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S3 June 2014 IAL (MA)

Q1a) 165, 8

bi) To choose the first person, use random numbers to select a number between 1 and 6 then pick the corresponding person. Then pick every 6^{th} person onwards.

ii) The list is alphabetical, so has not been sorted by gender.

c) Label all Males 1-180 and all Females 1-120. Then use random numbers to select 30 males and 20 females.

No. of each group required $= \left(\frac{\text{group size}}{\text{population size}}\right) \times$ sample size

d) any 1
- Sampling errors can be estimated with stratified
- Stratified is not biased as members are not chosen randomly.
- Gives more accurate estimates than Quota as it is a random process.

Q2a) $X \sim U[\alpha - 3, 2\alpha + 3]$

$$E(\bar{X}) = \frac{\alpha - 3 + 2\alpha + 3}{2} = \frac{3\alpha}{2}$$

$E(\bar{X}) \neq \alpha$ hence $\bar{X}$ is a biased estimater

$$\text{bias} = E(\bar{X}) - \alpha = \frac{3\alpha}{2} - \alpha = \boxed{\frac{\alpha}{2}}$$

b) unbiased $\therefore$ $E(Y) = \alpha$

$$E(k\bar{X}) = \alpha$$

$$kE(\bar{X}) = \alpha$$

$$k\left(\frac{3\cancel{\alpha}}{2}\right) = \cancel{\alpha}$$

$$\therefore \boxed{k = \frac{2}{3}}$$

c) sample mean $= \bar{X} = \dfrac{3+5+8+12+4+13+10+8+5+12}{10}$

$$= 8$$

Y estimates α, so $\hat{\alpha} = \frac{2}{3}\bar{X} = \frac{2}{3}(8) = \frac{16}{3}$

$$X_{max} = 2\alpha + 3 = 2\left(\frac{16}{3}\right) + 3 = \boxed{\frac{41}{3}}$$

upper limit of uniform distribution

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Q3) $H_0: \mu_A = \mu_B$

critical value : ± 2.3263
(1%, 1-tail)

$H_1: \mu_A > \mu_B$

$$\text{Test Statistic} = \frac{\bar{X} - \bar{Y} - (\mu_X - \mu_Y)}{\sqrt{\frac{\sigma_X^2}{n_X} + \frac{\sigma_Y^2}{n_Y}}}$$

$$= \frac{532 - 520}{\sqrt{\frac{35^2}{80} + \frac{28^2}{100}}} = 2.4939\ldots$$

$2.49 > 2.3263$

$\therefore$ Result is significant
Reject H_0.
Evidence suggests that the grocer's belief is correct.

Q4a)

Man	x	w	R_x	R_w	d	d^2
A	123	78	1	2	1	1
B	128	93	2	7	5	25
C	137	85	3	4	1	1
D	143	83	4	3	1	1
E	149	75	5	1	4	16
F	153	98	6	9	3	9
G	154	88	7	6	1	1
H	159	87	8	5	3	9
I	162	95	9	8	1	1
J	168	99	10	10	0	0
						64

$$\sum d^2 = 64$$

$$\therefore r_s = 1 - \frac{6(64)}{10(99)} = \boxed{0.612}$$

b) $H_0: \rho = 0$ critical value : ± 0.5636 (5%, 1-tail)

$H_1: \rho > 0$

$0.612 > 0.5636$

$\therefore$ Result is significant.

Reject H_0.

Evidence suggests that there is a positive correlation between systolic blood pressure and weight.

c) $H_0: \rho = 0$ critical value : ± 0.5494

$H_1: \rho > 0$ (5%, 1-tail)

$0.5114 < 0.5494$ $\therefore$ Result is insignificant

Accept H_0.

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Evidence suggests no positive correlation exists between systolic blood pressure and weight.

d) x and w are likely not jointly normally distributed so PMCC unsuitable - use conclusion from (b) that there is a positive correlation.

Q5a) H_0: There is no association between type of drink prefered and gender.

H_1: There is an association between type of drink prefered and gender.

$$\text{Expected no.} = \frac{\text{Row Total} \times \text{Column Total}}{\text{Grand Total}}$$

EXPECTED :

	Tea	Coffee	Hot Chocolate	
M	46.53	34.31	13.16	94
F	52.47	38.69	14.84	106
	99	73	28	$\boxed{200}$

O_i	E_i	$\frac{(O-E)^2}{E}$
57	46.53	2.356
26	34.31	2.013
11	13.16	0.355
42	52.47	2.089
47	38.69	1.785
17	14.84	0.314
		8.91

