Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Sample Assessment Materials for first teaching September 2018

(Time: 1 hour 30 minutes) | Paper Reference **WST02/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Statistics S2

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

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S2 sample 2018

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Answer ALL questions. Write your answers in the spaces provided.

1. The number of cars caught speeding per day, by a particular camera, has a Poisson distribution with mean 0.8

(a) Find the probability that in a given 4 day period exactly 3 cars will be caught speeding by this camera.

(3)

A car has been caught speeding by this camera.

(b) Find the probability that the period of time that elapses before the next car is caught speeding by this camera is less than 48 hours.

(3)

Given that 4 cars were caught speeding by this camera in a two day period,

(c) find the probability that 1 was caught on the first day and 3 were caught on the second day.

(5)

Each car that is caught speeding by this camera is fined £60

(d) Using a suitable approximation, find the probability that, in 90 days, the total amount of fines issued will be more than £5000

(5)

(a) $0.8 \rightarrow 1\text{day}$. $x = 3.2$

$x \rightarrow 4\text{days}$.

$X \sim Po(3.2)$.

$P(X=3)$.

$$\frac{e^{-3.2} \times 3.2^3}{3!}$$

$= 0.2226$

(b) $48\text{hrs} \rightarrow 2\text{days}$.

$0.8 \rightarrow 1\text{day}$

$x \rightarrow 2\text{days}$ $x = 1.6$.

$Y \sim Po(1.6)$.

$P(Y \geq 1) \Rightarrow 1 - P(Y=0)$.

$$1 - \left(\frac{e^{-1.6} \times 1.6^0}{0!}\right) = 0.7981$$

(c) $X \sim Po(0.8)$ $Y \sim Po(1.6)$

$$\frac{P(X=1) \times P(X=3)}{P(Y=4)}$$

$$= \frac{0.359463 \times 0.03894}{0.05513}$$

$= 0.25$

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Question 1 continued

(d) $0.8 \rightarrow 1$ day.
$x \rightarrow 90$ days.
$x = 72$

$X \sim Po(72)$. $\frac{5000}{60} = 83.3$ cars

$X \sim N(72, 72)$.

$P(X > 83.3) = P(X > 83.5)$

$= P\left(Z \geq \frac{83.5 - 72}{\sqrt{72}}\right)$

$= P(Z \geq 1.36)$

$= 1 - 0.9131 = 0.0869$

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2. A continuous random variable X has cumulative distribution function

$$F(x) = \begin{cases} 0 & x < 1 \\ \frac{1}{5}(x-1) & 1 \leqslant x \leqslant 6 \\ 1 & x > 6 \end{cases}$$

(a) Find $P(X > 4)$ **(2)**

(b) Write down the value of $P(X \neq 4)$ **(1)**

(c) Find the probability density function of X, specifying it for all values of x **(2)**

(d) Write down the value of $E(X)$ **(1)**

(e) Find $\mathrm{Var}(X)$ **(2)**

(f) Hence or otherwise find $E(3X^2 + 1)$ **(3)**

(a) $1 - F(4)$

$1 - 3/5 = 2/5$.

b) 1

(c) differentiate

$\frac{1}{5}(x-1) = \frac{1}{5}x - \frac{1}{5}$

$\frac{dy}{dx} = \frac{1}{5}$.

$f(x) = \begin{cases} 1/5 & 1 \leq x \leq 6. \\ 0 & \text{otherwise}. \end{cases}$

(d) $\frac{6+1}{2} = 7/2 = 3.5$.

e) $\mathrm{Var}(X) = E(X^2) - (E(X))^2$

$=$

$\frac{(b-a)^2}{12} = \frac{(6-1)^2}{12} = \frac{25}{12}$

(f) $E(3X^2 + 1)$

$= 3E(X^2) + 1$

$\frac{25}{12} = x - (3.5)^2$

$x = \frac{43}{3}$.

$3\left(\frac{43}{3}\right) + 1 = \underline{\underline{44}}$

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3. Explain what you understand by

(a) a statistic, **(1)**

(b) a sampling distribution. **(1)**

A factory stores screws in packets. A small packet contains 100 screws and a large packet contains 200 screws. The factory keeps small and large packets in the ratio 4:3 respectively.

(c) Find the mean and the variance of the number of screws in the packets stored at the factory. **(3)**

A random sample of 3 packets is taken from the factory and Y_1, Y_2 and Y_3 denote the number of screws in each of these packets.

(d) List all the possible samples. **(2)**

(e) Find the sampling distribution of $\bar{Y}$ **(4)**

(a) A quantity calculated solely from a random sample.

(b) Probability distribution of a statistic.

(c)

x	100	200
$P(X=x)$	$4/7$	$3/7$

$E(X) = \left(100 \times \frac{4}{7}\right) + \left(200 \times \frac{3}{7}\right)$

$= \frac{1000}{7}$

$E(X^2) = (100^2 \times 4/7) + (200^2 \times 3/7)$

$= 160{,}000/7$.

$\frac{160{,}000}{7} - \left(\frac{1000}{7}\right)^2 = \frac{120{,}000}{49}$.

(d) $(100, 100, 100)$

$3\times(100, 100, 200)$

$3\times(100, 200, 200)$

$(200, 200, 200)$.

(e)

X	100	$400/3$	$500/3$	200
$P(X=x)$	$\frac{64}{343}$	$\frac{144}{343}$	$\frac{108}{343}$	$\frac{27}{343}$
	0.1866	0.4196	0.3149	0.07872

$(100, 100, 100)$ $\left(\frac{4}{7}\right)^3 = \frac{64}{343}$

$3\times(100, 200, 100)$ $3\times\left(\frac{4}{7}\right)^2 \times \frac{3}{7} = \frac{144}{343}$

$3\times(100, 200, 200)$ $3\times\frac{4}{7}\times\left(\frac{3}{7}\right)^2 = \frac{108}{343}$

$(200, 200, 200)$ $\left(\frac{3}{7}\right)^3 = \frac{27}{343}$

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4. Accidents occur randomly at a crossroads at a rate of 0.5 per month. A researcher records the number of accidents, X, which occur at the crossroads in a year.

