Statistics S2 Mark scheme

Question	Scheme	Marks
1(a)	$X \sim \text{Po}(3.2)$	B1
	$\text{P}(X=3) = \dfrac{e^{-3.2}3.2^3}{3!}$	M1
	$= 0.2226$ awrt 0.223	A1
		(3)
(b)	$Y \sim \text{Po}(1.6)$	B1
	$\text{P}(Y \geq 1) = 1 - \text{P}(Y=0)$ $= 1 - e^{-1.6}$	M1
	$= 0.7981$ awrt 0.798	A1
		(3)
(c)	$X \sim \text{Po}(0.8)$	
	$\dfrac{\text{P}(X=1)\times\text{P}(X=3)}{\text{P}(Y=4)} = \dfrac{\left(e^{-0.8}\times 0.8\right)\times\left(\dfrac{e^{-0.8}0.8^3}{3!}\right)}{\dfrac{e^{-1.6}1.6^4}{4!}}$ $= \dfrac{0.3594\times 0.0383}{0.05513}$	M1 M1 M1 A1
	$= 0.25$	A1
		(5)
(d)	$A \sim \text{Po}(72)$ approximated by $\text{N}(72,72)$	B1
	$\dfrac{5000}{60} = 83.33$	M1
	$\text{P}(A \geq 84) = \text{P}\left(Z \geq \dfrac{83.5-72}{\sqrt{72}}\right)$	M1 M1
	$= \text{P}(Z \geq 1.355\ldots)$	
	$= 0.0869$ awrt 0.087/0.088	A1
		(5)
		(16 marks)

Notes:

(a)

B1: For writing or using Po(3.2)

M1: $\dfrac{e^{-\lambda}\lambda^3}{3!}$

(b)

B1: For writing or using Po(1.6)

M1: $1 - \text{P}(Y=0)$ or $1 - e^{-\lambda}$

Question 1 notes *continued*	
(c)	
M1:	Using Po(0.8) with $X = 1$ or $X = 3$ (may be implied by 0.359... or 0.0383...)
M1:	$\left(e^{-\lambda} \times \lambda\right) \times \left(\frac{e^{-\lambda}\lambda^3}{3!}\right)$ (consistent lambda) awrt 0.0138 implies 1st 2 M marks
M1:	Correct use of conditional probability with denominator $= \frac{e^{-1.6}1.6^4}{4!}$
A1:	Fully correct expression
A1:	0.25 (allow awrt 0.250)
(d)	
B1:	Writing or using N(72,72)
M1:	For exact fraction **or** awrt 83.3 (may be implied by 84) (Note: Use of N(4320,4320) can score B1 and 1st M1)
M1:	Using 84 +/– 0.5
M1:	Standardising using 82.5, 83, $83.\dot{3}$ (awrt 83.3), 83.5, 83.8, 84 or 84.5, ‘their mean’ **and** ‘their sd’

Question	Scheme		Marks
2(a)	$P(X>4) = 1 - F(4)$ $= 1 - \frac{3}{5}$		M1
	$= \frac{2}{5}$ oe		A1
			(2)
(b)	1		B1
			(1)
(c)	$f(x) = \frac{dF(x)}{dx} = \frac{1}{5}$		M1
	$f(x) = \begin{cases} \frac{1}{5} & 1 \le x \le 6 \\ 0 & \text{otherwise} \end{cases}$		A1
			(2)
(d)	$E(X) = 3.5$		B1
			(1)
(e)	Variance $= \frac{(6-1)^2}{12}$	or $\int_1^6 \frac{1}{5}x^2\,dx - (3.5)^2$	M1
	$= \frac{25}{12}$	awrt 2.08	A1
			(2)
(f)	$E(X^2) = \text{Var}(X) + [E(X)]^2$ $= \frac{25}{12} + 3.5^2$ or $\int_1^6 \frac{1}{5}x^2\,dx$ $= \frac{43}{3}$	or $\int_1^6 \frac{1}{5}(3x^2+1)\,dx$	M1
	$E(3X^2+1) = 3E(X^2) + 1$	$= \left[\frac{3x^3}{15} + \frac{x}{5}\right]_1^6$	dM1
	$= 44$	$= 44$	A1cao
			(3)
			(11 marks)

Notes:

(a)
M1: Writing or using $1 - F(4)$ o.e.

