

Please check the examination details below before entering your candidate information

Candidate surname

Other names

**Pearson Edexcel
Level 3 GCE**

Centre Number

--	--	--	--	--

Candidate Number

--	--	--	--

Wednesday 13 May 2020

Morning (Time: 2 hours)

Paper Reference **8MA0/01**

Mathematics
Advanced Subsidiary
Paper 1: Pure Mathematics

You must have:

Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear.
Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 14 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

P62645RA

©2020 Pearson Education Ltd.

1/1/1/1/1/



P 6 2 6 4 5 R A 0 1 4 0



Pearson

1. A curve has equation

$$y = 2x^3 - 4x + 5$$

Find the equation of the tangent to the curve at the point $P(2, 13)$.

Write your answer in the form $y = mx + c$, where m and c are integers to be found.

Solutions relying on calculator technology are not acceptable.

(5)

tangent: same gradient, same coordinate, one point of intersection \leftrightarrow one root

$$\text{differentiate } y(x): y = 2x^3 - 4x + 5 \Rightarrow \frac{dy}{dx} = 3 \times 2x^{3-1} - 4x^{1-1}$$

$$= 6x^2 - 4$$

$$\text{so gradient @ } P = 6(2)^2 - 4 = 20$$

$$\text{use } y - y_0 = m(x - x_0): y - 13 = 20(x - 2)$$

$$y - 13 = 20x - 40$$

$$\underline{y = 20x - 27}$$



2. [In this question the unit vectors \mathbf{i} and \mathbf{j} are due east and due north respectively.]

A coastguard station O monitors the movements of a small boat.

At 10:00 the boat is at the point $(4\mathbf{i} - 2\mathbf{j})$ km relative to O .

At 12:45 the boat is at the point $(-3\mathbf{i} - 5\mathbf{j})$ km relative to O .

The motion of the boat is modelled as that of a particle moving in a straight line at constant speed.

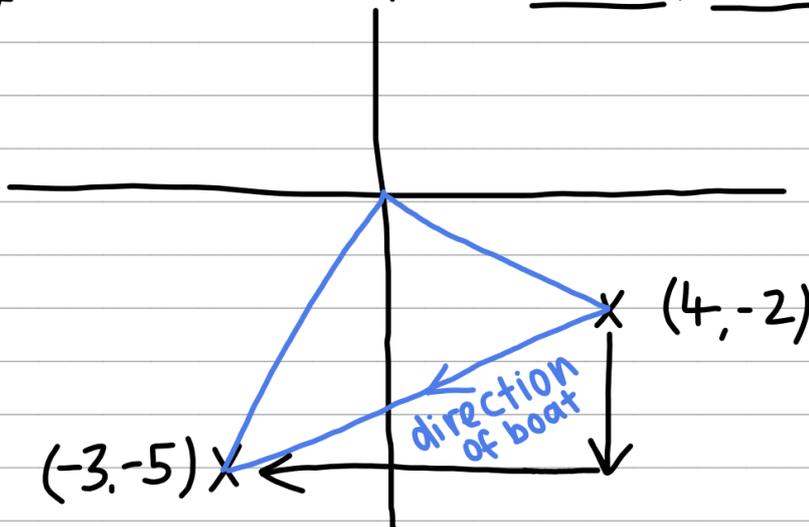
- (a) Calculate the bearing on which the boat is moving, giving your answer in degrees to one decimal place.

(3)

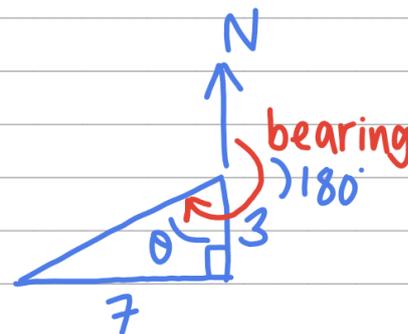
- (b) Calculate the speed of the boat, giving your answer in km h^{-1}

(3)

a) bearing is measured from north, clockwise



$$\text{direction vector: } \begin{pmatrix} -3 \\ -5 \end{pmatrix} - \begin{pmatrix} 4 \\ -2 \end{pmatrix} = \begin{pmatrix} -7 \\ -3 \end{pmatrix}$$



$$\text{angle needed: } \tan \theta = \frac{7}{3} \\ = 66.801^\circ$$

$$\text{total bearing} = 180^\circ + 66.8^\circ = \underline{\underline{246.8^\circ}}$$



