes after water started flowing into the tank the height of the water was 3.24 m

(b) use the model to find an equation linking h with t, giving your answer in the form

$$h^{\frac{3}{2}} = At + B$$

where A and B are constants to be found.

(5)

(c) Hence find the time taken, from when water started flowing into the tank, for the tank to be completely full.

(2)

Question 11 continued

a) $V = 20 \times 10 \times h$

$V = 200h$

$\frac{dV}{dh} = 200$ (1)

$\frac{dV}{dt} = \frac{k}{\sqrt{h}}$

$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt} = \frac{1}{200} \times \frac{k}{\sqrt{h}}$ (1) $= \frac{\lambda}{\sqrt{h}}$ (1) (where $\lambda = \frac{k}{200}$)

b) when $t=0$, $h=1.44$

when $t=8$, $h=3.24$

$\frac{dh}{dt} = \frac{\lambda}{\sqrt{h}} \Rightarrow \int h^{1/2}\, dh = \int \lambda\, dt$ (1)

$\frac{2}{3}h^{3/2} = \lambda t + c$ (1)

sub in $t=0$, $h=1.44$

$\frac{2}{3}(1.44)^{3/2} = 0\lambda + c \Rightarrow c = 1.152$ (1)

sub in $t=8$, $h=3.24$

$\frac{2}{3}(3.24)^{3/2} = 8\lambda + 1.152 \Rightarrow \lambda = 0.342$ (1)

$\frac{2}{3}h^{3/2} = 0.342t + 1.152$

$h^{3/2} = 0.513t + 1.728$ (1)

c) tank is full when $h=5$

$(5)^{3/2} = 0.513t + 1.728$ (1)

$t = \frac{5\sqrt{5} - 1.728}{0.513} \Rightarrow t = 18.4$ minutes (3sf) (1)

Question 11 continued

Question 11 continued

(Total for Question 11 is 10 marks)

12.

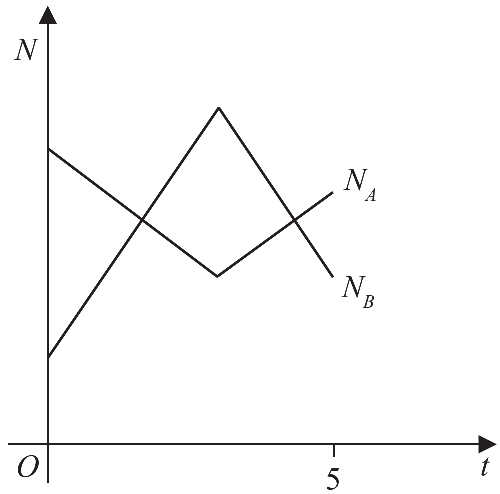

Figure 2

The number of subscribers to two different music streaming companies is being monitored.

The number of subscribers, N_A, in thousands, to **company A** is modelled by the equation

$$N_A = |t - 3| + 4 \qquad t \geqslant 0$$

where t is the time in years since monitoring began.

The number of subscribers, N_B, in thousands, to **company B** is modelled by the equation

$$N_B = 8 - |2t - 6| \qquad t \geqslant 0$$

where t is the time in years since monitoring began.

Figure 2 shows a sketch of the graph of N_A and the graph of N_B over a 5-year period.

Use the equations of the models to answer parts (a), (b), (c) and (d).

(a) Find the initial difference between the number of subscribers to **company A** and the number of subscribers to **company B**.

(2)

When $t = T$ **company A** reduced its subscription prices and the number of subscribers increased.

(b) Suggest a value for T, giving a reason for your answer.

(2)

(c) Find the range of values of t for which $N_A > N_B$ giving your answer in set notation.

(5)

(d) State a limitation of the model used for **company B**.

