Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Time 2 hours

Paper reference **9MA0/01**

Mathematics

Advanced
PAPER 1: Pure Mathematics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 16 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

1. The point $P(-2, -5)$ lies on the curve with equation $y = f(x)$, $x \in \mathbb{R}$

Find the point to which P is mapped, when the curve with equation $y = f(x)$ is transformed to the curve with equation

(a) $y = f(x) + 2$ **(1)**

(b) $y = |f(x)|$ **(1)**

(c) $y = 3f(x - 2) + 2$ **(2)**

(a) $y = f(x) + 2$ $f(x) = -5$ from question
$y = -5 + 2$
$y = -3$ the x value is not changed
P becomes $(-2, -3)$ 1

(b) $y = |f(x)|$
$y = |-5|$ ← $|a|$ takes the magnitude of a
$y = 5$
P becomes $(-2, 5)$ 1

(c) $y = 3f(x-2) + 2$

$x = -2$ ← $x - a$ changes the x-value by $+a$
$x' = -2 + 2$
$x' = 0$ 1

$y' = 3(-5) + 2$
$y' = -13$

P becomes $(0, -13)$ 1

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Question 1 continued

(Total for Question 1 is 4 marks)

2. $$f(x) = (x - 4)(x^2 - 3x + k) - 42 \quad \text{where } k \text{ is a constant}$$

Given that $(x + 2)$ is a factor of $f(x)$, find the value of k.

(3)

$f(-2) = 0$ 1 $\leftarrow$ $(x+2)$ is a factor of $f(x)$

$(-2-4)((-2)^2 - 3(-2) + k) - 42 = 0$

$-6(4 + 6 + k) = 42$

$-6(10 + k) = 42$ 1

$-60 - 6k = 42$

$-6k = 102$

$k = -17$ 1

Question 2 continued

(Total for Question 2 is 3 marks)

3. A circle has equation

$$x^2 + y^2 - 10x + 16y = 80$$

(a) Find

(i) the coordinates of the centre of the circle,

(ii) the radius of the circle.

(3)

Given that P is the point on the circle that is furthest away from the origin O,

(b) find the exact length OP

(2)

(a) (i) $x^2 + y^2 - 10x + 16y = 80$ 'complete the square' on x and y terms.

$(x-5)^2 + (y+8)^2 - 5^2 - 8^2 = 80$

$(x-5)^2 + (y+8)^2 = 169$ 1 $\quad x^2 + bx + c \downarrow \left(x+\frac{b}{2}\right)^2 + c - \left(\frac{b}{2}\right)^2$

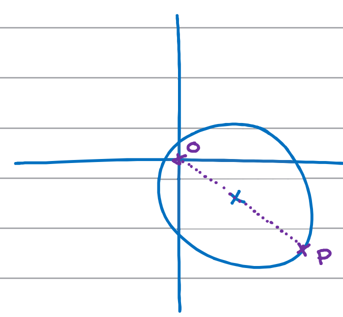
$\therefore$ centre $= (5, -8)$ 1

A circle with centre (a, b) and radius r has equation: $(x-a)^2 + (y-b)^2 = r^2$

(ii) radius $= \sqrt{169}$
$= 13$ 1

(b) furthest point will be origin $\rightarrow$ centre + radius.

length $= \underbrace{\sqrt{5^2 + (-8)^2}}_{\text{origin to centre}} + \underbrace{13}_{\text{radius}}$ 1



$= \sqrt{89} + 13$ 1

"exact" so leave in this form.

Question 3 continued

(Total for Question 3 is 5 marks)

4. (a) Express $\lim_{\delta x \to 0} \sum_{x=2.1}^{6.3} \frac{2}{x}\delta x$ as an integral.

(1)

(b) Hence show that

$$\lim_{\delta x \to 0} \sum_{x=2.1}^{6.3} \frac{2}{x}\delta x = \ln k$$

where k is a constant to be found.

(2)

(a)

$$\lim_{\delta x \to 0} \sum_{x=2.1}^{6.3} \frac{2}{x}\delta x = \int_{2.1}^{6.3} \frac{2}{x}\, dx \quad 1$$

(b)

$$\int_{2.1}^{6.3} \frac{2}{x}\, dx = \left[2\ln x\right]_{2.1}^{6.3}$$

$$= (2\ln 6.3) - (2\ln 2.1) \quad 1$$

$$= 2\ln\left(\frac{6.3}{2.1}\right)$$

$$= 2\ln 3$$

$$= \ln 3^2$$

$$= \ln 9 \quad 1$$

$$\therefore k = 9$$

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Question 4 continued

(Total for Question 4 is 3 marks)

5. The height, h metres, of a tree, t years after being planted, is modelled by the equation

$$h^2 = at + b \qquad 0 \leqslant t < 25$$

where a and b are constants.