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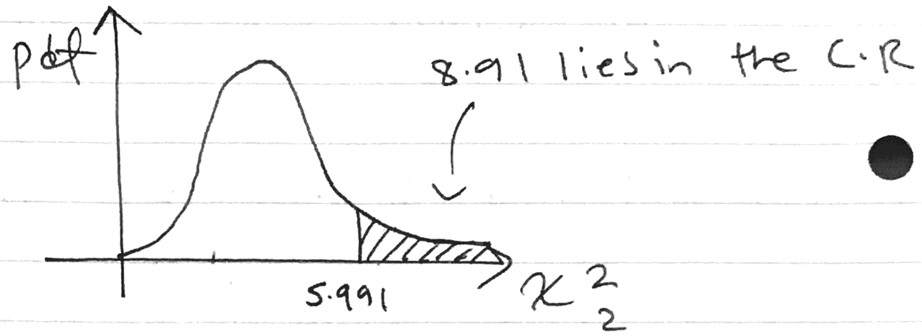
$$X^2 = \sum \frac{(O-E)^2}{E} = 8.91$$

$$\nu = (\text{rows} - 1)(\text{columns} - 1) = (2-1)(3-1) = 2$$

$\therefore$ critical value $= \chi^2_2(5\%) = 5.991$

$$8.91 > 5.991$$



$\therefore$ Result is significant.
Reject H_0.
Evidence suggests there is an association between gender & type of drink preferred.

b) $\chi^2_2(0.5\%) = 10.597$

$$8.91 < 10.597$$

With a 0.5% significance level, our conclusion would change to: "there is no association between gender and type of drink preferred".

Q6a) $p = \dfrac{\text{total failed tasks}}{\text{total tasks}} = \dfrac{1(21) + 2(45) + 3(42) + \cdots}{8(125)}$

$= \dfrac{300}{1000} = \boxed{0.3}$

b) $r = 125 \times P(X=3)$, where $X \sim B[8, 0.3]$
X = no. of tasks failed / 8.

$\therefore r = 125\left[\binom{8}{3}(0.3^3)(0.7^5)\right] = \boxed{31.76}$

$s = 125 - (\sum E_i) = \boxed{1.41}$

c) $\underline{H_0}$: Binomial distribution is a suitable model for these data.
$\underline{H_1}$: Binomial distribution is not a suitable model for these data.

Remember, expected frequencies must be greater than 5 for the test statistic X^2 to be approximated well by the chi-squared distribution(s).

So pool last two groups to give:

No. failed	0	1	2	3	4	≥ 5
E_i	7.21	24.71	37.06	31.76	17.02	7.24
O_i	2	21	45	42	12	3
$\frac{(O-E)^2}{E}$	3.765	0.557	1.701	3.294	1.481	2.483

$X^2 = \sum \dfrac{(O-E)^2}{E} = 13.28 = (13.3)$

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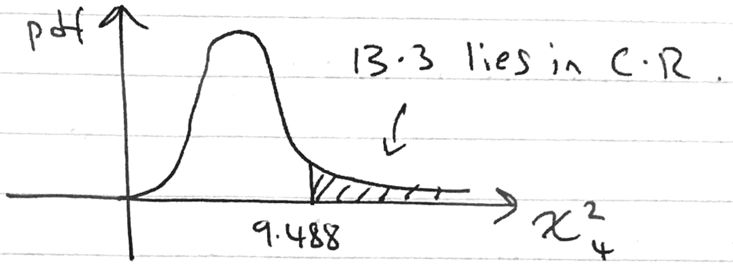
subtract an additional 1 as p was calculated in (a).

$\nu = 6 - 1 - 1 = 4$

$\therefore$ critical value $= \chi^2_4(5\%) = 9.488$

$13.3 > 9.488$



$\therefore$ Result is significant.
Reject H_0.
Evidence suggests that Binomial distribution is not a suitable model for these data.

d) Binomial is not a suitable model so the assumption that p is constant is <u>not</u> valid. (Belief is not justified).

Q7a) $Y \sim N(40, 3^2)$ $\quad W \sim N(50, 2^2)$

$X = 4Y - 3W$

$E(X) = 4E(Y) - 3E(W)$

$= 4(40) - 3(50) = 10$

$Var(X) = 4^2 Var(Y) + 3^2 Var(W)$

$= 4^2(3^2) + 3^2(2^2) = 180$

$\therefore \; X \sim N(10, 180)$

so $P(X > 25) = P(Z > \frac{25-10}{\sqrt{180}})$

$= P(Z > 1.12) = 1 - P(Z < 1.12)$

$= 1 - 0.8686 = \boxed{0.1314}$

b) $A = Y_1 + Y_2 + Y_3$

$E(A) = 3E(Y) = 3(40) = 120$

$Var(A) = 3 \times Var(Y) = 3(3^2) = 27$

($Y_1 + Y_2 + Y_3 \neq 3Y$ so do <u>not</u> square the 3)

and so $A \sim N(120, 27)$

and $C \sim N(115, \sigma^2)$

$\therefore \ (A-C) \sim N(5, (27+\sigma^2))$

$$P(A-C < 0) = 0.2$$

$$P\left(Z < \frac{0-5}{\sqrt{\sigma^2+27}}\right) = 0.2$$

$$P\left(Z > \frac{5}{\sqrt{\sigma^2+27}}\right) = 0.2$$

but $P(Z > 0.8416) = 0.2$

hence $\dfrac{5}{\sqrt{\sigma^2+27}} = 0.8416$

$$\therefore \ \sqrt{\sigma^2+27} = \frac{5}{0.8416}$$

$$\sigma^2+27 = \left(\frac{5}{0.8416}\right)^2$$

$$\sigma^2 = \left(\frac{5}{0.8416}\right)^2 - 27$$

$$= \boxed{8.30}$$