Pearson Edexcel Level 3

GCE Mathematics

Advanced Subsidiary

Paper 1: Pure Mathematics

Time: 2 hours	Paper Reference(s) 8MA0/01

You must have:
Mathematical Formulae and Statistical Tables, calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information
- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 15 questions in this paper. The total mark is 100.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Answer ALL questions. Write your answers in the spaces provided.

1. Find

$\int(\frac{1}{2}x^2 - 9\sqrt{x} + 4)\,dx$

giving your answer in its simplest form.

$$\int\left(\frac{1}{2}x^2 - 9\sqrt{x} + 4\right)dx = \int\left(\frac{1}{2}x^2 - 9x^{\frac{1}{2}} + 4\right)dx$$
$$= \frac{1}{2}\times\frac{x^3}{3} - 9\times\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 4x + c$$
$$= \frac{x^3}{6} - 6x^{\frac{3}{2}} + 4x + c$$

M1 Attempts to integrate awarded for any correct power

A1 Correct two non fractional power terms e.g. $\frac{1}{2}\times\frac{x^3}{3} - \cdots + 4x$

A1 Correct fractional power term e.g. $\ldots - 9\times\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \cdots$

A1 Completely correct, simplified and including constant of integration seen on one line.

$\frac{x^3}{6} - 6x^{\frac{3}{2}} + 4x + c$

(Total for Question 1 is 4 marks)

2. Use a counterexample to show that the following statement is false.

"$n^2 - n + 5$ is a prime number, for $2 \leq n \leq 6$"

$$2^2 - 2 + 5 = 7$$

$$3^2 - 3 + 5 = 11$$

$$4^2 - 4 + 5 = 17$$

$$5^2 - 5 + 5 = 25$$

25 is not a prime number, so $n = 5$ is a counterexample.

M1 Tries at least one value in the interval

e.g. $3^2 - 3 + 5 = 11$

A1 States that when $n = 5$ it is false and provides evidence

(Total for Question 2 is 2 marks)

3. Given that the point A has position vector $x\mathbf{i} - \mathbf{j}$, the point B has position vector $-2\mathbf{i} + y\mathbf{j}$ and $\overrightarrow{AB} = -3\mathbf{i} + 4\mathbf{j}$, find:

a. the values of x and y

$$\begin{aligned}\overrightarrow{AB} &= -\overrightarrow{OA} + \overrightarrow{OB} \\ &= -(x\boldsymbol{i} - \boldsymbol{j}) + -2\boldsymbol{i} + y\boldsymbol{j} \\ &= -x\boldsymbol{i} + \boldsymbol{j} - 2\boldsymbol{i} + y\boldsymbol{j} \\ &= (-2 - x)\boldsymbol{i} + (y + 1)\boldsymbol{j} = -3\boldsymbol{i} + 4\boldsymbol{j}\end{aligned}$$

Equating the **i** and **j** coefficients:

$$-2 - x = -3 \Rightarrow x = 1$$
$$y + 1 = 4 \Rightarrow y = 3$$

M1 Attempts $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

A1 Equating the coefficients of **i** and **j** and attempts to find the value of x or y

A1 Both correct values of x and y

(3)

b. a unit vector in the direction of $\overrightarrow{AB}$.

Find the length of $\overrightarrow{AB}$:

$$\left|\overrightarrow{AB}\right| = \sqrt{(-3)^2 + 4^2} = 5$$

A unit vector in the direction of $\overrightarrow{AB}$ will be the vector $\frac{\overrightarrow{AB}}{|\overrightarrow{AB}|}$

$$\text{unit vector: } \frac{-3}{5}\boldsymbol{i} + \frac{4}{5}\boldsymbol{j}$$

M1 makes an attempt to use Pythagoras theorem to find $\left|\overrightarrow{AB}\right|$

A1 correct answer only

(2)

(Total for Question 3 is 5 marks)

4. The line l_1 has equation $2x - 3y = 9$

The line l_2 passes through the points $(3, -1)$and $(-1,5)$

Determine, giving full reasons for your answer, whether lines l_1 and l_2 are parallel, perpendicular or neither.