Question 2 continued

b) to find speed, need distance travelled

direction vector $\begin{pmatrix} -7 \\ -3 \end{pmatrix}$ so distance = $\sqrt{7^2 + 3^2} = \sqrt{58}$ km

time: 2hrs 45 \Rightarrow 2.75 hours

speed = $\frac{\sqrt{58}}{2.75} = 2.77 \text{ kmh}^{-1}$

$$\downarrow$$
$$\sqrt{(4 - -3)^2 + (-2 + 5)^2}$$

(Total for Question 2 is 6 marks)



3.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(i) Solve the equation

$$x\sqrt{2} - \sqrt{18} = x$$

writing the answer as a surd in simplest form.

(3)

(ii) Solve the equation

$$4^{3x-2} = \frac{1}{2\sqrt{2}}$$

(3)

i. rearrange to make x the subject:

$$x\sqrt{2} - x = \sqrt{18}$$

$$x(\sqrt{2} - 1) = \sqrt{18}$$

$$x = \frac{\sqrt{18}}{\sqrt{2} - 1}$$

surd in denominator \Rightarrow rationalise: $x = \frac{\sqrt{18}}{\sqrt{2} - 1} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1}$

$$= \frac{\sqrt{18}(\sqrt{2} + 1)}{2 - 1}$$

$$= \frac{\sqrt{36} + \sqrt{18}}{1}$$

$$= 6 + \sqrt{9 \times 2}$$

$$= 6 + 3\sqrt{2}$$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 3 continued

ii. LHS has a power of 2, RHS has power of 2 \Rightarrow Manipulate so

2 is 'base' of each side: $(2^2)^{3x-2} = \frac{1}{2 \times 2^{\frac{1}{2}}}$

$$\hookrightarrow 2^{6x-4} = \frac{1}{2^{\frac{3}{2}}} = 2^{-\frac{3}{2}}$$

$$\therefore 6x-4 = -\frac{3}{2}$$

$$6x = 2.5 = \frac{5}{2}$$

$$x = \frac{5}{12}$$

(Total for Question 3 is 6 marks)



4. In 1997 the average CO₂ emissions of new cars in the UK was 190 g/km.

In 2005 the average CO₂ emissions of new cars in the UK had fallen to 169 g/km.

Given A g/km is the average CO₂ emissions of new cars in the UK n years after 1997 and using a linear model,

(a) form an equation linking A with n .

(3)

In 2016 the average CO₂ emissions of new cars in the UK was 120 g/km.

(b) Comment on the suitability of your model in light of this information.

(3)

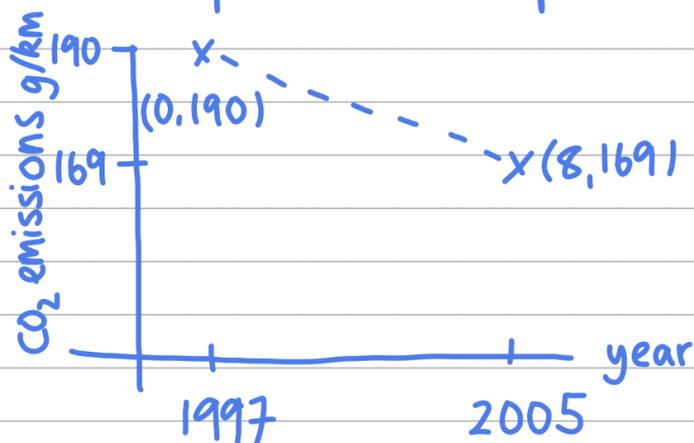
a) linear model: form $A = mn + c$

↪ function of number of years

it can help to visualise problem:

let 1997 be $n=0$

let 2005 be $n=8$



↪ so we want equation of dotted line

$$\text{gradient: } \frac{190 - 169}{-8} = -2.625$$

$$\text{intercept: } 190 = c$$

$$\text{so } \underline{A = -2.625n + 190}$$

b) given new data point, we need to see how it compares with

model's prediction:



Question 4 continued

$$A = -2.625 \times 19 + 190$$

$$= 140.125 \text{ g/km}$$

$140.125 \gg 120 \Rightarrow$ the model overestimates A & so is not suitable

(Total for Question 4 is 6 marks)



5.