Given that

- the height of the tree was 2.60 m, exactly 2 years after being planted
- the height of the tree was 5.10 m, exactly 10 years after being planted

(a) find a complete equation for the model, giving the values of a and b to 3 significant figures.
(4)

Given that the height of the tree was 7 m, exactly 20 years after being planted

(b) evaluate the model, giving reasons for your answer.
(2)

(a) $h^2 = at + b$

①: $h = 2.6,\ t = 2$ ← from first bullet point
$2.6^2 = 2a + b$ ①

②: $h = 5.1,\ t = 10$ ← from second bullet point
$5.1^2 = 10a + b$

①: $6.76 = 2a + b$
②: $26.01 = 10a + b$ ① solve simultaneously

② − ①: $19.25 = 8a$
$2.40625 = a$

$6.76 = 2 \times 2.40625$
$6.76 = 4.8125 + b$
$1.9475 = b$

$a = 2.41$ ① rounded to 2.d.p
$b = 1.95$

$\therefore h^2 = 2.41t + 1.95$ ①

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Question 5 continued

b) Height of the tree when $t = 20$ years :

$h^2 = 2.41(20) + 1.95$

$h^2 = 48.2 + 1.95$

$h^2 = 50.15$

$h = \sqrt{50.15}$

1

$= 7.08$ m $\therefore$ the model is good as 7.08 m is close to 7 m.

1

(Total for Question 5 is 6 marks)

6.

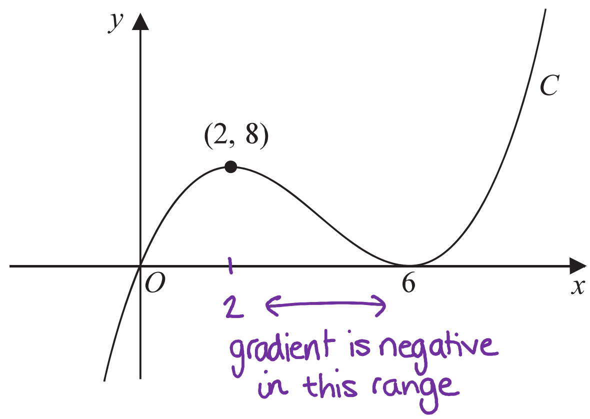


Figure 1

Figure 1 shows a sketch of a curve C with equation $y = \mathrm{f}(x)$ where $\mathrm{f}(x)$ is a cubic expression in x.

The curve

- passes through the origin
- has a maximum turning point at $(2, 8)$
- has a minimum turning point at $(6, 0)$

(a) Write down the set of values of x for which

$$\mathrm{f}'(x) < 0$$

(1)

The line with equation $y = k$, where k is a constant, intersects C at only one point.

(b) Find the set of values of k, giving your answer in set notation.

(2)

(c) Find the equation of C. You may leave your answer in factorised form.

(3)

(a) $2 < x < 6$ 1

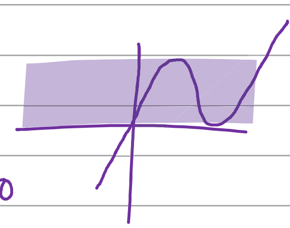
$\mathrm{f}'(x) < 0$ means the gradient is negative.
Negative gradient = line going down. ↘

(b) $k > 8$ or $k < 0$ 1

$\{k : k > 8\} \cup \{k : k < 0\}$ 1

$y = k$ is a horizontal line through the y-axis.

has to be outside of shaded region to intersect only once. →



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CHOOSE ONE OF THESE METHODS.

Question 6 continued

(c) Method 1 : Recognise curve has form $y = ax(x-6)^2$ ① states form of curve

$(2,8) \rightarrow \quad 8 = 2a(2-6)^2$ ①

$8 = 32a$

$a = \frac{1}{4}$

$\therefore y = \frac{1}{4}x(x-6)^2$ ①

Method 2 : Solving Simultaneous Equations

$y = ax^3 + bx^2 + cx$ ← no $+d$ because the curve goes through the origin.

when $x = 2$, $y = 8$:

$8 = a(2^3) + b(2^2) + c(2)$

① $4 = 4a + 2b + c$

when $x = 6$, $y = 0$:

$0 = a(6^3) + b(6^2) + c(6)$

② $0 = 216a + 36b + 6c$ ① for 2 sim. eq.

$f'(x) = 3ax^2 + 2bx + c$

when $x = 6$, $f'(x) = 0$: ← $(6,0)$ is a turning point

$0 = 3a(6^2) + 2b(6) + c$

③ $0 = 108a + 12b + c$

Solve ①, ②, ③ simultaneously : ← use a calculator or solve by hand.

$4 = 4a + 2b + c$

$0 = 216a + 36b + 6c$

$0 = 108a + 12b + c$

$a = \frac{1}{4}$, $b = -3$, $c = 9$ ① for solving sim. eq.

