Pearson Edexcel Level 3

GCE Mathematics

Advanced

Paper 2: Pure Mathematics

PMT Mock 3

Time: 2 hours	Paper Reference(s) 9MA0/02

You must have:
Mathematical Formulae and Statistical Tables, calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 14 questions in this paper. The total mark is 100.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. Given that a is a positive constant,

a. Sketch the graph with equation

$$y = |a - 2x|$$

Show on your sketch the coordinates of each point at which the graph crosses the x-axis and y-axis.

(2)

b. Solve the inequality $|a - 2x| > x + 2a$

(3)

(Total for Question 1 is 5 marks)

2. Solve

$$4^{x-3} = 6$$

giving your answer in the form $a + b\log_2 3$, where a and b are constants to be found.

(4)

(Total for Question 2 is 4 marks)

3. Given that

$$y = \frac{1}{3}x^3$$

use differentiation from first principle to show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x^2$$

(3)

(Total for Question 3 is 3 marks)

4. A sequence a_1, a_2, a_3 is defined by

$$a_n = \sin^2\left(\frac{n\pi}{3}\right)$$

Find the exact values of

a. i) a_1
   ii) a_2
   iii) a_3

(3)

b. Hence find the exact value of

$$\sum_{n=1}^{100}\left\{n + \sin^2\left(\frac{n\pi}{3}\right)\right\}$$

(3)

(Total for Question 4 is 6 marks)

5. The table below shows corresponding values of x and y for $y = \log_3(x)$
The values of y are given to 2 decimal places as appropriate.

x	3	4.5	6	7.5	9
y	1	1.37	1.63	1.83	2

a. Obtain an estimate for $\int_3^9 \log_3(x) \ \mathrm{d}x$, giving your answer to two decimal places.

(3)

Use your answer to part (a) and making your method clear, estimate

b. i) $\int_3^9 \log_3\sqrt{x} \ \mathrm{d}x$

ii) $\int_3^{18} \log_3(9x^3) \ \mathrm{d}x$

(3)

(Total for Question 5 is 6 marks)

6.

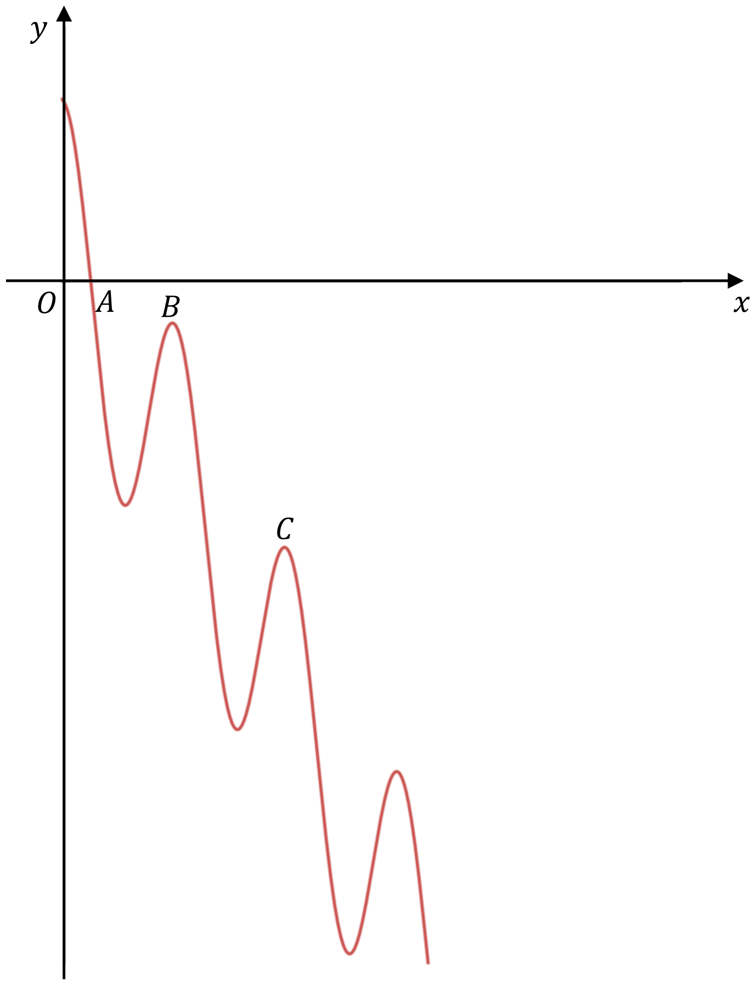

Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$f(x) = 4\cos 2x - 2x + 1 \qquad x > 0$$

and where x is measured in radians.

The curve crosses the x-axis at the point A, as shown in figure 1.

Given that x-coordinate of A is α

a. show that α lies between 0.7 and 0.8

(2)

Given that x-coordinates of B and C are β and γ respectively and they are two smallest values of x at which local maxima occur

b. find, using calculus, the value of β and the value of γ, giving your answers to 3 significant figures.

(5)

c. taking $x_0 = 0.7$ or 0.8 as a first approximation to α, apply the Newton-Raphson method once to $f(x)$ to obtain a second approximation to α. Show, your method and give your answer to 2 significant figures.