Find the line l_1 in the form $y = mx + c$ (to find the gradient)

$$l_1 : y = \frac{2}{3}x - 3$$
$$m_2 = \frac{2}{3}$$

Find the gradient of the line l_2:

$$m_2 = \frac{-1 - 5}{3 - -1}$$
$$= \frac{-6}{4} = -\frac{3}{2}$$

Two lines are perpendicular if $m_1 \times m_2 = -1$

$$m_1 \times m_2 = \frac{2}{3} \times -\frac{3}{2}$$
$$= -1 \text{ so the two lines are perpendicular}$$

B1 States the gradient of line l_1 correctly

M1 attempts to find gradient of line joining $(3, -1)$and $((-1,5)$

M1 attempts the product of m_1 and m_2

A1 states that lines l_1 and l_2 are perpendicular.

(Total for Question 4 is 4 marks)

5. A student is asked to solve the equation

$$\log_3 x - \log_3 \sqrt{x-2} = 1$$

The student's attempt is shown

$$\log_3 x - \log_3 \sqrt{x-2} = 1$$
$$x - \sqrt{x-2} = 3^1$$
$$x - 3 = \sqrt{x-2}$$
$$(x-3)^2 = x - 2$$
$$x^2 - 7x + 11 = 0$$
$$x = \frac{7+\sqrt{5}}{2} \quad \text{or} \quad x = \frac{7-\sqrt{5}}{2}$$

a. Identify the error made by this student, giving a brief explanation.

B1 The student goes wrong in line 2, where the subtraction should be a division as $\log x - \log y = \log \frac{x}{y}$.

(1)

b. Write out the correct solution.

Using the subtraction law for logarithms:

$$\log_3 \frac{x}{\sqrt{x-2}} = 1$$

Taking each side of the equation to the power of 3:

$$\frac{x}{\sqrt{x-2}} = 3^1$$

Rearranging to find x

$$x = 3\sqrt{x-2}$$
$$x^2 = 9(x-2)$$
$$x^2 - 9x + 18 = 0$$
$$(x-6)(x-3) = 0$$
$$x = 6 \quad , \quad x = 3$$

M1 using the subtraction law and power law correctly.

M1 student obtains a quadratic and correctly factorises their quadratic

A1 both values of x are correct

(3)
(Total for Question 5 is 4 marks)

6.

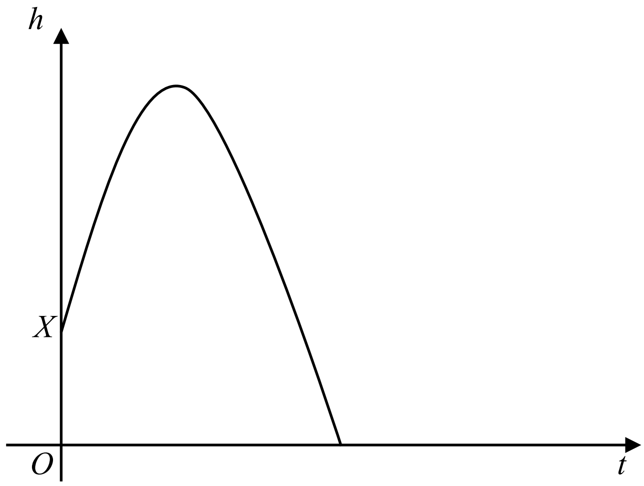


Figure 1

A stone is thrown over level ground from the top of a tower, X.

The height, h, in meters, of the stone above the ground level after t seconds is modelled by the function.

$$\mathrm{h}(t) = 7 + 21t - 4.9t^2, \; t \geq 0$$

A sketch of h against t is shown in Figure 1.

Using the model,

a. give a physical interpretation of the meaning of the constant term 7 in the model.

B1 The tower is 7 m tall, the stone is at height 7m at time 0

(1)

b. find the time taken after the stone is thrown for it to reach ground level.

Need to find the points such that $h(t) = 0$:

$$t = \frac{-21 \pm \sqrt{(21)^2 - 4(-4.9)(7)}}{2(-4.9)}$$

$t = -0.31079 \ldots$ or $t = 4.59650 \ldots$

t must be positive as the stone move forwards

$t = 4.60$ seconds

M1 Recognising that the values of t such that $h(t) = 0$ need to be found

M1 uses the quadratic formula to find the values of t

A1 states that $t = 4.60$ sec is the correct answer OR crosses out the other answer OR explains that t must be positive

(3)

c. Rearrange $\text{h}(t)$ into the form $A - B(t - C)^2$, where A, B and C are constants to be found.