$y = \frac{1}{4}x^3 - 3x^2 + 9x$ ①

(Total for Question 6 is 6 marks)

7. (i) Given that p and q are integers such that

$$pq \text{ is even}$$

use algebra to prove by contradiction that at least one of p or q is even.

(3)

(ii) Given that x and y are integers such that

- $x < 0$
- $(x + y)^2 < 9x^2 + y^2$

show that $y > 4x$

(2)

(i) There exists integers p and q such that pq is even and p and q are both odd. 1

write out the contradiction.

Let $p = 2m+1$ and $q = 2n+1$

we know that $2\times$ any number is even by definition, therefore $2m+1$ is odd by definition.

$pq = (2m+1)(2n+1)$

$= 4mn + 2n + 2m + 1$ 1

$= 2(2mn + n + m) + 1$

1 This is of the form $2a+1$, so is odd.

This is a contradiction, therefore if pq is even, then at least one of p and q must be even.

(ii) $(x+y)^2 < 9x^2 + y^2$

$x^2 + 2xy + y^2 < 9x^2 + y^2$ $\quad -y^2, -x^2$

$2xy < 8x^2$ 1 $\quad \div x$

$2y < 8x$ $\quad -8x$

$2y - 8x < 0$

$x < 0$ so $8x < 0$. $2y - 8x > 0$, so $2y > 8x$.

$2y > 8x$ $\quad \div 2$

$y > 4x$ 1

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Question 7 continued

(Total for Question 7 is 5 marks)

8.

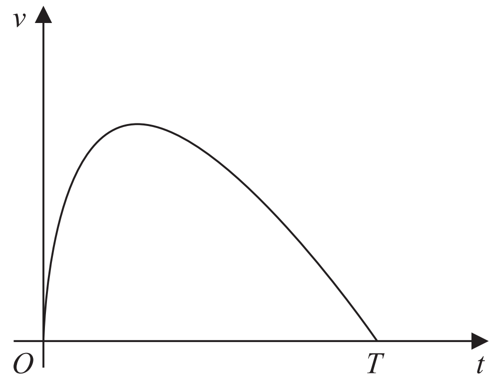

Figure 2

A car stops at two sets of traffic lights.

Figure 2 shows a graph of the speed of the car, $v\,\text{ms}^{-1}$, as it travels between the two sets of traffic lights.

The car takes T seconds to travel between the two sets of traffic lights.

The speed of the car is modelled by the equation

$$v = (10 - 0.4t)\ln(t + 1) \qquad 0 \leqslant t \leqslant T$$

where t seconds is the time after the car leaves the first set of traffic lights.

According to the model,

(a) find the value of T

(1)

(b) show that the maximum speed of the car occurs when

$$t = \frac{26}{1 + \ln(t + 1)} - 1$$

(4)

Using the iteration formula

$$t_{n+1} = \frac{26}{1 + \ln(t_n + 1)} - 1$$

with $t_1 = 7$

(c) (i) find the value of t_3 to 3 decimal places,

(ii) find, by repeated iteration, the time taken for the car to reach maximum speed.

(3)

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Question 8 continued

(a) $(10 - 0.4t)\ln(t+1) = 0$ ← $v = 0$ when $t = 0$ and when $t = T$.

$10\ln(t+1) - 0.4t\ln(t+1) = 0$ ⤵ $+ 0.4t\ln(t+1)$

$10\ln(t+1) = 0.4t\ln(t+1)$ ⤵ $\div \ln(t+1)$ this is okay because we know $v=0$ when $t=0$, so $T>0$. Then $T+1>0$, so $\ln(t+1) \neq 0$.

$10 = 0.4t$

$25 = t$

$\therefore T = 25$ (1)

(b) $v = (10 - 0.4t)\ln(t+1)$

let $v = f(t)g(t)$

then $v' = f(t)g'(t) + f'(t)g(t)$

$f(t) = 10 - 0.4t$ $\quad f'(t) = -0.4$

$g(t) = \ln(t+1)$ $\quad g'(t) = \frac{1}{t+1}$

$\frac{dv}{dt} = \ln(t+1) \times -0.4 + (10 - 0.4t) \times \frac{1}{t+1}$ (2)

$0 = -0.4\ln(t+1) + \frac{10 - 0.4t}{t+1}$ (1) ← max speed when gradient is 0 (at turning point)