(c)
M1: For differentiating to get $\frac{1}{5}$

Question 2 notes *continued*	
A1:	Both lines correct with ranges
(e) **M1:**	$\dfrac{(6-1)^2}{12}$ or $\int_1^6 \dfrac{1}{5}x^2\ \mathrm{d}x$ – 'their 3.5'^{2}
(f) **M1:**	"Their Var(X)" + ["their E(X)"]^2 (which must follow from the 1st method in (e)) **or** $\int_1^6 \dfrac{1}{5}x^2\ \mathrm{d}x$ **and** integrating $x^n \to \dfrac{x^{n+1}}{n+1}$ (may be seen in (e)) **or** writing $\int_1^6 \dfrac{1}{5}(3x^2+1)\ \mathrm{d}x$ (May be implied by $\dfrac{43}{3}$ seen)
dM1:	**Using** 3 × 'their E(X^2)' + 1 **or** $\int_1^6 \dfrac{1}{5}(3x^2+1)\ \mathrm{d}x$ and integrating $x^n \to \dfrac{x^{n+1}}{n+1}$

Question	Scheme	Marks
3(a)	(A random variable) that is a function of a (random) sample involving no unknown quantities/parameters **or** A quantity calculated solely from a random sample	B1
		(1)
(b)	If all possible samples are chosen from a population;	B1
	then the values of a statistic and the associated probabilities is a sampling distribution	
	or a probability distribution of a statistic	
		(1)
(c)	Mean $= 100 \times \frac{4}{7} + 200 \times \frac{3}{7}$ $= \frac{1000}{7}$ awrt 143	B1
	Variance $= 100^2 \times \frac{4}{7} + 200^2 \times \frac{3}{7} - \left(\frac{1000}{7}\right)^2$	M1
	$= \frac{120000}{49}$ awrt 2450 (to 3sf)	A1
		(3)
(d)	(100,100,100)	B2
	(100,100,200) (100,200,100) (200,100,100) or 3 x (100,100,200)	
	(100,200,200) (200,100,200) (200,200,100) or 3 x (100,200,200)	
	(200,200,200)	
		(2)
(e)	(100,100,100) $\left(\frac{4}{7}\right)^3 = \frac{64}{343}$ awrt 0.187	B1 both
	(200,200,200) $\left(\frac{3}{7}\right)^3 = \frac{27}{343}$ awrt 0.0787	
	(100,100,200) $3 \times \left(\frac{4}{7}\right)^2 \times \left(\frac{3}{7}\right) = \frac{144}{343}$ awrt 0.420 (allow 0.42)	M1
	(100,200,200) $3 \times \left(\frac{4}{7}\right) \times \left(\frac{3}{7}\right)^2 = \frac{108}{343}$ awrt 0.315	A1

Question	Scheme	Marks
3(e) *continued*		
	m | 100 | $400/3$ awrt 133 | $500/3$ awrt 167 | 200	
	$\mathrm{P}(M=m)$ | $\frac{64}{343}$ or awrt 0.187 | $\frac{144}{343}$ or awrt 0.420 (allow 0.42) | $\frac{108}{343}$ or awrt 0.315 | $\frac{27}{343}$ or awrt 0.0787	A1
		(4)
		(11 marks)

m	100	$400/3$ awrt 133	$500/3$ awrt 167	200
$\mathrm{P}(M=m)$	$\frac{64}{343}$ or awrt 0.187	$\frac{144}{343}$ or awrt 0.420 (allow 0.42)	$\frac{108}{343}$ or awrt 0.315	$\frac{27}{343}$ or awrt 0.0787

Notes:

(a)

B1: For a definition which includes each of the following 3 aspects

A **function**[1] of a (random) **sample**[2] involving **no unknown quantities/parameters**[3]

1. function/quantity/calculation/value/random variable
2. sample/observations/data
3. no unknown parameters/no unknown values/solely (from a sample)

(b)

B1: Requires all underlined words: <u>All</u> <u>values</u> of a <u>statistic</u> with their associated <u>probabilities</u>

or

<u>probability distribution</u> of a <u>statistic</u>

(c)

M1: $100^2\times\frac{4}{7}+200^2\times\frac{3}{7}-(\text{their mean})^2$

(d)

B1: Any 2 of (100,100,100), (100,100,200) any order, (100,200,200) any order, (200,200,200)