Expand $A - B(t - C)^2$:

$$A - B(t - C)^2 = A - B(t^2 - 2Ct + C^2)$$
$$= -Bt^2 + 2Ct + (A - C^2B)$$

Equating the coefficients:

$$-Bt^2 + 2BCt + (A - C^2B) = -4.9t^2 + 21t + 7$$
$$B = 4.9$$
$$2BC = 21 \Rightarrow 2C = \frac{21}{4.9} \Rightarrow C = \frac{15}{7}$$
$$A - C^2B = 7 \Rightarrow A - 22.5 = 7 \Rightarrow A = 29.5$$

B1 Achieves $7 + 21t - 4.9t^2 = -4.9(t \pm k)^2 \pm \cdots$ or states that $B = 4.9$

M1 deals correctly with first two terms of $7 + 21t - 4.9t^2$

Scored for $7 + 21t - 4.9t^2 = -4.9\left(t - \frac{15}{7}\right)^2 \pm \cdots$ or states with $B = 4.9$ and $C = \frac{15}{7}$

A1 All coefficients correct

(3)

d. Using your answer to part **c** or otherwise, find the maximum height of the stone above the ground, and the time after which this maximum height is reached.

When completing the square, the maximum height is given by $C = 29.5$. $t = \frac{15}{7}$

B1 Maximum height

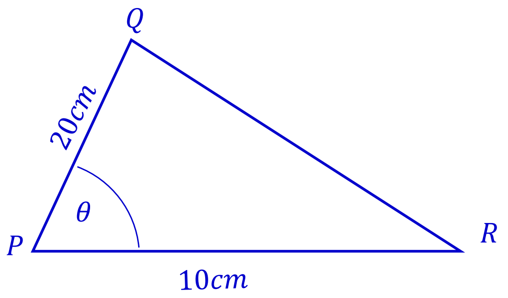
B1 time

(2)

(Total for Question 6 is 9 marks)

7. In a triangle PQR, $PQ = 20$ cm, $PR = 10$ cm and angle $QPR = \theta$, where θ is measured in degrees. The area of triangle PQR is 80 cm^2.

a. Show that the two possible values of $\cos\theta = \pm\frac{3}{5}$



area of a triangle $= \frac{1}{2}ab\sin C$

$a = PQ, b = PR, C = QPR$

$80 = \frac{1}{2} \times 20 \times 10 \times \sin\theta$

$\sin\theta = \frac{4}{5}$

using $\sin^2\theta + \cos^2\theta = 1$ (or a triangle method)

$\cos^2\theta = 1 - \left(\frac{4}{5}\right)^2$

$\cos^2\theta = \frac{9}{25}$

$\cos\theta = \pm\frac{3}{5}$

M1 use the area formula and attempts to find the value of $\sin\theta$

A1 correct value of $\sin\theta$

M1 uses their value of $\sin\theta$ to find two values of $\cos\theta$ with the correct formula

A1 correct value of $\cos\theta$

(4)

Given that QR is the longest side of the triangle,

b. Find the exact perimeter of the triangle PQR, giving your answer as a simplified surd.

cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

$a = QR, b = PQ, c = PR, A = QPR$

$$\cos\theta = \frac{3}{5} \Rightarrow 53.13, \cos\theta = -\frac{3}{5} \Rightarrow 126.87$$

Choosing a negative value of $\cos\theta$ will give the longest side.

$QR^2 = 10^2 + 20^2 - 2 \times 10 \times 20 \times \cos\theta$

$QR^2 = 10^2 + 20^2 - 2 \times 10 \times 20 \times -\frac{3}{5}$

$QR^2 = 740$

$QR = \pm 2\sqrt{185}$

QR is a length so must be positive:

$QR = 2\sqrt{185}$

$Perimeter: 10 + 20 + 2\sqrt{185} = 30 + 2\sqrt{185}$

M1 uses a suitable method of finding the longest side by choosing a negative value of $\cos\theta$ and proceeds to find QR using a cosine rule.

A1 the value of QR is correct $QR = 2\sqrt{185}$

A1 perimeter = 30 +$2\sqrt{185}$

(3)

(Total for Question 7 is 7 marks)

8.