$\frac{10 - 0.4t}{t+1} = 0.4\ln(t+1)$ ⤵ $\times (t+1)$

$10 - 0.4t = 0.4\ln(t+1) \times (t+1)$ ⤵ $+ 0.4t$

$10 = 0.4t\ln(t+1) + 0.4\ln(t+1) + 0.4t$ ⤵ $\div 0.4$

$25 = t\ln(t+1) + \ln(t+1) + t$ ⤵ factorise

$25 = t(\ln(t+1) + 1) + \ln(t+1)$ ⤵ $-\ln(t+1)$

$25 - \ln(t+1) = t(\ln(t+1) + 1)$ ⤵ $\div (1 + \ln(t+1))$

$\frac{25 - \ln(t+1)}{1 + \ln(t+1)} = t$

Question 8 continued

$$\frac{26}{1+\ln(t+1)} - 1 = t \quad 1$$

(c) $$t_{n+1} = \frac{26}{1+\ln(t_n+1)} - 1$$

$$t_1 = 7$$

$$t_2 = \frac{26}{1+\ln(7+1)} - 1 \;1 = 7.298 \;1$$

$$t_3 = \frac{26}{1+\ln(7.298+1)+1} = 7\ 33 \quad 1$$

$t_3 = 7.33$ seconds

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Question 8 continued

(Total for Question 8 is 8 marks)

9.

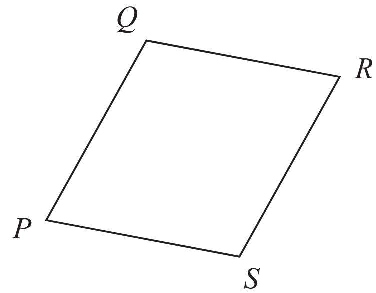


Figure 3

Figure 3 shows a sketch of a parallelogram *PQRS*.

Given that

- $\overrightarrow{PQ} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$ ← bold letters represent vectors
- $\overrightarrow{QR} = 5\mathbf{i} - 2\mathbf{k}$

(a) show that parallelogram *PQRS* is a rhombus. ← all 4 sides are the same length **(2)**

(b) Find the exact area of the rhombus *PQRS*. **(4)**

(a) $|\overrightarrow{PQ}| = \sqrt{2^2 + 3^2 + (-4)^2}$ ← $|v|$ is the magnitude (length) of v.

$= \sqrt{29}$

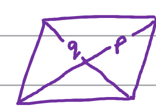
$|\overrightarrow{QR}| = \sqrt{5^2 + (-2)^2}$ 1

$= \sqrt{29}$

Since we know PQRS is a parallelogram, we only need to calculate the length of 2 of the 4 sides

$|\overrightarrow{PQ}| = |\overrightarrow{QR}|$ ∴ PQRS is a rhombus. 1

area of a rhombus $= \dfrac{p \times q}{2}$



(b) $\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR}$

$= (2i + 3j - 4k) + (5i - 2k)$

$= 7i + 3j - 6k$ 1

$\overrightarrow{QS} = -\overrightarrow{PQ} + \overrightarrow{PS}$ ← we're going 'backwards' along PQ.

$= -(2i + 3j - 4k) + (5i - 2k)$ ← $\overrightarrow{PS} = \overrightarrow{QR}$

$= 3i - 3j + 2k$ 1

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Question 9 continued

$$\text{Area} = \frac{|\overrightarrow{PR}| \times |\overrightarrow{QS}|}{2} \quad (1)$$

$$= \frac{\sqrt{7^2 + 3^2 + (-6)^2} \times \sqrt{3^2 + (-3)^2 + 2^2}}{2}$$

$$\therefore \text{Area} = \sqrt{517} \quad (1)$$

Question 9 continued

Question 9 continued

(Total for Question 9 is 6 marks)

10. A scientist is studying the number of bees and the number of wasps on an island.

The number of bees, measured in thousands, N_b, is modelled by the equation

$$N_b = 45 + 220\,e^{0.05t}$$

where t is the number of years from the start of the study.

According to the model,

(a) find the number of bees at the start of the study,

(1)

(b) show that, exactly 10 years after the start of the study, the number of bees was increasing at a **rate** of approximately 18 thousand per year.

(3)

The number of wasps, measured in thousands, N_w, is modelled by the equation

$$N_w = 10 + 800\,e^{-0.05t}$$

where t is the number of years from the start of the study.

When $t = T$, according to the models, there are an equal number of bees and wasps.

(c) Find the value of T to 2 decimal places.

(4)

(a) when $t = 0$:

$N_b = 45 + 220e^{0.05\times0}$

$= 45 + 220e^{0}$ ← $e^{0} = 1$

$= 45 + 220$

$= 265$

265 thousand 1

(b) $\frac{dN_b}{dt} = 0.05 \times 220 \times e^{0.05t}$ ← differentiate w.r.t time to get rate of change.

$= 11e^{0.05t}$ 1 ← $\frac{d}{dx}e^{ax} = ae^{ax}$

when $t = 10$.

$\frac{dN_b}{dt} = 11e^{0.05\times10}$ 1

$= 18.135...$

which is approximately 18 thousand bees per year 1

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Question 10 continued

(c) when $t = T$, $N_b = N_w$:

$$45 + 220e^{0.05t} = 10 + 800e^{-0.05t}$$ (1) $-10,\ -800e^{-0.05t}$

$$220e^{0.05t} + 35 - 800e^{-0.05t} = 0$$ $\times e^{0.05t}$

(1) $$220(e^{0.05t})^2 + 35e^{0.05t} - 800 = 0$$

Do this to remove the $e^{-0.05t}$ term.