B1: All correct, allow 3 × (100,100,200) and 3 × (100,200,200) and (100,100,100) and (200,200,200)

(Note: Allow other notation for 100 and 200 e.g. Small and Large)

(e)

B1: Both probabilities for (100,100,100) and (200,200,200) correct

M1: $3\times p^2\times(1-p)$

A1: Either correct

A1: All means correct **and** all probabilities correct (table not required but means must be associated with correct probabilities)

Question	Scheme	Marks
4(a)	$X \sim \text{Po}(6)$	M1
	$\text{P}(5 \le X < 7) = \text{P}(X \le 6) - \text{P}(X \le 4)$ or $\dfrac{e^{-6}6^5}{5!} + \dfrac{e^{-6}6^6}{6!}$ $= 0.6063 - 0.2851$	M1
	$= 0.3212$ awrt 0.321	A1
		(3)
(b)	H_0: $\lambda = 9$ H_1: $\lambda < 9$	B1
	$X \sim \text{Po}(9)$ therefore $\text{P}(X \le 4) = 0.05496...$ or CR $X \le 3$	B1
	Insufficient evidence to reject H_0 **or** Not Significant **or** 4 does not lie in the critical region.	dM1
	There is no evidence that the mean number of accidents at the crossroads has reduced/decreased.	A1cso
		(4)
		(7 marks)

Notes:

(a)

M1: Writing or using Po(6)

M1: Either $\text{P}(X \le 6) - \text{P}(X \le 4)$ or $\dfrac{e^{-\lambda}\lambda^5}{5!} + \dfrac{e^{-\lambda}\lambda^6}{6!}$

(b)

B1: Both hypotheses correct (λ or μ) allow 0.5 instead of 9

B1: Either awrt 0.055 **or** critical region $X \le 3$

dM1: For a correct comment (dependent on previous B1)

Contradictory non-contextual statements such as "not significant" so "reject H_0" score M0.

(May be implied by a correct contextual statement)

A1: Cso requires correct contextual conclusion with underlined words and all previous marks in (b) to be scored.

Question	Scheme	Marks
5(a)	$\int_{-1}^{2} k(x^2+a)\mathrm{d}x + \int_{2}^{3} 3k\ \mathrm{d}x = 1$	M1
	$\left[k\left(\frac{x^3}{3}+ax\right)\right]_{-1}^{2} + [3kx]_2^3 = 1$	dM1
	$k\left(\frac{8}{3}+2a+\frac{1}{3}+a\right)+9k-6k=1$	A1
	$6k+3ak=1$ $\int_{-1}^{2} k(x^3+ax)\mathrm{d}x + \int_{2}^{3} 3kx\ \mathrm{d}x \left[=\frac{17}{12}\right]$	M1
	$\left[k\left(\frac{x^4}{4}+\frac{ax^2}{2}\right)\right]_{-1}^{2} + \left[\frac{3kx^2}{2}\right]_2^3 = \frac{17}{12}$	dM1
	$k\left(4+2a-\frac{1}{4}-\frac{a}{2}\right)+\frac{27k}{2}-6k=\frac{17}{12}$	A1
	$\frac{45k}{4}+\frac{3ak}{2}=\frac{17}{12}$ $135k+18ak=17$ $99k=11$	ddM1
	$a=1, k=\frac{1}{9}$	A1
		(8)
(b)	2	B1
		(1)
		(9 marks)

Notes:

(a)

M1: Writing or using $\int_{-1}^{2} k(x^2+a)\mathrm{d}x + \int_{2}^{3} 3k\ \mathrm{d}x = 1$ ignore limits.

dM1: Attempting to integrate at least one $x^n \to \frac{x^{n+1}}{n+1}$ **and** sight of correct limits (dependent on previous M1).

A1: Correct equation – need not be simplified.

M1: $\int_{-1}^{2} k(x^3+ax)\mathrm{d}x + \int_{2}^{3} 3kx\ \mathrm{d}x$ ignore limits.

dM1: Setting $=\frac{17}{12}$ **and** attempting to integrate at least one $x^n \to \frac{x^{n+1}}{n+1}$ **and** sight of correct limits (dependent on previous M1).