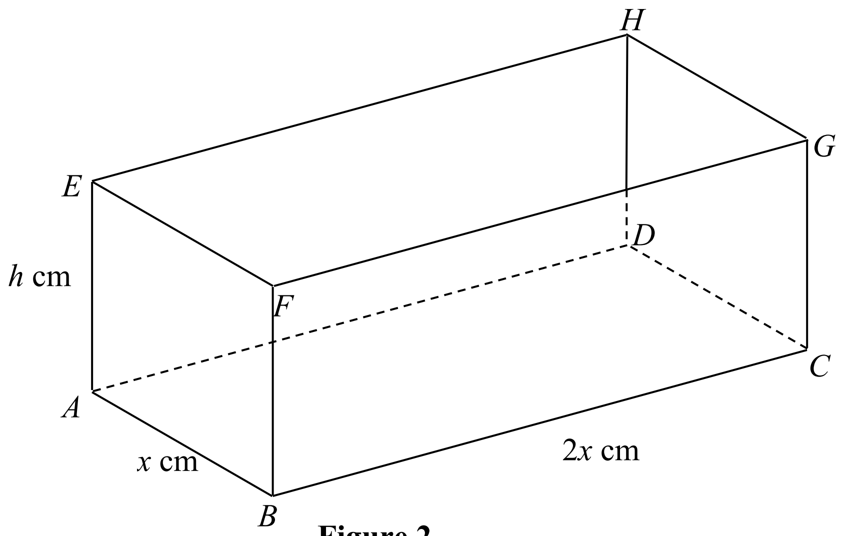


Figure 2

Figure 2 shows a solid cuboid *ABCDEFGH*.

$AB = x$ cm, $BC = 2x$ cm, $AE = h$ cm

The total surface area of the cuboid is 180 cm^2.

The volume of the cuboid is V cm^3.

a. Show that $V = 60x - \frac{4x^3}{3}$

Surface area: $2(2x^2 + hx + 2xh) = 180$

$4x^2 + 6xh = 180 \Rightarrow h = \frac{180-4x^2}{6x}$

Substituting into the formula for the volume:

$V = h \times x \times 2x$

$V = \frac{180-4x^2}{6x} \times 2x^2$

$V = \frac{360x^2-8x^4}{6x} = 60x - \frac{4x^3}{3}$

M1 attempting an expression in terms of x and y for the total surface area

A1 correct expression equated to 180

M1 substitute h into $V = 2x^2h$ to form an expression in terms of x only

A1 correct solution only

(4)

Given that x can vary,

b. use calculus to find, to 3 significant figures, the value of x for which V is a maximum.

Justify that this value of x gives a maximum value of V.

Want to find the maximum of $V = 60x - \frac{4x^3}{3}$

$\frac{dV}{dx} = 60 - 4x^2$

When $\frac{dV}{dx} = 0$, $60 - 4x^2 = 0, so\ x^2 = 15$

$x = \pm\sqrt{15}$

Finding the second derivative and substituting in:

$\frac{d^2V}{dx^2} = -8x$

When $x = -\sqrt{15}$, $\frac{d^2V}{dx^2} = 8\sqrt{15}$ and when $x = \sqrt{15}$, $\frac{d^2V}{dx^2} = -8\sqrt{15}$

$\frac{d^2V}{dx^2} < 0$ at a local maximum, so $x = \sqrt{15}$

$x = 3.87$ to 3 significant figures

(could also consider that x is a length so must be positive, but will still need to show that the second derivative is negative)

B1 Finding the first derivative (does not have to be correct)

M1 equate their $\frac{dV}{dx}$ to 0 and solve for x^2 or x

A1 Correct x values

M1 finding the second derivative and considering the sign

A1 correct reason and conclusion

(5)

c. Find the maximum value of V, giving your answer to the nearest cm^3.

Substitute $x = \sqrt{15}$ into the formula for V

$$V = 60\sqrt{15} - \frac{4(\sqrt{15})^3}{3}$$

$$V = 40\sqrt{15} = 154.9193 = 155cm^3 \text{to the nearest } cm^3$$

M1 substitutes their x value into the given expression for V.

A1 correct answer only

(2)

(Total for Question 8 is 11 marks)

9. $\mathrm{f}(x) = -2x^3 - x^2 + 4x + 3$

a. Use the factor theorem to show that $(3 - 2x)$ is a factor of $\mathrm{f}(x)$.

The factor theorem states that $(x - a)$ is a factor of $f(x)$ if and only if $f(a) = 0$.

$(3 - 2x) = 0 \Rightarrow x = \frac{3}{2}$

$f\left(\frac{3}{2}\right) = -2\left(\frac{3}{2}\right)^3 - \left(\frac{3}{2}\right)^2 + 4\left(\frac{3}{2}\right) + 3$

$f\left(\frac{3}{2}\right) = 0$, so $(3 - 2x)$ is a factor

M1 substituting $\frac{3}{2}$ into $\mathrm{f}(x)$

A1 Requires a correct statement and conclusion.