(2)

(Total for Question 6 is 9 marks)

7. a. Use the binomial theorem to expand

$$(8-3x)^{\frac{2}{3}}$$

in ascending powers of x, up to and including the term x^3, as a fully simplifying each term.

(4)

Edward, a student decides to use the expansion with $x = \frac{1}{3}$ to find an approximation for $(7)^{\frac{2}{3}}$. Using the answer to part (a) and without doing any calculations,

b. explain clearly whether Edward's approximation will be an overestimate, or, an underestimate.

(1)

(Total for Question 7 is 5 marks)

8.

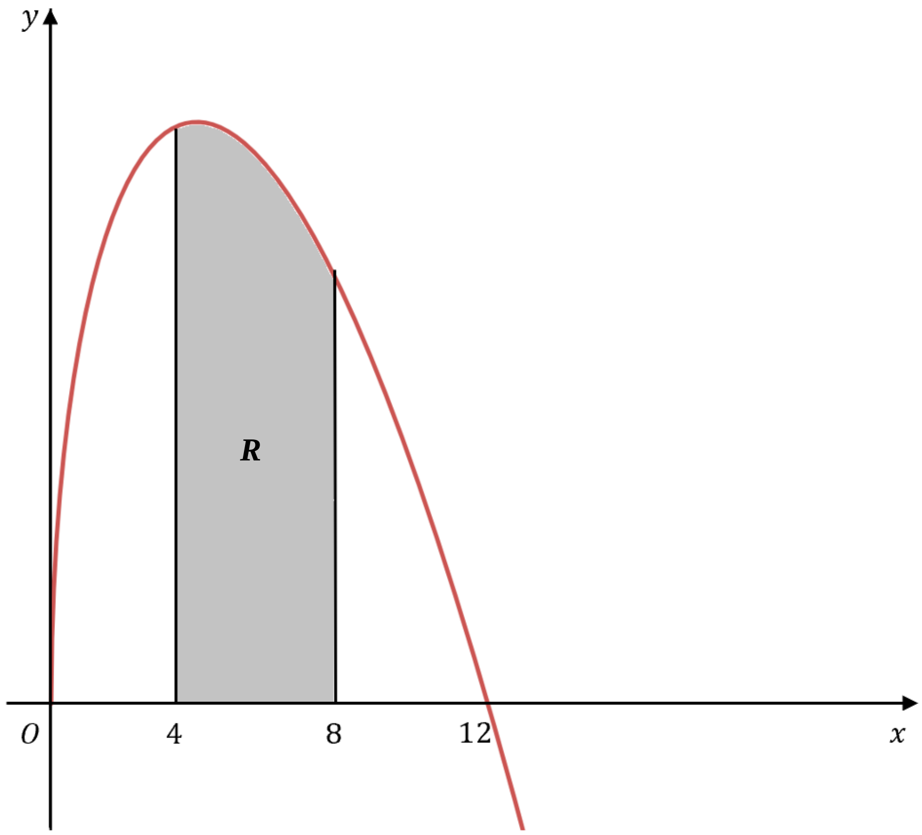

Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = \frac{12x - x^2}{\sqrt{x}}, \quad x > 0$$

The region R, shows shaded in figure 2, is bounded by the curve, the line with equation $x = 4$, the x-axis and the line with equation $x = 8$.

Show that the area of the shaded region R is $\frac{128}{5}(3\sqrt{2} - 2)$.

(5)

(Total for Question 8 is 5 marks)

9. $$\mathrm{f}(\theta) = 4\cos\theta + 5\sin\theta \qquad \theta \in R$$

a. Express $\mathrm{f}(\theta)$ in the form $R\cos(\theta - \alpha)$ where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give the exact value of R and give the value of α, in radians, to 3 decimal places.

(3)

Given that

$$\mathrm{g}(\theta) = \frac{135}{4 + \mathrm{f}(\theta)^2} \qquad \theta \in R$$

b. find the range of g.

(2)

(Total for Question 9 is 5 marks)

10. The functions f and g are defined with their respective domains by

$$f(x) = 4 - x^2 \qquad x \in R \quad x \geq 0$$

$$g(x) = \frac{2}{x+1} \qquad x \in R \quad x \geq 0$$

a. Write down the range of f.

(1)

b. Find the value of fg(3)

(2)

c. Find $g^{-1}(x)$

(3)

(Total for Question 10 is 6 marks)

11. Prove, using algebra that

$$n^2 + 1$$

is not divisible by 4.

(4)

(Total for Question 11 is 4 marks)

12. A curve has equation $y = \frac{2xe^x}{x+k}$ where k is a positive constant.

i. Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{e^x(2x^2+2kx+2k)}{(x+k)^2}$

(3)

ii. Given that the curve has exactly one stationary point find the value of k.

(3)

(Total for Question 12 is 6 marks)

13. Relative to a fixed origin O

- the point P has position vector $(0, -1, 2)$
- the point Q has position vector $(1, 1, 5)$
- the point R has position vector $(3, 5, m)$

where m is a constant.