$e^{0.05t} \times e^{-0.05t} = e^0 = 1$

This is a quadratic $220x^2 + 35x - 800 = 0$ with $x = e^{0.05t}$. Solve with calculator.

$$e^{0.05t} = 1.829,\ -1.988$$ ignore negative result because e^n cannot be negative

$$0.05t = \ln(1.829)$$ (1)

$$t = 12.08 \quad (2dp)$$

$\therefore T = 12.08$ years (1)

Question 10 continued

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Question 10 continued

(Total for Question 10 is 8 marks)

11.

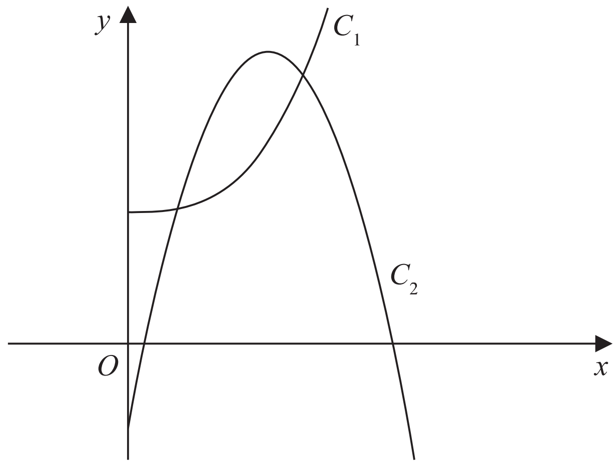

Figure 4

Figure 4 shows a sketch of part of the curve C_1 with equation

$$y = 2x^3 + 10 \qquad x > 0$$

and part of the curve C_2 with equation

$$y = 42x - 15x^2 - 7 \qquad x > 0$$

(a) Verify that the curves intersect at $x = \frac{1}{2}$

(2)

The curves intersect again at the point P

(b) Using algebra and showing all stages of working, find the exact x coordinate of P

(5)

(a) when $x = \frac{1}{2}$:

C_1: $y = 2(\frac{1}{2})^3 + 10$

$= \frac{41}{4}$

C_2: $y = 42(\frac{1}{2}) - 15(\frac{1}{2})^2 - 7$ 1

$= \frac{41}{4}$

$\therefore$ C_1 and C_2 intersect at $(\frac{1}{2}, \frac{41}{4})$ 1

(b) $2x^3 + 10 = 42x - 15x^2 - 7$ 1

$2x^3 + 15x^2 - 42x + 17 = 0$

$2x - 1$ is a factor of this equation – this could be deduced by inspection, trial-and-error or any other valid method.

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Question 11 continued

$$
\begin{array}{r|l}
 & x^2 + 8x - 17 \\
\hline
2x - 1 & 2x^3 + 15x^2 - 42x + 17 \\
 & 2x^3 - x^2 \\
 & 0 + 16x^2 \\
 & \quad 16x^2 - 8x \\
 & \quad 0 - 34x \\
 & \quad\quad -34x + 17 \\
 & \quad\quad 0 + 0
\end{array}
$$

(1)

← you don't have to do long division - inspection or other valid algebraic methods are accepted.

$2x^3 + 15x^2 - 42x + 17 = 0 \Rightarrow (2x-1)(x^2 + 8x - 17) = 0$ (1)

$2x - 1 = 0 \Rightarrow x_1 = \frac{1}{2}$

solve $x^2 + 8x - 17$ using a calculator, or the quadratic equation.

from $x^2 + 8x - 17$:

$x_2 = -4 + \sqrt{33}$

$x_3 = -4 - \sqrt{33}$ (1)

($x = \frac{1}{2}$ is the other intercept)

The point P is on the positive side of the y-axis, therefore:

$x = -4 + \sqrt{33}$ (1)

Question 11 continued

Question 11 continued

(Total for Question 11 is 7 marks)

12. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Show that

$$\int_1^{e^2} x^3 \ln x \, dx = ae^8 + b$$

where a and b are rational constants to be found.

(5)

$u = \ln x \Rightarrow \frac{du}{dx} = \frac{1}{x}$

$du = \frac{1}{x} dx$

integration by parts:

$\int u \, dv = uv - \int v \, du$

$dv = x^3 \Rightarrow v = \frac{1}{4}x^4$

choose $\ln x$ as u because it is much easier to differentiate than integrate.