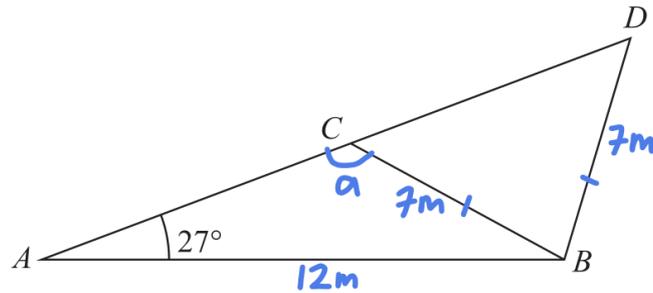


Figure 1

Not to scale

Figure 1 shows the design for a structure used to support a roof.

The structure consists of four steel beams, AB , BD , BC and AD .

Given $AB = 12\text{ m}$, $BC = BD = 7\text{ m}$ and angle $BAC = 27^\circ$

(a) find, to one decimal place, the size of angle ACB .

(3)

The steel beams can only be bought in whole metre lengths.

(b) Find the minimum length of steel that needs to be bought to make the complete structure.

(3)

a) use sine rule : $\frac{\sin a}{A} = \frac{\sin b}{B}$ ← ratio of sine of angle & opposite side

$$\therefore \frac{\sin a}{12} = \frac{\sin 27^\circ}{7}$$

$$a = \sin^{-1}\left(\frac{12 \sin 27^\circ}{7}\right)$$

$$= 51.1^\circ \text{ (primary) or } 180 - 51.1 = 128.9^\circ \text{ (secondary)}$$

$$a \text{ is obtuse } \Rightarrow a = 128.9^\circ$$

b) use cosine rule to find AD : $a^2 = b^2 + c^2 - 2bc \cos A$

$$\angle DCB = 180^\circ - 128.9^\circ = 51.1^\circ$$

$$\triangle DCB \text{ is isosceles } \therefore \angle CBD = 180 - 2 \times 51.1^\circ = 77.8^\circ$$



Question 5 continued

$$\angle CBA = 180^\circ - 27^\circ - 128.9^\circ = 24.1^\circ$$

$$\therefore \angle ABD = 24.1^\circ + 77.8^\circ = 101.9^\circ$$

$$\text{(check: } \angle ABD = 180^\circ - 27^\circ - 51.1^\circ = 101.9^\circ)$$

$$\begin{aligned} \Rightarrow AD^2 &= 7^2 + 12^2 - 2 \times 12 \times 7 \cos 101.9^\circ \\ &= 227.64 \end{aligned}$$

$$\therefore AD = 15.088 \text{ m}$$

$$\begin{aligned} \text{total length} &= 12 + 7 + 7 + 15.09 \\ &= 41.09 \end{aligned}$$

round up to whole metres: 42m bought

(Total for Question 5 is 6 marks)



6. (a) Find the first 4 terms, in ascending powers of x , in the binomial expansion of

$$(1 + kx)^{10}$$

where k is a non-zero constant. Write each coefficient as simply as possible. (3)

Given that in the expansion of $(1 + kx)^{10}$ the coefficient of x^3 is 3 times the coefficient of x ,

- (b) find the possible values of k . (3)

a) use binomial formula: $(x+y)^n = \sum_0^n \binom{n}{k} x^k y^{n-k}$

$$(1+kx)^{10} = \sum_0^{10} \binom{10}{t} (kx)^t \times 1^{10-t} = 1 + \binom{10}{1} (kx)^1 + \binom{10}{2} (kx)^2 + \binom{10}{3} (kx)^3$$

1st 4 terms

$$= 1 + 10kx + 45k^2x^2 + 120k^3x^3$$

b) x^3 coeff. = $3 \times x$ coeff.