(2)

b. Hence show that $\mathrm{f}(x)$ can be written in the form $\mathrm{f}(x) = (3 - 2x)(x + a)^2$ where a is an integer to be found.

Can divide $f(x)$ by $(3 - 2x)$ or expand and equate coefficients:

$$(3 - 2x)(x + a)^2 = -2x^3 - x^2 + 4x + 3$$
$$(3 - 2x)(x^2 + 2ax + a^2) = -2x^3 - x^2 + 4x + 3$$
$$-2x^3 + (3 - 4a)x^2 + (6a - 4a^2)x + 3a = -2x^3 - x^2 + 4x + 3$$

$$3a = 3 \Rightarrow a = 1$$

So $f(x) = (3 - 2x)(x + 1)^2$

M1 attempts to divide $\mathrm{f}(x)$ by $(3 - 2x)$ and expect to see $(3 - 2x)(x^2 + \cdots \pm 1)$

A1 correct quadratic factor is $(x^2 + 2x + 1)$

M1 attempts to factorise their $x^2 + 2x + 1$

A1 correct answer only $(3 - 2x)(x + 1)^2$

(4)

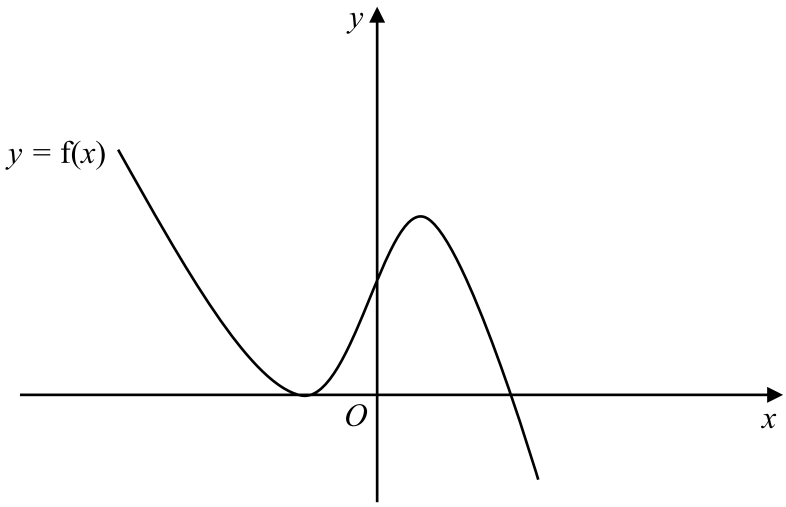


Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$.

c. Use your answer to part (b), and the sketch, to deduce the values of x for which

i. $\mathrm{f}(x) \leq 0$

ii. $\mathrm{f}\left(\frac{x}{2}\right) = 0$

$i. f(x) = 0 \Rightarrow x = \frac{3}{2}, x = 1$ (repeated root)

$f(x) \leq 0$ at $x \geq \frac{3}{2}$ and $x = -1$

M1 one correct answer

A1 both correct answers

ii. The graph of $f\left(\frac{x}{2}\right)$ is the graph of $f(x)$ 'stretched' by a scale factor of 2 along the x-axis.

The roots will therefore be at $x = -2, x = 3$

B1 both correct

(3)

(Total for Question 9 is 9 marks)

10. Prove, from the first principles, that the derivative of $5x^2$ is $10x$.

Formula for differentiation from first principles:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Substituting $f(x) = 5x^2$ and simplifying:

$$f'(x) = \lim_{h \to 0} \frac{5(x+h)^2 - 5x^2}{h}$$

$$= \lim_{h \to 0} \frac{5x^2 + 10xh + 5h^2 - 5x^2}{h}$$

$$= \lim_{h \to 0} \frac{10xh + 5h^2}{h}$$

$$= \lim_{h \to 0} 10x + 5h$$

As $h \to 0$, $10x + 5h \to 0$, so $f'(x) = 10x$

B1 substitutes the function

M1 attempts to expand $5(x+h)^2$ looking for two correct terms

A1 simplifies correctly

A1 correctly computes the limit

(Total for Question 10 is 4 marks)

