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Question Number	Scheme	Marks
Q1	$H_0: \mu = 80$, $H_1: \mu > 80$	B1,B1
	$z = \dfrac{83-80}{\frac{15}{\sqrt{100}}} = 2$	M1A1
	$2 > 1.6449$ (accept 1.645 or better)	B1
	Reject H_0 or significant result or in the critical region	M1
	Managing director's claim is supported.	A1
		7

1^{st} B1 for H_0. They must use μ not *x, p,* λ or $\bar{x}$ etc

2^{nd} B1 for H_1 (must be > 80). Same rules about μ.

1^{st} M1 for attempt at standardising using 83, 80 and $\frac{15}{\sqrt{100}}$. Can accept $\pm$.

May be implied by $z = \pm 2$

1^{st} A1 for + 2 only

3^{rd} B1 for ± 1.6449 seen (or probability of 0.0228 or better)

2^{nd} M1 for a correct statement about "significance" or rejecting H_0 (or H_1) based on their *z* value
and their 1.6449 (provided it is a recognizable critical value from normal tables) or their probability (< 0.5) and significance level of 0.05.
Condone their probability > 0.5 compared with 0.95 for the 2^{nd} M1

2^{nd} A1 for a correct contextualised comment. Must mention "director" and "claim" or "time" and "use of Internet". No follow through.

2^{nd} M1A1

If no comparison or statement is made but a correct contextualised comment is given the M1 can be implied.
If a comparison is made it must be compatible with statement otherwise M0
e.g. comparing 0.0228 with 1.6449 is M0 or comparing probability 0.9772 with 0.05 is M0
comparing -2 with - 1.6449 is OK provided a correct statement accompanies it
condone -2 >-1.6449 provided their statement correctly rejects H_0.

Critical Region

They may find a critical region for $\bar{X}$: $\bar{X} > 80 + \frac{15}{\sqrt{100}} \times 1.6449$ = awrt 82.5

1^{st} M1 for $80 + \frac{15}{\sqrt{100}} \times$ (*z* value)

3^{rd} B1 for 1.645 or better

1^{st} A1 for awrt 82.5

The rest of the marks are as per the scheme.

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Question Number	Scheme	Marks
Q2	[$P \sim N(90,9)$ and $J \sim N(91,12)$]	
(a)	$(J-P) \sim N(1,21)$	M1, A1
	$P(J<P) = P(J-P<0)$	
	$= P\left(Z < \frac{0-1}{\sqrt{21}}\right)$	dM1
	$= P(Z<-0.2182...)$	
	$= 1-0.5871 = 0.4129$ awrt (**0.413 ~ 0.414**)	A1
	calculator (0.4136….)	(4)
(b)	$X = (J_1+J_2+....+J_{60})-(P_1+P_2+....+P_{60})$	M1
	$E(X) = 60\times91-60\times90 = 60$ [stated as E(X) = 60 or X~N(60, …)]	B1
	$Var(X) = 60\times9+60\times12 = 1260$	A1
	$P(X>120) = P\left(Z > \frac{120-60}{\sqrt{1260}}\right)$	M1
	$= P(Z>1.69030...)$	
	$= 1-0.9545 = 0.0455$ awrt (**0.0455**)	A1
		(5)
		9

(a)

1st M1 for attempting J - P and E(J - P) or P - J and E(P - J)

1st A1 for variance of 21 (Accept 9 + 12). Ignore any slip in μ here.

2nd dM1 for attempting the correct probability and standardising with their mean and sd.
This mark is dependent on previous M so if J - P (or P - J) is not being used score M0
If their method is not crystal clear then they must be attempting P(Z< -ve value) or P(Z > +ve value) i.e. their probability after standardisation should lead to a prob. < 0.5 so e.g. P(J - P < 0) leading to 0.5871 is M0A0 unless the M1 is clearly earned.

2nd A1 for awrt 0.413 or 0.414

The first 3 marks may be implied by a correct answer

(b)

1st M1 for a clear attempt to identify a correct form for X. This may be implied by correct variance of 1260

B1 for E(X) = 60. Can be awarded even if they are using X = 60J - 60P. Allow P - J and -60

1st A1 for a correct variance. If 1260 is given the M1 is scored by implication.