$$120k^3 = 3 \times 10k$$

$$= 30k$$

$k \neq 0 \Rightarrow$ divide by k : $120k^2 = 30$

$$k^2 = \frac{1}{4}$$

$$\therefore k = \pm \frac{1}{2}$$



7. Given that k is a positive constant and $\int_1^k \left(\frac{5}{2\sqrt{x}} + 3 \right) dx = 4$

(a) show that $3k + 5\sqrt{k} - 12 = 0$

(4)

(b) Hence, using algebra, find any values of k such that

$$\int_1^k \left(\frac{5}{2\sqrt{x}} + 3 \right) dx = 4$$

a) $\int_1^k \left(\frac{5}{2} x^{-\frac{1}{2}} + 3 \right) dx = \left[\left(\frac{5}{2} \div \left(-\frac{1}{2} + 1 \right) \right) x^{-\frac{1}{2} + 1} + \frac{3}{1} x^1 \right]_1^k$ (4)

$$= \left[\left(\frac{5}{2} \div \frac{1}{2} \right) x^{\frac{1}{2}} + 3x \right]_1^k$$

$$= \left[5x^{\frac{1}{2}} + 3x \right]_1^k$$

$$= 5\sqrt{k} + 3k - 5 - 3$$

$$\Rightarrow 4 = 5\sqrt{k} + 3k - 8$$

$$0 = 5\sqrt{k} + 3k - 12$$

b) use $x = \sqrt{k}$: $3x^2 + 5x - 12 = 0$

$$(3x - 4)(x + 3) = 0$$

$$\Rightarrow x = \sqrt{k} = \frac{4}{3} \text{ or } \sqrt{k} = -3$$

reject negative root so $\underline{k = \frac{16}{9}}$

$$\rightarrow \int_1^{\frac{4}{3}} \left(\frac{5}{2} x^{-\frac{1}{2}} + 3 \right) dx = \left[5\sqrt{x} + 3x \right]_1^{\frac{4}{3}}$$

$$= 5\left(\frac{4}{3}\right) + 3\left(\frac{16}{9}\right) - 5 - 3$$

↑ takes +ve root

$$= 34 \neq 4$$



8. The temperature, $\theta^\circ\text{C}$, of a cup of tea t minutes after it was placed on a table in a room, is modelled by the equation

$$\theta = 18 + 65e^{-\frac{t}{8}} \quad t \geq 0$$

Find, according to the model,

- (a) the temperature of the cup of tea when it was placed on the table, (1)
- (b) the value of t , to one decimal place, when the temperature of the cup of tea was 35°C . (3)
- (c) Explain why, according to this model, the temperature of the cup of tea could not fall to 15°C . (1)

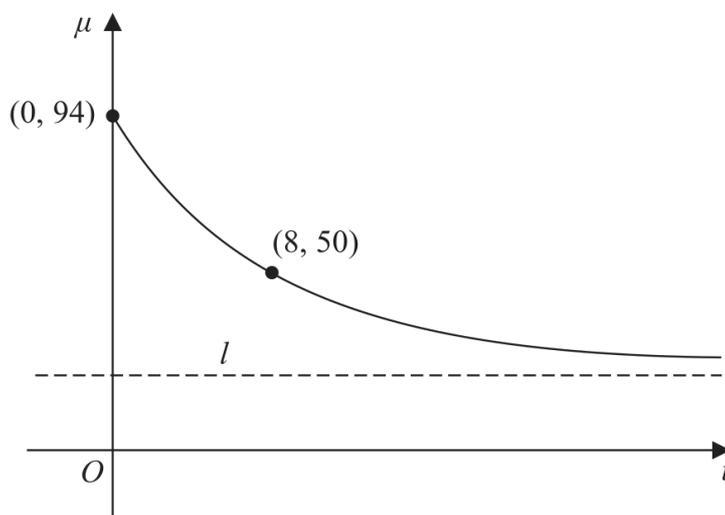


Figure 2

The temperature, $\mu^\circ\text{C}$, of a second cup of tea t minutes after it was placed on a table in a different room, is modelled by the equation

$$\mu = A + Be^{-\frac{t}{8}} \quad t \geq 0$$

where A and B are constants.

Figure 2 shows a sketch of μ against t with two data points that lie on the curve.

The line l , also shown on Figure 2, is the asymptote to the curve.