11. The first 3 terms, in ascending powers of x, in the binomial expansion of $(1 + kx)^{10}$ are given by

$$1 + 15x + px^2$$

where k and p are constants.

a. Find the value of k

From the binomial expansion, $10kx = 15x$

$k = \frac{3}{2}$

M1 attempts to use the binomial expansion to find a value for k

A1 correct answer only

(2)

b. Find the value of p

From the binomial expansion, $\frac{10(9)}{2}k^2x^2 = px^2$

$45k^2 = p$

$45\left(\frac{3}{2}\right)^2 = p$

$\frac{405}{4} = p$

M1 Uses the binomial expansion to find an equation for p using their k

A1 correct answer only **(2)**

c. Given that, in the expansion of $(1 + kx)^{10}$, the coefficient of x^4 is q, find the value of q.

From the binomial expansion, the x^4 term of $(1 + bx)^n$ is $\frac{n(n-1)(n-2)(n-3)}{4!}b^4x^4$

Substituting $n = 10, b = k$:

$qx^2 = \frac{10\times9\times8\times7}{4!} \times (kx)^4$

$q = \frac{10\times9\times8\times7}{4!}\left(\frac{3}{2}\right)^4$

$q = \frac{8505}{4}$

M1 identifies the correct term and uses their value of k to find the value of q

A1 correct answer only

(2)

(Total for Question 11 is 6 marks)

12. a. Explain mathematically why there are no values of θ that satisfy the equation

$$(3\cos\theta - 4)(2\cos\theta + 5) = 0$$

Set each bracket to 0 to find values of $\cos\theta$

$3\cos\theta - 4 = 0 \Rightarrow \cos\theta = \frac{4}{3}$

$2\cos\theta + 5 = 0 \Rightarrow \cos\theta = -\frac{5}{2}$

$\cos\theta$ takes values between -1 and 1, so there does not exist such a θ to satisfy these equations.

B1 attempts to find the values of $\cos\theta$

B1 requires a correct statement and conclusion

(2)

b. Giving your solutions to one decimal place, where appropriate, solve the equation

$$3\sin y + 2\tan y = 0 \quad \text{for } 0 \le y \le \pi$$

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

Substituting $\tan y = \frac{\sin y}{\cos y}$:

$3\sin y + 2\frac{\sin y}{\cos y} = 0$

$3\sin y\cos y + 2\sin y = 0$
$\sin y(3\cos y + 2) = 0$
$\sin y = 0 \Rightarrow y = 0, \pi$
$\cos y = \frac{-2}{3} \Rightarrow$ no solutions

M1 uses a trigonometric substitution in the given equation
M1 correct work leading to 2 factors
A1 All solutions correct

(3)
(Total for Question 12 is 5 marks)

13.

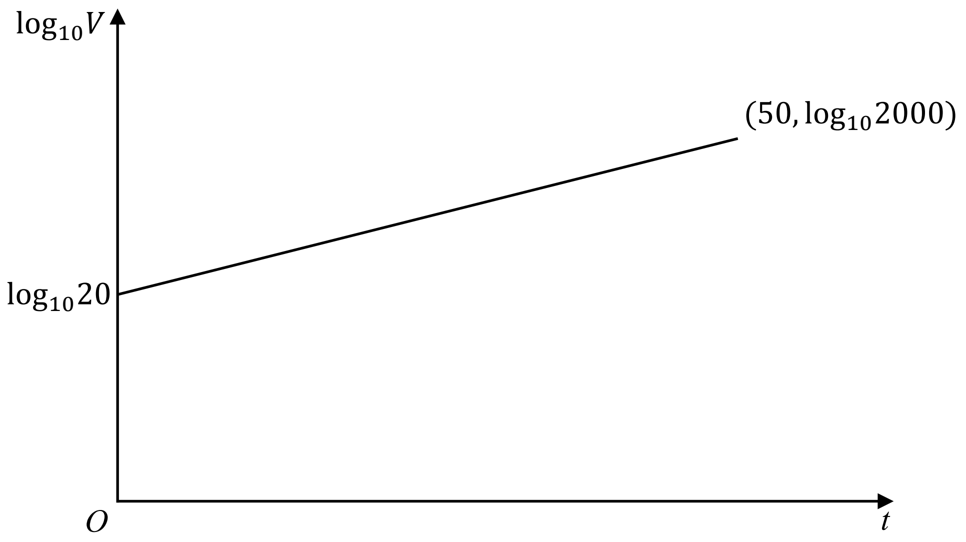


Figure 4

The value of a sculpture, £V, is modelled by the equation $V = Ap^t$, where A and p are constants and t is the number of years since the value of the painting was first recorded on 1st January 1960.