2nd M1 for attempting a correct probability and standardising with 120 and their 60 and 1260
If the answer is incorrect a full expression must be seen following through their values for M1 e.g. $P\left(Z > \frac{120 - \text{their } 60}{\sqrt{\text{their variance}}}\right)$. If using -60, should get $P\left(Z < \frac{-120 - -60}{\sqrt{\text{their variance}}}\right)$

Use of means

Attempt to use $\bar{J} - \bar{P}$ for 1st M1, E($\bar{J} - \bar{P}$) = 1 for B1 and Var($\bar{J} - \bar{P}$) = 0.35 for A1
Then 2nd M1 for standardisation with 2, and their 1 and 0.35

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Question Number	Scheme			Marks
Q3 (a)	$E \sim \mathrm{N}(0, 0.5^2)$	or	$X \sim \mathrm{N}(w, 0.5^2)$	
	$\mathrm{P}(\lvert E\rvert < 0.6) = \mathrm{P}\left(\lvert Z\rvert < \frac{0.6}{0.5}\right)$	or	$\mathrm{P}(\lvert X-w\rvert < 0.6) = \mathrm{P}\left(\lvert Z\rvert < \frac{0.6}{0.5}\right)$	M1
	$= \mathrm{P}(\lvert Z\rvert < 1.2)$			
	$= 2\times 0.8849 - 1 = 0.7698$		awrt **0.770**	A1
				(2)
(b)	$\bar{E} \sim \mathrm{N}\left(0, \frac{1}{64}\right)$	or	$\bar{X} \sim \mathrm{N}\left(w, \frac{0.5^2}{16}\right)$	M1
	$\mathrm{P}(\lvert \bar{E}\rvert < 0.3) = \mathrm{P}\left(\lvert Z\rvert < \frac{0.3}{\frac{1}{8}}\right)$	or	$\mathrm{P}(\lvert \bar{X}-w\rvert < 0.3) = \mathrm{P}\left(\lvert Z\rvert < \frac{0.3}{\frac{1}{8}}\right)$	M1, A1
	$= \mathrm{P}(\lvert Z\rvert < 2.4)$			
	$= 2\times 0.9918 - 1 = 0.9836$		awrt **0.984**	A1
				(4)
(c)	$35.6 \pm 2.3263 \times \frac{1}{8}$			M1 B1
	(**35.3, 35.9**)			A1,A1
				(4)
				10

(a)
1st M1 for identifying a correct probability (they must have the 0.6) and attempting to standardise. Need | |. This mark can be given for 0.8849 - 0.1151 seen as final answer.
1st A1 for awrt 0.770. NB an answer of 0.3849 or 0.8849 scores M0A0 (since it implies no | |)

M1 may be implied by a correct answer

(b)
1st M1 for a correct attempt to define $\bar{E}$ or $\bar{X}$ but must attempt $\frac{\sigma^2}{n}$.
Condone labelling as E or X
This mark may be implied by standardisation in the next line.
2nd M1 for identifying a correct probability statement using $\bar{E}$ or $\bar{X}$. Must have 0.3 and | |
1st A1 for correct standardisation as printed or better
2nd A1 for awrt 0.984

The M marks may be implied by a correct answer.

Sum of 16, not means
1st M1 for correct attempt at suitable sum distribution with correct variance ($= 16\times\frac{1}{4}$)
2nd M1 for identifying a correct probability. Must have 4.8 and | |
1st A1 for correct standardisation i.e. need to see $\frac{4.8}{\sqrt{4}}$ or better

(c)
M1 for $35.6 \pm z \times \frac{0.5}{\sqrt{16}}$
B1 for 2.3263 or better. Use of 2.33 will lose this mark but can still score ¾
1st A1 for awrt 35.3
2nd A1 for awrt 35.9

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Question Number	Scheme	Marks
Q4 (a)	<table><tr><td>Distance rank</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr><tr><td>Depth rank</td><td>1</td><td>2</td><td>4</td><td>3</td><td>6</td><td>7</td><td>5</td></tr><tr><td>$\lvert d\rvert$</td><td>0</td><td>0</td><td>1</td><td>1</td><td>1</td><td>1</td><td>2</td></tr><tr><td>d^2</td><td>0</td><td>0</td><td>1</td><td>1</td><td>1</td><td>1</td><td>4</td></tr></table>	M1 M1
	$\sum d^2 = 8$	M1A1
	$r_s = 1-\frac{6\times 8}{7\times 48}$	M1
	$=\frac{6}{7}=0.\dot{8}57142\dot{}$ awrt **0.857**	A1 (6)
(b)	$H_0 : \rho = 0, H_1 : \rho > 0$	B1
	Critical value at 1% level is 0.8929	B1
	$r_s < 0.8929$ so not significant evidence to reject H_0,	M1
	The researcher's claim is not correct (at 1% level). or insufficient evidence for researcher's claim or there is insufficient evidence that water gets deeper further from inner bank. or no (positive) correlation between depth of water and distance from inner bank	A1ft (4)
		10