Using the equation of this model and the information given in Figure 2

- (d) find an equation for the asymptote l . (4)

a) placed on table @ $t = 0$: $\theta = 18 + 65e^0$

temperature = 83°C



Question 8 continued

$$b) \theta = 35 = 18 + 65e^{-\frac{t}{8}}$$

$$\rightarrow 65e^{-\frac{t}{8}} = 17$$

$$\rightarrow e^{-\frac{t}{8}} = \frac{17}{65}$$

$$\text{take ln of both sides: } -\frac{t}{8} = \ln\left(\frac{17}{65}\right)$$

$$t = -8 \ln\left(\frac{17}{65}\right)$$

$$= 10.729\dots$$

$$= 10.7 \text{ (1 d.p.)}$$

c) as $t \rightarrow \infty$, $e^{-\frac{t}{8}} \rightarrow 0$ from above so $\theta \rightarrow 18^\circ\text{C}$ from above.

hence, the minimum temperature (18°C) is $> 15^\circ\text{C}$

$$d) \mu = A + Be^{-\frac{t}{8}}$$

$$\text{given points } (0, 94) \text{ \& } (8, 50): 94 = A + B^0 \Rightarrow A + B = 94 \text{ ①}$$

$$50 = A + B e^{-\frac{8}{8}} \Rightarrow A + Be^{-1} = 50$$

$$\text{①} - \text{②}: 94 - 50 = B(1 - e^{-1})$$

$$44 = B(1 - e^{-1})$$

$$44e = B(e - 1)$$

$$B = \frac{44e}{(e - 1)}$$



Question 8 continued

$$\begin{aligned}A+B=94 &\Rightarrow A=94-\frac{44e}{(e-1)} \\ &= \frac{94e-44e-94}{e-1} \\ &= \frac{50e-94}{e-1}\end{aligned}$$

as $t \rightarrow \infty$, $Be^{-\frac{t}{8}} \rightarrow 0$ so asymptote given by

$$A = \frac{50e-94}{e-1} (= l)$$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



9.

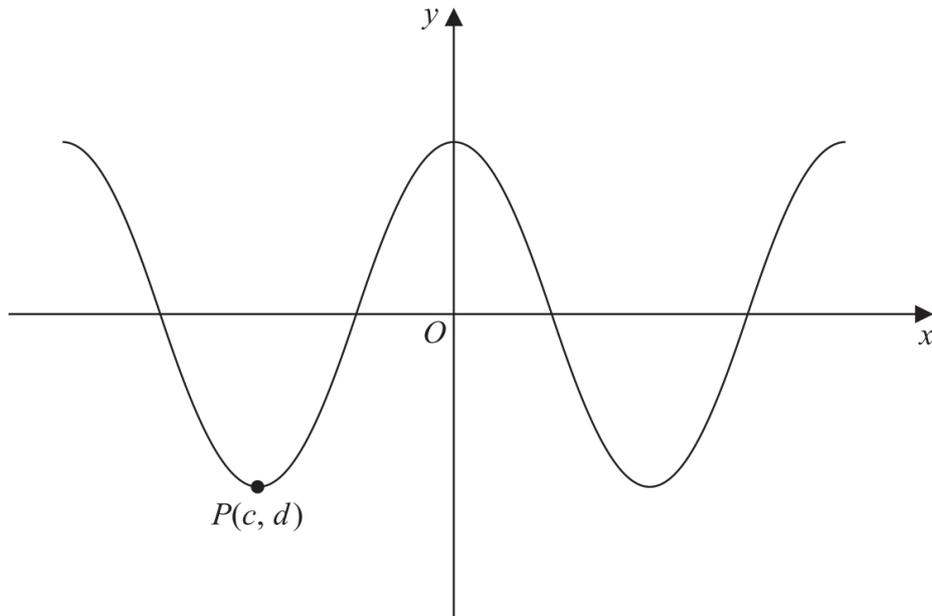


Figure 3

Figure 3 shows part of the curve with equation $y = 3 \cos x^\circ$.

The point $P(c, d)$ is a minimum point on the curve with c being the smallest negative value of x at which a minimum occurs.