Given that P, Q and R lie on a straight line,

a. find the value of m

(3)

The line segment OQ is extended to a point T so that $\overrightarrow{RT}$ is parallel to $\overrightarrow{OP}$

b. Show that $|\overrightarrow{OT}| = 9\sqrt{3}$.

(3)

(Total for Question 13 is 6 marks)

14. a. Express $\frac{1}{(3-x)(1-x)}$ in partial fractions.

(2)

A scientist is studying the mass of a substance in a laboratory.
The mass, *x* grams, of a substance at time *t* seconds after a chemical reaction starts is modelled by the differential equation

$$2\frac{\mathrm{d}x}{dt} = (3-x)(1-x) \quad t \geq 0,\ 0 \leq x < 1$$

Given that when $t = 0, x = 0$

b. solve the differential equation and show that the solution can be written as

$$x = \frac{3(e^t - 1)}{3e^t - 1}$$

(5)

c. Find the mass, *x* grams, which has formed 2 seconds after the start of the reaction. Give your answer correct to 3 significant figures.

(1)

d. Find the limiting value of *x* as *t* increases.

(1)

(Total for Question 14 is 9 marks)

15. The first three terms of a geometric series where θ is a constant are

$$-8\sin\theta, \quad 3-2\cos\theta \qquad \text{and} \qquad 4\cot\theta$$

a. Show that $4\cos^2\theta + 20\cos\theta + 9 = 0$

(3)

Given that θ lies in the interval $90^0 < \theta < 180^0$,

b. Find the value of θ.

(2)

c. Hence prove that this series is convergent.

(2)

d. Find S_∞, in the form $a(1-\sqrt{3})$

(2)

(Total for Question 15 is 9 marks)

16.

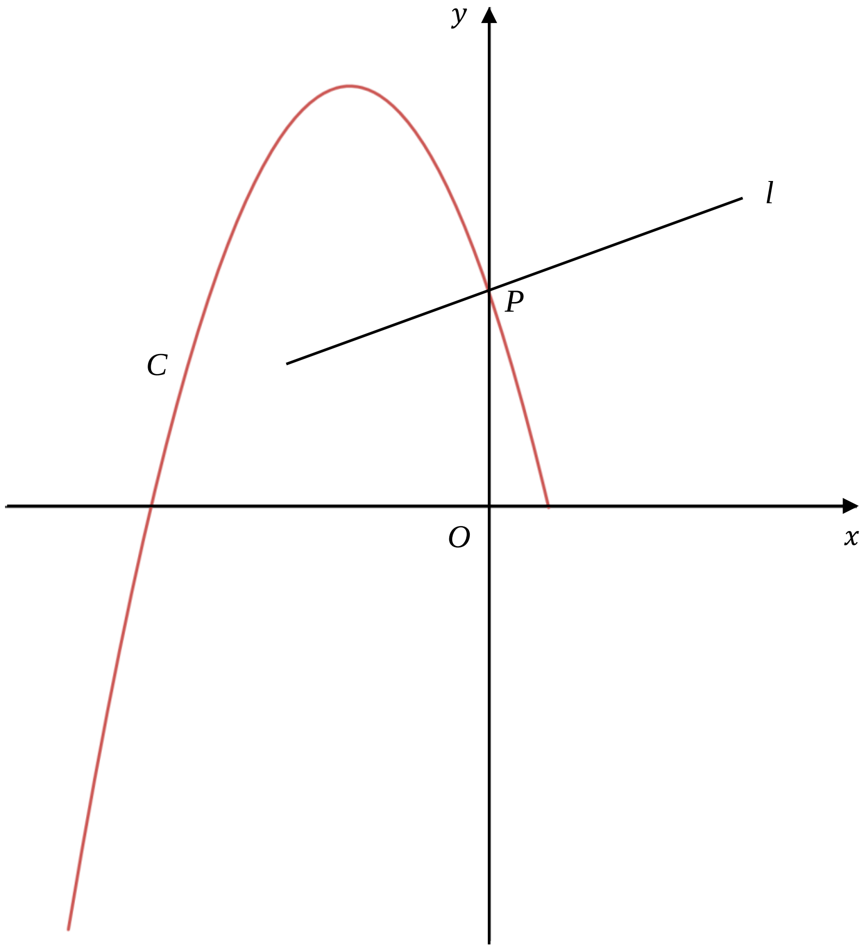

Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$x = -3 + 6\sin\theta$, $y = 9\cos 2\theta$ $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{4}$

where θ is a parameter.

a. Find an expression for $\frac{dy}{dx}$ in terms of θ

(2)

The line l is normal to C at the point P where $\theta = \frac{\pi}{6}$

b. Show that an equation for l is

$$y = \frac{1}{3}x + \frac{9}{2}$$

(3)

c. The cartesian equation for the curve C can be written in the form

$$y = a - \frac{1}{2}(x + b)^2$$

where a and b are integers to be found.

(2)

The straight line with equation

$$y = \frac{1}{3}x + k$$

where k is a constant intersects C at two distinct points.

d. Find the range of possible values for k.

(5)

(Total for Question 16 is 12 marks)