The line l shown in Figure 4 illustrates the linear relationship between t and $\log_{10}V$ for $t \geq 0$.

The line l passes through the point $(0, \log_{10}20)$ and $(50, \log_{10}2000)$.

a. Write down the equation of the line l.

$$m = \frac{\log_{10}2000 - \log_{10}20}{50-0} = \frac{\log_{10}\frac{2000}{20}}{50} = \frac{2}{50} = 0.04$$

Substitute into the equation of the line:

$\log_{10}V = mt + c$ or $\log_{10}V - k = m(t - t_0)$

$$\log_{10}V = \frac{1}{25}t + \log_{10}20$$

$$\log_{10}V = \frac{1}{25}t + \log_{10}20$$

M1 attempts to find the gradient of line l

M1 uses the equation of a straight line in the form

A1 correct equation of line l

(3)

b. Using your answer to part **a** or otherwise, find the values of A and p.

$\log_{10} V = \frac{1}{25}t + \log_{10} 20$

$V = Ap^t$

Take logarithms of $V = Ap^t$ (or exponentials of the equation of the line)

$\log_{10}V = \log_{10}(Ap^t)$

$\log_{10} V = \log_{10} p^t + \log_{10} A$

$\log_{10} V = t\log_{10} p + \log_{10} A$

Equate the two equations:

$\frac{1}{25}t + \log_{10} 20 = t\log_{10} p + \log_{10} A$

$\frac{1}{25} = \log_{10} p \Rightarrow p = 10^{\frac{1}{25}}$

$A = 20$

M1 attempts to rearrange their equation by taking exponentials or takes the log of both sides of the given equation

M1 completes rearrangement correctly so that both equations are in directly comparable form.

A1 states that $A = 20$

A1 states that $p = 10^{\frac{1}{25}}$

(4)

c. With reference to the model, interpret the values of the constant A and p.

B1 A is the initial value of the sculpture.

B1 p is the annual proportional increase in the value of the sculpture.

(2)

d. Use your model to predict the value of the sculpture, on 1st January 2020, giving your answer to the nearest pounds.

$2020 - 1960 = 60$, so substitute $t = 60$

$$V = 20\left(10^{\frac{1}{25}}\right)^{60} = 20 \times 10^{2.4} = 5023.7728$$

$$V = £5024$$

B1 substitute 60 into their formula from part *b*, correct answer

(1)
(Total for Question 13 is 10 marks)

14. A curve with centre C has equation

$$x^2 + y^2 + 2x - 6y - 40 = 0$$

a. i. State the coordinates of C

Complete the square with respect to x and y

$(x+1)^2 - 1 + (y-3)^2 - 9 - 40 = 0$

$(x+1)^2 + (y-3)^2 = 50$

$C = (-1,3)$

M1 attempts to complete the square

A1 gives the centre correctly

ii. Find the radius of the circle, giving your answer as $r = n\sqrt{2}$.

radius $= \sqrt{50} = 5\sqrt{2}$

A1 correct answer only

(3)

b. The line l is a tangent to the circle and has gradient -7.

Find two possible equations for l, giving your answers in the form $y = mx + c$.

Find an equation for the radius: it passes through the centre $(-1,3)$ with gradient $\frac{1}{7}$ (as by circle theorems the tangent and radius are perpendicular)

$y = \frac{1}{7}x + c$

$3 = \frac{1}{7}(-1) + c \Rightarrow c = \frac{22}{7}$

Substitute the equation of the line into the equation of the circle:

$(x-1)^2 + \left(\frac{1}{7}x + \frac{22}{7} - 3\right)^2 = 50$

$(x^2 + 2x + 1) + \left(\frac{1}{49}x^2 + \frac{2}{49}x + \frac{1}{49}\right) = 50$

$49x^2 + 98x + 49 + x^2 + 2x + 1 = 2450$

$50x^2 + 100x + 50 - 2450 = 0$

$x^2 + 2x - 48 = 0$

$(x+8)(x-6) = 0 \Rightarrow x = -8 \; or \; x = 6$

Substituting into the equation of the line:

$x = -8 \Rightarrow y = 2, \quad x = 6 \Rightarrow y = 4$

Computing equations of tangents with gradient -7 and passing through $(-8,2)$ or $(6,4)$