(a) State the value of c and the value of d . (1)

(b) State the coordinates of the point to which P is mapped by the transformation which transforms the curve with equation $y = 3 \cos x^\circ$ to the curve with equation

(i) $y = 3 \cos \left(\frac{x^\circ}{4} \right)$

(ii) $y = 3 \cos(x - 36)^\circ$ (2)

(c) Solve, for $450^\circ \leq \theta < 720^\circ$,

$$3 \cos \theta = 8 \tan \theta$$

giving your solution to one decimal place.

In part (c) you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable. (5)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 9 continued

a) minimum of $\cos x = -1 \Rightarrow$ minimum of $3\cos x = -3 = d$

P is the first minimum for $x < 0 \therefore c = -180^\circ$

$$P(-180^\circ, -3)$$

b) i. $y = 3\cos\left(\frac{x}{4}\right) \Rightarrow$ x 'stretched' $\times 4$, no change to y

$$\hookrightarrow (-720^\circ, -3)$$

ii. $y = 3\cos(x - 36^\circ) \Rightarrow$ translation in x -direction $+36^\circ$

$$\hookrightarrow (-144^\circ, -3)$$

c) When solving these types of question, list the relevant trig. identities that could help you.

$$\tan \theta = \frac{\sin \theta}{\cos \theta}, \sin^2 \theta + \cos^2 \theta = 1$$

$$3\cos \theta = 8\tan \theta$$

$$= 8 \frac{\sin \theta}{\cos \theta}$$

$$\cos \theta$$

$$\times \cos \theta : 3\cos^2 \theta = 8\sin \theta$$

Form quadratic in $\sin \theta$ (you want an equation with only one trig. function)

$$3(1 - \sin^2 \theta) = 8\sin \theta$$



Question 9 continued

$$3\sin^2\theta + 8\sin\theta - 3 = 0$$

$$(3\sin\theta - 1)(\sin\theta + 3) = 0$$

$$\sin\theta \neq -3 \text{ so } \sin\theta = \frac{1}{3}$$

min. value is -1

$$\theta = 19.47^\circ$$

always note what range you need

but range is $450^\circ \leq \theta < 720^\circ$

$$\theta = 180 - 19.47^\circ$$

$$= 160.53^\circ$$

$\sin\theta$ repeats every 360°

to bring into range: $160.53^\circ + 360^\circ = \underline{520.5^\circ}$ (1 d.p.)

$19.47^\circ + 360^\circ$ also not in range

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



10.

$$g(x) = 2x^3 + x^2 - 41x - 70$$

(a) Use the factor theorem to show that $g(x)$ is divisible by $(x - 5)$. (2)

(b) Hence, showing all your working, write $g(x)$ as a product of three linear factors. (4)

The finite region R is bounded by the curve with equation $y = g(x)$ and the x -axis, and lies below the x -axis.

(c) Find, using algebraic integration, the exact value of the area of R . (4)

a) factor theorem: if $f(x)$ is divisible by $(x-a)$, then

$$f(a) = 0$$

$$g(5) = 2 \times 5^3 + 5^2 - 41 \times 5 - 70 = 250 + 25 - 205 - 70$$

$$= 0$$

$g(5) = 0 \Rightarrow (x-5)$ is a factor, so $g(x)$ is divisible by $(x-5)$

b) divide $g(x)$ by $(x-5)$ to get a quadratic

$$\begin{array}{r}
 2x^2 + 11x + 14 \\
 (x-5) \overline{) 2x^3 + x^2 - 41x - 70} \\
 \underline{-(2x^3 - 10x^2)} \\
 11x^2 - 41x - 70 \\
 \underline{-(11x^2 - 55x)} \\
 14x - 70
 \end{array}$$