$\int x^3 dx = \frac{1}{4}x^4$

$\int u \, dv = uv - \int v \, du$

$\int_1^{e^2} x^3 \ln x \, dx = \left[\ln x \times \frac{1}{4}x^4\right]_1^{e^2} - \int_1^{e^2} \frac{1}{x} \times \frac{x^4}{4} \, dx$ (1) (1)

$\frac{1}{x} \times \frac{x^4}{4} = \frac{x^3}{4}$

$= \left[\frac{x^4}{4}\ln x - \frac{x^4}{16}\right]_1^{e^2}$ (1)

$= \left(\frac{e^{2^4}}{4}\ln(e^2) - \frac{e^{2^4}}{16}\right) - \left(\frac{1^4}{4}\ln 1 - \frac{1^4}{16}\right)$ (1)

$\ln e^2 = 2$

$\ln 1 = 0$

$= \left(\frac{2e^8}{4} - \frac{e^8}{16}\right) - \left(-\frac{1^4}{16}\right)$

$= \frac{7}{16}e^8 + \frac{1}{16}$ (1)

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Question 12 continued

(Total for Question 12 is 5 marks)

13. (i) In an arithmetic series, the first term is a and the common difference is d.

Show that

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

(3)

(ii) James saves money over a number of weeks to buy a printer that costs £64

He saves £10 in week 1, £9.20 in week 2, £8.40 in week 3 and so on, so that the weekly amounts he saves form an arithmetic sequence.

Given that James takes n weeks to save exactly £64

(a) show that

$$n^2 - 26n + 160 = 0$$

(2)

(b) Solve the equation

$$n^2 - 26n + 160 = 0$$

(1)

(c) Hence state the number of weeks James takes to save enough money to buy the printer, giving a brief reason for your answer.

(1)

(i) $S = a + (a+d) + (a+d+d) + \ldots + (a+(n-1)d)$ 1

reverse order of terms

$S = (a+(n-1)d) + (a+(n-2)d) + \ldots + a$

add sequences by adding pairs of terms in each position

$2S = (a + a + (n-1)d) + (a + 2d + a + (n-2)d) \ldots$ 1

for reversing + adding

$2S = 2a + (n-1)d + 2a + (n-1)d \ldots$

$2S = n \times (2a + (n-1)d)$

$S = \frac{1}{2}n(2a + (n-1)d)$ 1 as required.

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Question 13 continued

(ii) (a) $a = 10$ ← first term

$d = 9.2 - 10 = -0.8$ ← common difference

$64 = \frac{n}{2}(2\times10 + (n-1)\times -0.8)$ 1

$128 = n(20 - 0.8n + 0.8)$

$128 = 20.8n - 0.8n^2$

$0.8n^2 - 20.8n + 128 = 0$ $\div 0.8$

$n^2 - 26n + 160 = 0$ as required. 1

(b) $(n-10)(n-16) = 0$ ← or use calculator / quadratic equation

$\therefore n = 10$ and $n = 16$ 1

(c) 10 weeks – by 10 weeks he will have saved enough money, so he wouldn't need to save for 6 more weeks. 1

Question 13 continued

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Question 13 continued

(Total for Question 13 is 7 marks)

14. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Given that

$$2\sin(x - 60°) = \cos(x - 30°)$$

show that

$$\tan x = 3\sqrt{3}$$

(4)

(b) Hence or otherwise solve, for $0 \leqslant \theta < 180°$

$$2\sin 2\theta = \cos(2\theta + 30°)$$

giving your answers to one decimal place.

(4)

from formula book:

$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

$\cos(A \pm B) = \sin A\sin B \mp \cos A\cos B$

(a) $\sin(x-60) = \sin x\cos 60 - \sin 60\cos x$

$\cos(x-30) = \sin x\sin 30 + \cos x\cos 30$ 1

$2\sin x\cos 60 - 2\sin 60\cos x = \sin x\sin 30 + \cos x\cos 30$ 1

$\sin 60 = \frac{\sqrt{3}}{2}$ $\quad \sin 30 = \frac{1}{2}$ $\quad \cos 60 = \frac{1}{2}$ $\quad \cos 30 = \frac{\sqrt{3}}{2}$

$\left(2\sin x \times \frac{1}{2}\right) - \left(2 \times \frac{\sqrt{3}}{2} \times \cos x\right) = \frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x$ 1

$\sin x - \sqrt{3}\cos x = \frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x$

collect sin and cos terms

$\frac{1}{2}\sin x = \left(\frac{\sqrt{3}}{2} + \sqrt{3}\right)\cos x$

$\times 2$

$\sin x = \sqrt{3} + 2\sqrt{3}\cos x$

$\sin x = 3\sqrt{3}\cos x$

$\frac{\sin x}{\cos x} = \tan x$ $\qquad \frac{\cos x}{\cos x} = 1$

$\tan x = 3\sqrt{3}$ 1

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Question 14 continued

(b) Using part (a):

$x - 60 = 2\theta$ $\quad\quad$ $x - 30 = 2\theta + 30$

$x = 2\theta + 60$ (1) $\quad\quad$ $x = 2\theta + 60$

So we can use $\tan(x) = 3\sqrt{3}$ with $x = 2\theta + 60$

$\tan(2\theta + 60) = 3\sqrt{3}$

$2\theta + 60 = \tan^{-1}(3\sqrt{3})$

$2\theta + 60 = 79.1$ (1)

$2\theta = 19.1$

$\theta = 9.6$ (1) ← tan repeats every $90°$

$\theta = 9.6°, 99.6°$ (1)

$9.6 + 90 = 99.6$

the next value (189.6) will be outside the given range of $0 \leq \theta \leq 180$

Question 14 continued

Question 14 continued

(Total for Question 14 is 8 marks)

15.