$l_1: 2 = -7(-8) + c \Rightarrow c = -54$

$l_2: 4 = -7(6) + c \Rightarrow c = 46$

$l_1: y = -7x - 54, \quad l_2: y = -7x + 46$

M1 attempts to write an equation of radius/diameter of circle

A1 correct equation

M1 substitutes their equation of radius into equation of the circle and obtain a quadratic equation in terms of x

M1 Factorise their quadratic equation and reaching a value of x

A1 both x values are correct

A1 substituting x values into equation of radius and obtaining y values correctly

M1 attempts to write an equation of the tangents with their x and y values

A1 both equation of tangents are correct

(8)

(Total for Question 14 is 11 marks)

15.

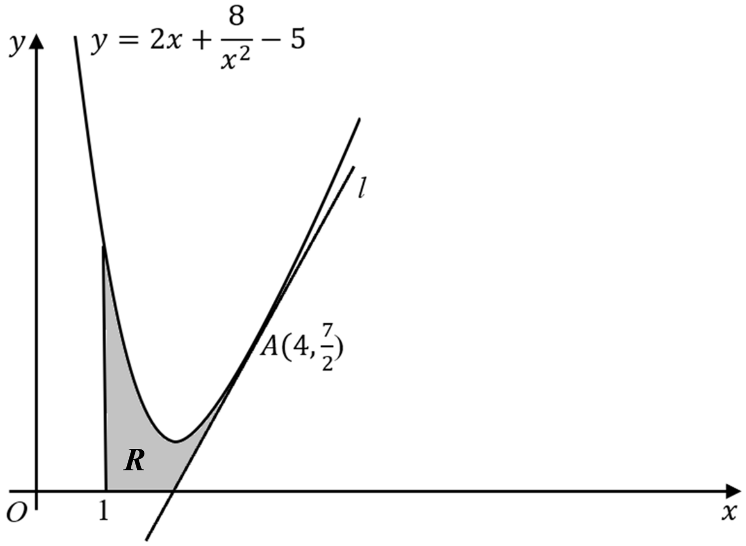


Figure 5

Figure 5 shows a sketch of part of the curve $y = 2x + \frac{8}{x^2} - 5, \;\; x > 0$.

The point $A(4, \frac{7}{2})$ lies on C. The line l is the tangent to C at the point A.

The region ***R***, shown shaded in figure 5 is bounded by the line l, the curve C, the line with equation $x = 1$ and the x-axis.

Find the exact area of ***R***.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

Finding the equation of the line l:

$\frac{d}{dx}(2x + 8x^{-2} - 5) = 2 - 16x^{-3}$

$\frac{d}{dx}_{x=4} = 2 - 16(4)^{-3} = \frac{7}{4}$

$y = \frac{7}{4}x + c \Rightarrow \frac{7}{2} = \frac{7}{4}(4) + c \Rightarrow c = -\frac{7}{2}$

Finding the intersection of l with the x-axis:

$0 = \frac{7}{4}x - \frac{7}{2} \Rightarrow x = 2$

$Area\ R = \int_1^4 2x + 8x^{-2} - 5\,dx - \text{Triangle}\left((2,0), \left(4, \frac{7}{2}\right), (4,0)\right)$

$\int_1^4 2x + 8x^{-2} - 5\ dx = \left[\frac{2x^2}{2} + \frac{8x^{-1}}{-1} - 5x\right]_1^4$

$= \left(\frac{2(16)}{2} + \frac{8(4)^{-1}}{-1} - 5(4)\right) - \left(\frac{2(1)^2}{2} + \frac{8(1)^{-1}}{-1} - 5(1)\right) = 6$

Area of triangle= $\frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2} \times 2 \times \frac{7}{2} = \frac{7}{2}$

Area R: $6 - \frac{7}{2} = \frac{5}{2}$

M1 attempts to differentiate $y = 2x + 8x^{-2} - 5$ with at least one index reduced by one

A1 correct derivative obtained

M1 correct method to find equation of tangent

A1 find correct intercept with x-axis

M1 complete strategy of finding the areas under the curve between $x = 1$ and $x = 4$ and area of triangle

M1 method for integration

A1 correct integration

M1 attempts to find a value for the area of shaded region

A1 correct value for the area of shaded region

(Total for Question 15 is 9 marks)

TOTAL FOR THE PAPER: 100 MARKS