(a)
1st M1 for an attempt to rank the depths against the distances
2nd M1 for attempting *d* for their ranks. Must be using ranks.
3rd M1 for attempting $\sum d^2$ (must be using ranks)
1st A1 for sum of 8 (or 104 for reverse ranking)
4th M1 for use of the correct formula with their $\sum d^2$. If answer is not correct an expression is required.
2nd A1 for awrt (±) 0.857. Sign should correspond to ranking (so use of 104 should get -0.857)

(b)
1st B1 for both hypotheses in terms of ρ, H_1 must be one tail and compatible with their ranking
2nd B1 for cv of 0.8929 (accept ±)
M1 for a correct statement relating their r_s with their cv but cv must be such that |cv|<1
A1ft for a correct contextualised comment. Must mention "researcher" and "claim" or "distance (from bank)" and "depth (of water)"
Follow through their r_s and their cv (provided it is |cv| <1)
Use of "association" is A0

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Question Number	Scheme	Marks
Q5		

Finances / Income	Worse	Same	Better	
Under £15 000	10.54	10.54	12.92	34
£15 000 and above	20.46	20.46	25.08	66
	31	31	38	100

M1
A1

H_0 : State of finances and income are independent (not associated)

H_1 : State of finances and income are not independent (associated)

B1

O_i	E_i	$\frac{(O_i-E_i)^2}{E_i}$	$\frac{O_i^2}{E_i}$
14	10.54	1.1358….	18.59..
11	10.54	0.0200….	11.48..
9	12.92	1.1893…	6.269..
17	20.46	0.5851…	14.12..
20	20.46	0.0103…	19.55..
29	25.08	0.6126…	33.53..

M1

A1

$\sum \frac{(O_i-E_i)^2}{E_i} = 3.553\ldots$ or $\sum \frac{O_i^2}{E_i} - 100 = 103.553... - 100 = 3.553...$ (awrt **3.55**) A1

$\nu = (3-1)(2-1) = 2$ B1

cv is 5.991 B1

3.553 < 5.991 so insufficient evidence to reject H_0 or not significant M1

There is no evidence of association between state of finances and income. A1

10

1st M1 for some use of $\frac{\text{Row Total} \times \text{Col.Total}}{\text{Grand Total}}$. May be implied by correct E_i

1st A1 for all expected frequencies correct

B1 for both hypotheses. Must mention "state" or "finances" and "income" at least once
Use of "relationship" or "correlation" or "connection"is B0

2nd M1 for at least two correct terms (as in 3rd or 4th column) or correct expressions with their E_i

2nd A1 for all correct terms. May be implied by a correct answer.(2 dp or better-allow eg 1.13…)

3rd M1 for a correct statement linking their test statistic and their cv . Must be χ^2 not normal.

4th A1 for a correct comment in context - must mention "state" or "finances" and "income" condone "relationship" or "connection" here but **not** "correlation". No follow through. e.g. "There is no evidence of a relationship between finances and income"

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Question Number	Scheme	Marks

Q6

Distance from centre of site (m)	0-1	1-2	2-4	4-6	6-9	9-12
$b-a$	1	1	2	2	3	3
No of artefacts	22	15	44	37	52	58
$P(a \le X < b)$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{4}$
$228 \times P(a \le X < b)$	19	19	38	38	57	57

M1

A1

A1

Class	O_i	E_i	$\frac{(O_i-E_i)^2}{E_i}$	$\frac{O_i^2}{E_i}$
0-1	22	19	$\frac{9}{19}=0.4736...$	25.57…
1-2	15	19	$\frac{16}{19}=0.8421...$	11.84…
2-4	44	38	$\frac{36}{38}=0.9473...$	50.94…
4-6	37	38	$\frac{1}{38}=0.0263...$	36.02…
6-9	52	57	$\frac{25}{57}=0.4385...$	47.43…
9-12	58	57	$\frac{1}{57}=0.0175...$	59.01…