Question 10 continued

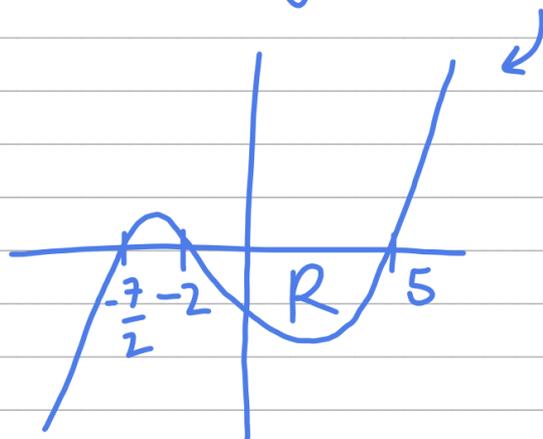
$$g(x) = (x-5)(2x^2+11x+14)$$

$$= (x-5)(2x+7)(x+2)$$

Sketch to help you find R

c) g 's roots: $-\frac{7}{2}, -2, 5$

so R bound by $x=-2, x=5$



$$\int_{-2}^5 g(x) dx = \int_{-2}^5 (2x^3 + x^2 - 41x - 70) dx$$

$$= \left[\frac{2}{4}x^4 + \frac{1}{3}x^3 - \frac{41}{2}x^2 - 70x \right]_{-2}^5$$

$$= \frac{1}{2}(625) + \frac{1}{3}(125) - \frac{41}{2}(25) - 70(5)$$

$$- \frac{1}{2}(16) - \frac{1}{3}(-8) + \frac{41}{2}(4) - 70(-2)$$

$$= -\frac{1525}{3} - \frac{190}{3}$$

$$\text{area} = 571 \frac{2}{3}$$



11. (i) A circle C_1 has equation

$$x^2 + y^2 + 18x - 2y + 30 = 0$$

The line l is the tangent to C_1 at the point $P(-5, 7)$.

Find an equation of l in the form $ax + by + c = 0$, where a , b and c are integers to be found.

(5)

(ii) A different circle C_2 has equation

$$x^2 + y^2 - 8x + 12y + k = 0$$

where k is a constant.

Given that C_2 lies entirely in the 4th quadrant, find the range of possible values for k .

(4)

i. complete the square to find centre of circle

$$x^2 + y^2 + 18x - 2y + 30 = 0$$

$$\rightarrow (x+9)^2 - 81 + (y-1)^2 - 1 + 30 = 0$$

\therefore centre $(-9, 1)$ (don't need radius)

We can use the fact that the radius & tangent are \perp to find

gradient of the tangent: $m_r \times m_t = -1$

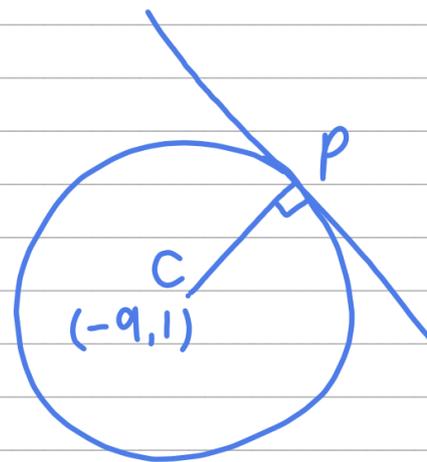
gradient of radius joining C to P:

$$\frac{7-1}{-5+9} = \frac{3}{2}$$

$$\therefore m_t = -\frac{2}{3}$$

using $y - y_0 = m(x - x_0)$: $y - 7 = -\frac{2}{3}(x + 5)$

$$3y - 21 = -2x - 10$$



Question 11 continued

$$\therefore l: 2x + 3y - 11 = 0$$

ii. lies in 4th quadrant \Rightarrow need centre of C_2

$$x^2 + y^2 - 8x + 12y + k = 0$$

$$\hookrightarrow (x-4)^2 - 16 + (y+6)^2 - 36 + k = 0$$

$$\Rightarrow (x-4)^2 + (y+6)^2 = 52 - k$$

centre $(4, -6)$

to lie entirely in one quadrant, can't cross axes

\Rightarrow radius must be less than shortest distance from

axes

$$\therefore r < 4 \Rightarrow 52 - k < 4^2$$

$$\therefore k > 36 \quad \text{lengths can't be negative}$$

$$r > 0 \Rightarrow 52 - k > 0$$

so in total, $36 < k < 52$



12. An advertising agency is monitoring the number of views of an online advert.

The equation

$$\log_{10} V = 0.072t + 2.379 \quad 1 \leq t \leq 30, t \in \mathbb{N}$$

is used to model the total number of views of the advert, V , in the first t days after the advert went live.

(a) Show that $V = ab^t$ where a and b are constants to be found.

Give the value of a to the nearest whole number and give the value of b to 3 significant figures.