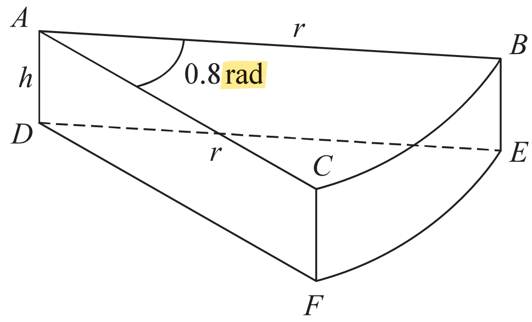

Figure 5

A company makes toys for children.

Figure 5 shows the design for a solid toy that looks like a piece of cheese.

The toy is modelled so that

- face ABC is a sector of a circle with radius r cm and centre A
- angle $BAC = 0.8$ radians
- faces ABC and DEF are congruent
- edges AD, CF and BE are perpendicular to faces ABC and DEF
- edges AD, CF and BE have length h cm

Given that the volume of the toy is $240\,\text{cm}^3$

(a) show that the surface area of the toy, $S\,\text{cm}^2$, is given by

$$S = 0.8r^2 + \frac{1680}{r}$$

making your method clear.

(4)

Using algebraic differentiation,

(b) find the value of r for which S has a stationary point.

(4)

(c) Prove, by further differentiation, that this value of r gives the minimum surface area of the toy.

(2)

(a) $\underbrace{\frac{1}{2} \times 0.8 \times r^2}_{\text{area of sector}} \times \underbrace{h}_{\text{height}} = \underbrace{240}_{\text{volume}}$ 1

$\frac{1}{2}\theta r^2$ = area of sector (when θ is in radians)

$0.4r^2h = 240$ $\div 0.4$

$r^2h = 600$

$h = \frac{600}{r^2}$ $\div r^2$ 1

P 6 9 6 0 1 A 0 4 2 4 8

Question 15 continued

total surface area = 2 × area of sector face + 2 × area of sector length + area of arc

$$S = 2\left(\tfrac{1}{2}\theta r^2\right) + 2(rh) + (r\theta \times h)$$

$$S = 0.8r^2 + 2rh + 0.8rh$$

$$h = \frac{600}{r^2}$$

$$S = 0.8r^2 + 2r\left(\frac{600}{r^2}\right) + 0.8r\left(\frac{600}{r^2}\right) \quad 1$$

$$S = 0.8r^2 + \frac{1200}{r} + \frac{480}{r}$$

$$S = 0.8r^2 + \frac{1680}{r} \quad 1$$

(b) $S = 0.8r^2 + 1680r^{-1}$ $\qquad \frac{1}{x} = x^{-1}$

$$\frac{dS}{dr} = 0.8 \times 2r^{2-1} + (-1) \times 1680r^{-1-1}$$

$$= 1.6r - 1680r^{-2} \quad 2$$

$$0 = 1.6r - \frac{1680}{r^2} \quad 1 \leftarrow \frac{dS}{dr} = 0 \text{ at stationary point}$$

$$1.6r = \frac{1680}{r^2}$$

$\times r^2$

$$1.6r^3 = 1680$$

$\div 1.6$

$$r^3 = 1050$$

$\sqrt[3]{\ }$

$$r = \sqrt[3]{1050}$$

$$r = 10.16 \quad 1$$

Question 15 continued

(c) $\frac{dS}{dr} = 1.6r - 1680r^{-2}$

$\frac{d^2S}{dr^2} = 1.6 \times 1r^{1-1} - (-2) \times 1680r^{-2-1}$

$= 1.6 + 3360r^{-3}$

$= 1.6 + \frac{3360}{r^3}$

when $r = 10.16$: ← from part (b), stationary point at $r = 10.16$

$1.6 + \frac{3360}{(10.16)^3} = 4.80$ 1

$\frac{d^2S}{dr^2} > 0$ when $r = 10.16$ therefore this is a minimum value of S. 1

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Question 15 continued

(Total for Question 15 is 10 marks)

16.