M1

A1

H_0: continuous uniform distribution is a good fit — B1

H_1: continuous uniform distribution is not a good fit

$\sum \frac{(O_i-E_i)^2}{E_i} = \frac{313}{114} = 2.75$ or $\sum \frac{O_i^2}{E_i} - 228 = 230.745... - 228 = \ldots$ (awrt **2.75**) — dM1A1

$\nu = 6-1=5$ — B1

$\chi^2_5(0.05) = 11.070$ (ft their ν i.e. $\chi^2_\nu(0.05)$) — B1ft

2.75<11.070, insufficient evidence to reject H_0 — M1

Continuous uniform distribution is a suitable model — A1

12

1st M1 for calculation of at least 3 widths and attempting proportions/probs. or for 1:2:3 ratio seen

1st A1 for correct probabilities

2nd A1 for all correct expected frequencies

2nd M1 for attempting $\frac{(O-E)^2}{E}$ or $\frac{O^2}{E}$, at least 3 correct expressions or values.
Follow through their E_i provided they are not all = 38

3rd A1 for a correct set of calcs - 3rd or 4th column. (2 dp or better and allow e.g. 0.94…)

3rd dM1 **dependent on 2nd M1** for attempting a correct sum or calculation (must see at least 3 terms and +)

The first three Ms and As can be implied by a test statistic of awrt 2.75

4th M1 for a correct statement based on their test statistic (> 1) and their cv (> 3.8)
Contradictory statements score M0 e.g. "significant" do not reject H_0.

5th A1 for a correct comment suggesting that continuous uniform model is suitable. No ft

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Question Number		Scheme	Marks
Q7	(a)	Label full time staff 1-6000, part time staff 1-4000	M1
		Use random numbers to select	M1
		Simple random sample of 120 full time staff and 80 part time staff	A1 (3)
	(b)	Enables estimation of statistics / errors for each strata <u>or</u> "reduce variability" <u>or</u> "more representative" <u>or</u> "reflects population structure" **NOT** "more accurate"	B1 (1)
	(c)	$H_0: \mu_f = \mu_p$, $H_1: \mu_f \neq \mu_p$ (accept μ_1, μ_2)	B1
		s.e. $= \sqrt{\frac{21}{80}+\frac{19}{80}}$, $z = \frac{52-50}{\sqrt{\frac{21}{80}+\frac{19}{80}}} = \left(2\sqrt{2}\right)$	M1,M1
		$= 2.828\ldots$ (awrt **2.83**)	A1
		Two tailed critical value z = 2.5758 (or prob of awrt 0.002 (<0.005) or 0.004 (<0.01))	B1
		[2.828 > 2.5758 so] significant evidence to reject H_0	dM1
		There is evidence of a difference in policy awareness between full time and part time staff	A1ft (7)
	(d)	Can use mean full time and mean part time	B1
		~ Normal	B1 (2)
	(e)	Have assumed $s^2 = \sigma^2$ or variance of sample = variance of population	B1 (1)
	(f)	2.53 < 2.5758, not significant <u>or</u> do not reject H_0	M1
		So there is insufficient evidence of a difference in mean awareness	A1ft (2)
	(g)	Training course has closed the gap between full time staff and part time staff's mean awareness of company policy.	B1 (1)
			17

(a)	1st M1	for attempt at labelling full-time and part-time staff. One set of correct numbers.
	2nd M1	for mentioning use of random numbers
	1st A1	for s.r.s. of 120 full-time and 80 part-time
(c)	1st M1	for attempt at s.e. - condone one number wrong . NB correct s.e. = $\sqrt{\frac{1}{2}}$
	2nd M1	for using their s.e. in correct formula for test statistic. Must be $\frac{\pm(52-50)}{\sqrt{\frac{p}{q}+\frac{r}{s}}}$
	3rd dM1	**dep. on 2nd M1** for a correct statement based on their normal cv and their test statistic
	2nd A1	for correct comment in context. Must mention "scores" or " policy awareness" and types of "staff". Award **A0** for a one-tailed comment. Allow ft
(d)	1st B1	for mention of mean(s) <u>or</u> use of $\bar{X}$, provided $\bar{X}$ clearly refers to full-time or part-time
	2nd B1	for stating that distribution can be assumed normal e.g. "mean score of the test is normally distributed" gets B1B1
(f)	M1	for correct statement (may be implied by correct contextualised comment)
	A1	for correct contextualised comment. Accept "no difference in mean scores". Allow ft
(g)	B1	for correct comment in context that implies training was effective. This must be supported by their (c) and (f). Condone one-tailed comment here.