(1)

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Question 12 continued

a) initial ⇒ $t=0$

$N_A = |0-3|+4 = 7 \qquad N_B = 8-|0-6| = 2$

$N_A - N_B = 5$ (1) ∴ difference is 5000 subscribers (1)

b) find vertex of graph:

$N_A = |t-3|+4$

vertex has $|T-3| = 0 \therefore T = 3$ (1)

This was the point where the number of subscribers for A started increasing. (1)

c) find critical values

$|t-3|+4 = 8-|2t-6|$ (1)

$|2t-6| = |2(t-3)|$
$= |2||(t-3)| = 2|t-3|$

$|t+3|+4 = 8-2|t-3|$

$3|t-3| = 4$ (1)

$|t-3| = \frac{4}{3}$

$t-3 = \frac{4}{3} \qquad t-3 = -\frac{4}{3}$

$t = \frac{13}{3}$ (1) $\qquad t = \frac{5}{3}$

choose the outside region: $t < \frac{5}{3}$ or $t > \frac{13}{3}$ (1)

$\left\{t \in \mathbb{R}^+ : t < \frac{5}{3}\right\} \cup \left\{t \in \mathbb{R}^+ : t > \frac{13}{3}\right\}$ (1)

$\mathbb{R}^+$ = positive real numbers

Question 12 continued

d) eventually, the number of subscribers for B will become negative. 1

Question 12 continued

(Total for Question 12 is 10 marks)

13. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Find the first three terms, in ascending powers of x, of the binomial expansion of

$$(3 + x)^{-2}$$

writing each term in simplest form.

(4)

(b) Using the answer to part (a) and using algebraic integration, estimate the value of

$$\int_{0.2}^{0.4} \frac{6x}{(3 + x)^2}\, dx$$

giving your answer to 4 significant figures.

(4)

(c) Find, using algebraic integration, the exact value of

$$\int_{0.2}^{0.4} \frac{6x}{(3 + x)^2}\, dx$$

giving your answer in the form $a \ln b + c$, where a, b and c are constants to be found.

(5)

a) $(3+x)^{-2} = \left(3\left(1+\frac{x}{3}\right)\right)^{-2} = 3^{-2}\left(1+\frac{x}{3}\right)^{-2}$

1 1 1

$= \frac{1}{9}\left(1 - \frac{2x}{3} + \frac{(-2)(-2-1)}{2!}\left(\frac{x}{3}\right)^2 + \dots\right)$

$= \frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}$ 1

b) $\int 6x(3+x)^{-2}\,dx \approx \int 6x\left(\frac{1}{9} - \frac{2x}{27} - \frac{x^2}{27}\right)dx$

$= \int\left(\frac{2x}{3} - \frac{4x^2}{9} + \frac{2x^3}{9}\right)dx$ 1

Question 13 continued

$$\int_{0.2}^{0.4}\left(\frac{2x}{3}-\frac{4x^2}{9}+\frac{2x^3}{9}\right)dx=\left[\frac{x^2}{3}-\frac{4x^3}{27}+\frac{x^4}{18}\right]_{0.2}^{0.4} \quad 1$$

$$=\left(\frac{(0.4)^2}{3}-\frac{4(0.4)^3}{27}+\frac{(0.4)^4}{18}\right)-\left(\frac{(0.2)^2}{3}-\frac{4(0.2)^3}{27}+\frac{(0.2)^4}{18}\right) \quad 1$$

$$=\frac{223}{6570}$$

$$=0.03304\ (4sf) \quad 1$$

c) $\displaystyle\int_{0.2}^{0.4}\frac{6x}{(3+x)^2}\,dx$ let $u=3+x \Rightarrow x=u-3$, $du=dx$

limits: $(0.2, 0.4)\rightarrow(3.2, 3.4)$

$$=\int_{3.2}^{3.4}\frac{6(u-3)}{u^2}\,du=\int_{3.2}^{3.4}\left(\frac{6}{u}-18u^{-2}\right)du \quad 1$$

$$=\left[6\ln|u|+18u^{-1}\right]_{3.2}^{3.4} \quad 1 \quad 1$$

$$=\left(6\ln 3.4+\frac{18}{3.4}\right)-\left(6\ln 3.2+\frac{18}{3.2}\right) \quad 1 \quad =6\ln\left(\frac{17}{16}\right)-\frac{45}{136} \quad 1$$

Question 13 continued

Question 13 continued

(Total for Question 13 is 13 marks)

14. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$2\tan\theta(8\cos\theta + 23\sin^2\theta) = 8\sin 2\theta(1 + \tan^2\theta)$$

may be written as

$$\sin 2\theta(A\cos^2\theta + B\cos\theta + C) = 0$$

where A, B and C are constants to be found.