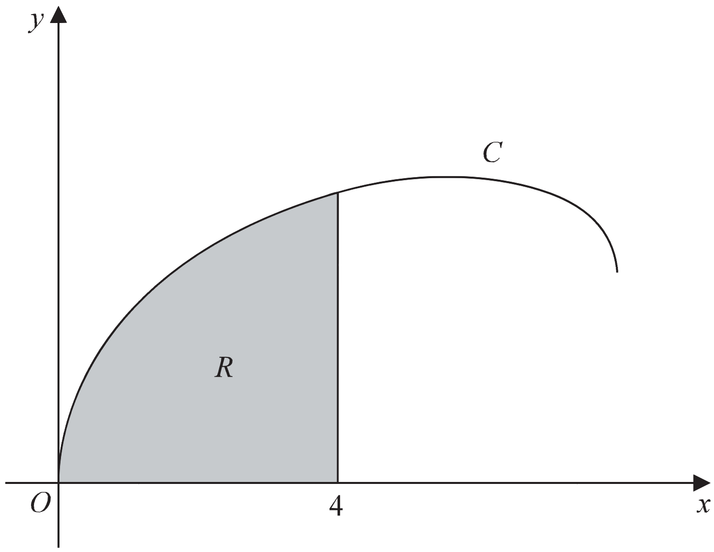

Figure 6

Figure 6 shows a sketch of the curve C with parametric equations

$$x = 8\sin^2 t \qquad y = 2\sin 2t + 3\sin t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The region R, shown shaded in Figure 6, is bounded by C, the x-axis and the line with equation $x = 4$

(a) Show that the area of R is given by

$$\int_0^a \left(8 - 8\cos 4t + 48\sin^2 t\cos t\right) dt$$

where a is a constant to be found.

(5)

(b) Hence, using algebraic integration, find the exact area of R.

(4)

(a) $R = \int_q^p y \frac{dx}{dt}\, dt$

$x = 8\sin^2 t$

$\frac{dx}{dt} = 8 \times 2\sin t\cos t$

$= 16\sin t \cos t$

$\frac{d}{dx}\sin^2 x = 2\sin x\cos x$ using the chain rule with $u = \sin x$

$y \times \frac{dx}{dt} = (2\sin 2t + 3\sin t) \times 16\sin t\cos t$ 1

Question 16 continued

$$y \times \frac{dx}{dt} = \left[2(\sin t\cos t + \cos t\sin t) + 3\sin t\right] \times 16\sin t\cos t$$

$$= (4\sin t\cos t + 3\sin t) \times 16\sin t\cos t$$

$$= (64\sin^2 t\cos t^2 + 48\sin^2 t\cos t)$$

$$R = \int_0^a y\frac{dx}{dt}\,dt = \int_0^a 64\sin^2 t\cos t^2 + 48\sin^2 t\cos t \ dt \quad (1)$$

We need to find a way to simplify this into the required form.

$$\cos 4t = 2\cos^2 2t - 1$$
$$= 2(1-\sin^2 2t) - 1$$
$$= 2 - 2\sin^2 2t - 1$$
$$= 1 - 2\sin^2 2t$$
$$= 1 - 2(\sin 2t\sin 2t)$$
$$= 1 - 2(2\sin t\cos t \times 2\sin t\cos t)$$
$$= 1 - 2(4\sin^2 t\cos^2 t)$$
$$= 1 - 8\sin^2 t\cos^2 t \quad (1)$$

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

using $\sin(2x) = 2\sin x\cos x$

$$8\sin^2 t\cos^2 t = 1 - \cos 4t$$

$64\sin^2 t\cos^2 t = 8(1-\cos 4t)$

$$R = \int_0^a 8 - 8\cos 4t + 48\sin^2 t\cos t \ dt \quad (1)$$

$$a = \frac{\pi}{4} \quad (1)$$

Finding the new domain :

$$R = \int_0^4 y\,dx = \int y\frac{dx}{dt}\,.\,dt$$

$x = 8\sin^2 t \longrightarrow \frac{dx}{dt} = 16\sin t\cos t$

when $x = 0$, $8\sin^2 t = 0$, Hence, $t = 0$

when $x = 4$, $8\sin^2 t = 4 \longrightarrow \sin^2 t = \frac{1}{2}$

$t = \sin^{-1}\sqrt{\frac{1}{2}} = \pi/4$

Question 16 continued

(b) $\int_0^{\frac{\pi}{4}} 8 - 8\cos 4t + 48\sin^2 t\cos t \, dt = 8t - 2\sin 4t + 16\sin^3 t$ (2)

$\left[8t - 2\sin 4t + 16\sin^3 t\right]_0^{\frac{\pi}{4}} = \left[8\left(\frac{\pi}{4}\right) - 2\sin\left(4\times\frac{\pi}{4}\right) + 16\sin^3\left(\frac{\pi}{4}\right)\right]$

$- \left[8(0) - 2\sin(4\times 0) + 16\sin^3(0)\right]$ (1)

$= 2\pi + 4\sqrt{2}$ (1)

(Total for Question 16 is 9 marks)

TOTAL FOR PAPER IS 100 MARKS