(4)

(b) Interpret, with reference to the model, the value of ab .

(1)

Using this model, calculate

(c) the total number of views of the advert in the first 20 days after the advert went live.
Give your answer to 2 significant figures.

(2)

$$a) \log_{10} V = 0.072t + 2.379$$

$$\text{raise both sides: } V = 10^{0.072t + 2.379} \quad (\text{base} = 10)$$

$$= 10^{0.072t} \times 10^{2.379}$$

$$\therefore a = 10^{2.379} \quad \& \quad b = 10^{0.072}$$

by calculator, nearest whole value: $a = 239$, $b = 1.18$ (3s.f.)

$$\Rightarrow V = 239 \times 1.18^t$$

b) we get $V = ab$ when $t = 1$: $V = ab^1$. Thus, the value of ab is the total number of views of the ad. 1 day after it went live

$$c) t = 20: V = 239 \times 1.18^{20} \\ = 6545...$$



Question 12 continued

$$\Rightarrow V = 6500 \text{ views}$$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



P 6 2 6 4 5 R A 0 3 3 4 0

13. (a) Prove that for all positive values of a and b

$$\frac{4a}{b} + \frac{b}{a} \geq 4 \quad (4)$$

(b) Prove, by counter example, that this is not true for all values of a and b .

(1)

a) for all real numbers, their value squared is always ≥ 0

$$\frac{4a}{b} + \frac{b}{a} = 4 \Rightarrow 4a^2 + b^2 - 4ab = 0 \Rightarrow (2a-b)^2 = 0$$

you can 'reverse engineer' to find how to prove statement

starting point for proof

$$\therefore (2a-b)^2 \geq 0$$

$$4a^2 + b^2 - 4ab \geq 0$$

as $a, b > 0$, dividing by either doesn't change direction of

$$\text{inequality: } \frac{4a^2}{ab} + \frac{b^2}{ab} \geq \frac{4ab}{ab}$$

$$\Rightarrow \frac{4a}{b} + \frac{b}{a} \geq 4$$

(a) uses $a, b > 0$

b) so counter example must use negative value

$$\text{e.g. } a=5, b=-1: \frac{4a}{b} + \frac{b}{a} = -20 - \frac{1}{5} < 4$$



14. A curve has equation $y = g(x)$.

Given that

- $g(x)$ is a cubic expression in which the coefficient of x^3 is equal to the coefficient of x
- the curve with equation $y = g(x)$ passes through the origin
- the curve with equation $y = g(x)$ has a stationary point at $(2, 9)$

(a) find $g(x)$,

(7)

(b) prove that the stationary point at $(2, 9)$ is a maximum.

(2)

tick off properties as you go to keep track

a) cubic: $g(x) = ax^3 + bx^2 + cx + d$

$$x^3 \text{ coeff.} = x \text{ coeff.} \Rightarrow g(x) = ax^3 + bx^2 + ax + d$$

$$\text{passes through origin} \Rightarrow d = 0, g(x) = ax^3 + bx^2 + ax$$

$$\text{passes through } (2, 9) \Rightarrow 9 = 8a + 4b + 2a$$

$$\Rightarrow 10a + 4b = 9 \quad \textcircled{1}$$

$$(2, 9) \text{ is a stationary point} \Rightarrow g'(2) = 0$$

$$g'(x) = 3ax^2 + 2bx + a$$

$$\Rightarrow 0 = 12a + 4b + a$$

$$13a + 4b = 0 \quad \textcircled{2}$$

$$\textcircled{2} - \textcircled{1} : 3a = -9$$

$$a = -3$$

$$\Rightarrow b = \frac{9 + 10(3)}{4}$$

$$= \frac{39}{4}$$



Question 14 continued

$$\text{so } g(x) = -3x^3 + \frac{39}{4}x^2 - 3x$$

b) for a maximum, $g''(x) < 0$

$$\begin{aligned} g''(x) &= 2 \times 3x - 3 + 2 \times \frac{39}{4} \\ &= -18x + \frac{39}{2} \end{aligned}$$

$$\begin{aligned} g''(2) &= -18(2) + \frac{39}{2} \\ &= -\frac{33}{2} < 0 \text{ hence point is a max.} \end{aligned}$$