(3)

(b) Hence, solve for $360° \leqslant x \leqslant 540°$

$$2\tan x(8\cos x + 23\sin^2 x) = 8\sin 2x(1 + \tan^2 x) \qquad x \in \mathbb{R} \qquad x \neq 450°$$

(4)

a) $2\dfrac{\sin\theta}{\cos\theta}(8\cos\theta + 23(1-\cos^2\theta)) = 8\sin 2\theta \times \sec^2\theta$ $\quad \times\cos^2\theta$

$2\sin\theta\cos\theta(8\cos\theta + 23 - 23\cos^2\theta) = 8\sin 2\theta$ (1)

$\sin 2\theta(8\cos\theta + 23 - 23\cos^2\theta) = 8\sin 2\theta$ (1)

$\sin 2\theta(23\cos^2\theta - 8\cos\theta - 15) = 0$ (1)

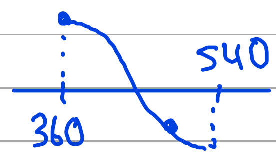
b) $\sin 2x(23\cos^2 x - 8\cos x - 15) = 0$

$\sin 2x = 0$ or $23\cos^2 x - 8\cos x - 15 = 0$

$\therefore x = 360°$ $\quad (\cos x - 1)(23\cos x + 15) = 0$

or $x = 540°$ (1)

$\cos x = 1 \Rightarrow x = 360°$



$\cos x = -\dfrac{15}{23}$ (1) $\Rightarrow x = 491°$ (3sf) (1)

$x = \{360, 491, 540\}$ (1)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 14 continued

Question 14 continued

Question 14 continued

(Total for Question 14 is 7 marks)

15. A student attempts to answer the following question:

> Given that x is an obtuse angle, use algebra to prove by contradiction that
>
> $$\sin x - \cos x \geqslant 1$$

The student starts the proof with:

> Assume that $\sin x - \cos x < 1$ when x is an obtuse angle
>
> $$\Rightarrow (\sin x - \cos x)^2 < 1$$
>
> $$\Rightarrow \ldots$$

The start of the student's proof is reprinted below.

Complete the proof.

(3)

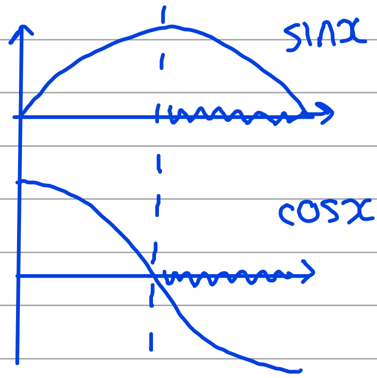
Assume that $\sin x - \cos x < 1$ when x is an obtuse angle

$$\Rightarrow (\sin x - \cos x)^2 < 1$$

$\sin^2 x - 2\sin x\cos x + \cos^2 x < 1$ ①

$-2\sin x\cos x < 0$

$\sin x\cos x > 0$ ①



If x is obtuse, then $\sin x > 0$ and $\cos x < 0$

therefore $\sin x\cos x < 0$. This is a contradiction

$\therefore \sin x - \cos x \geq 1$ ①

P 7 2 8 0 5 A 0 4 6 4 8

Question 15 continued

Question 15 continued

(Total for Question 15 is 3 marks)

TOTAL FOR PAPER IS 100 MARKS