Question 5 notes *continued*	
A1:	A correct equation – need not be simplified.
ddM1:	Attempting to solve two simultaneous equations in *a* and *k* by eliminating 1 variable (dependent on 1st and 3rd M1s).
A1:	Both *a* and *k* correct.

Question	Scheme	Marks
6(a)	$P(X = 5) = {}^{20}C_5(0.3)^5(0.7)^{15}$ or $0.4164 - 0.2375$	M1
	$= 0.17886\ldots$ awrt 0.179	A1
		(2)
(b)	Mean = 6	B1
	sd = $\sqrt{20\times0.7\times0.3}$	M1
	$= 2.049\ldots$ awrt 2.05	A1
		(3)
(c)	H_0: $p = 0.3$ H_1: $p > 0.3$	B1
	$X \sim B(20,0.3)$	M1
	$P(X \geq 8) = 0.2277$ or $P(X \geq 10) = 0.0480$, so CR $X \geq 10$	A1
	Insufficient evidence to reject H_0 **or** Not Significant **or** 8 does not lie in the critical region.	dM1
	There is no evidence to support the Director (of Studies') belief/There is no evidence that the proportion of parents that do not support the new curriculum is greater than 30%	A1 cso
		(5)
(d)	$X \sim B(2n, 0.25)$	
	$X \sim B(8, 0.25)$ $P(X \geq 4) = 0.1138$	M1
	$X \sim B(10, 0.25)$ $P(X \geq 5) = 0.0781$	
	$2n = 10$ $n = 5$	A1 A1
		(3)
		(13 marks)

Notes:

(a)

M1: ${}^{20}C_5(p)^5(1-p)^{15}$ or using $P(X \leq 5) - P(X \leq 4)$

(b)

M1: Use of $20\times0.7\times0.3$ (with or without the square root).

(c)

B1: Both hypotheses correct (p or π).

M1: Using $X \sim B(20,0.3)$ (may be implied by 0.7723, 0.2277, 0.8867 or 0.1133)

A1: Awrt 0.228 **or** CR $X \geq 10$

dM1: A correct comment (dependent on previous M1)

A1: Cso requires correct contextual conclusion with underlined words and all previous marks in (c) to be scored.

Question 6 notes *continued*	
(d)	
M1:	For 0.1138 or 0.0781 or 0.8862 or 0.9219 seen.
A1:	B(10, 0.25) selected (may be implied by $n = 10$ or $2n = 10$ or $n = 5$). An answer of 5 with no incorrect working seen scores 3 out of 3. Special Case: Use of a normal approximation.
M1:	For $\frac{(n-0.5)-\frac{n}{2}}{\sqrt{\frac{3}{8}n}} = z$ with $1.28 \le z \le 1.29$, 1st A1 for n=4.2/4.3, 2nd A1 for n=5

Question	Scheme	Marks
7	$Y \sim \mathrm{N}\left(\frac{n}{5}, \frac{4n}{25}\right)$	B1
	$\mathrm{P}(Y \geq 30) = \mathrm{P}\left(Z > \frac{29.5 - n/5}{\frac{2}{5}\sqrt{n}}\right)$	M1 M1 A1
	$\frac{29.5 - n/5}{\frac{2}{5}\sqrt{n}} = 2$	B1
	$n + 4\sqrt{n} - 147.5 = 0$ or $0.04n^2 - 12.44n + 870.25 = 0$	dM1
	$\sqrt{n} = 10.3...$ $n = 106.26\ldots$ or $n = 204.73\ldots$	A1
	$n = 106$	A1 cao
		(8 marks)

Notes:

B1: Writing or using $\mathrm{N}\left(\frac{n}{5}, \frac{4n}{25}\right)$

M1: Writing or using 30 +/– 0.5

M1: Standardising using 29, 29.5, 30 or 30.5 and their mean and their sd

A1: Fully correct standardisation (allow +/-)

B1: For z = +/– 2 or awrt 2.00 must be compatible with their standardisation

dM1: (Dependent on 2nd M1) getting quadratic equation **and** solving leading to a value of $\sqrt{n}$ or n

A1: Awrt 10.3 **or** awrt (106 **or** 107 **or** 204 **or** 205)

A1: For 106 only (must reject other solutions if stated)

(Note: $\frac{29.5 - n/5}{\frac{2}{5}\sqrt{n}} = -2$ leading to an answer of 106 may score B1M1M1A1B0M1A1A1)