(a) Find $P(5 \leqslant X < 7)$
(3)

A new system is introduced at the crossroads. In the first 18 months, 4 accidents occur at the crossroads.

(b) Test, at the 5% level of significance, whether or not there is reason to believe that the new system has led to a reduction in the mean number of accidents per month. State your hypotheses clearly.
(4)

a) $0.5 \rightarrow$ 1 mont.
x – 12 months – $\underline{x = 6}$

$X \sim Po(6)$.

$p(5 \leq X < 7)$.

$p(X \leq 6) - p(X \leq 4)$

$0.6063 - 0.2851$

$\underline{= 0.3212}$

(b) 0.5 – 1 mont.
x = 18 months
$\underline{x = 9}$

$X \sim Po(9)$.

$H_0: \lambda = 9$.
$H_1: \lambda < 9$.

$p(X \leq 4) = 0.0550$. x
$p(X \leq 3) = 0.0212$ ✓.

Critical region is $x \leq 3$.

∴ Insufficient evidence to reject H_0.

∴ No evidence that the mean no. of accidents at the crossroads has ~~cr~~ reduced.

Leave blank

5. The continuous random variable X has probability density function f(x) given by

$$f(x) = \begin{cases} k(x^2 + a) & -1 < x \leqslant 2 \\ 3k & 2 < x \leqslant 3 \\ 0 & \text{otherwise} \end{cases}$$

where k and a are constants.

Given that $E(X) = \dfrac{17}{12}$

(a) find the value of k and the value of a **(8)**

(b) Write down the mode of X **(1)**

(a) $\int_{-1}^{2} (kx^2 + ka)\,dx + \int_{2}^{3} 3k\,dx = 1$

$k\left[\frac{x^3}{3} + ax\right]_{-1}^{2} + \left[3kx\right]_{2}^{3} = 1$

$k\left(\frac{8}{3} + 2a\right) - k\left(-\frac{1}{3} - a\right) + (9k - 6k) = 1$

$k(6 + 3a) = 1.$

$\cancel{6a}\ 6k + 3ak = 1$

$k\int_{-1}^{2} x^3 + ax\,dx + \int_{2}^{3} 3kx\,dx = \frac{17}{12}$

$k\left[\frac{x^4}{4} + \frac{ax^2}{2}\right]_{-1}^{2}$

$+ \left[\frac{3kx^2}{2}\right]_{2}^{3} = \frac{17}{12}$

$k\left[(4 + 2a) - \left(\frac{1}{4} + \frac{1}{2}a\right)\right]$

$+ \left(\frac{27}{2}k - 6k\right) = \frac{17}{12}.$

$4k + 2ak - \frac{1}{4}k - \frac{1}{2}ka + \frac{27}{2}k - 6k = \frac{17}{12}$

$\frac{45}{4}k + \frac{3}{2}ak = \frac{17}{12}.$

$(6k + 3ak)\ \cancel{\times 1.5} = 1$

$\frac{45}{4}k + \frac{3}{2}ak = \frac{17}{12}$

Using simultaneous eqns.

$135k + 18ak = 17.$

$99k = 11$

$k = 1/9.$

$\therefore a = 1.$

(b) Mode = 2.

Leave blank

6. The Headteacher of a school claims that 30% of parents do not support a new curriculum. In a survey of 20 randomly selected parents, the number, X, who do not support the new curriculum is recorded.

Assuming that the Headteacher's claim is correct, find

(a) the probability that $X = 5$

(2)

(b) the mean and the standard deviation of X

(3)

The Director of Studies believes that the proportion of parents who do not support the new curriculum is greater than 30%. Given that in the survey of 20 parents 8 do not support the new curriculum,

(c) test, at the 5% level of significance, the Director of Studies' belief. State your hypotheses clearly.

(5)

The teachers believe that the sample in the original survey was biased and claim that only 25% of the parents are in support of the new curriculum. A second random sample, of size $2n$, is taken and exactly half of this sample supports the new curriculum.

A test is carried out at a 10% level of significance of the teachers' belief using this sample of size $2n$

Using the hypotheses $H_0: p = 0.25$ and $H_1: p > 0.25$

(d) find the minimum value of n for which the outcome of the test is that the teachers' belief is rejected.

(3)

(a) $X \sim B(20, 0.3)$

$p(X=5)$.

$\binom{20}{5}(0.3)^5(0.7)^{15}$

$= \underline{0.179}$

(b) Mean $= np = 6$

Variance $= np(1-p)$

$= 6(0.7) = \sqrt{21/5}$

$= 2.05$

(c) $H_0: p = 0.3$

$H_1: p > 0.3$.

$p(X \geq 9) = 1 - p(X \leq 8) = 1 - 0.8867$

$= 0.1133$ ×

$P(X \geq 10) = 1 - p(X \leq 9) = 1 - 0.9520$

$= 0.048$ ✓

$CR = P(X \geq 10)$

∴ Doesn't lie in critical region

∴ Reject H_0 ∴ No evidence to support Directors claim.

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Question 6 continued

(d) $X \sim B(2n, 0.25)$.

$H_0: p = 0.25$
$H_1: p > 0.25$.

$n = 3$
$X \sim B(6, 0.25)$

$P(X \geq 3) = 1 - P(X \leq 2)$
$= 0.1694 > 0.1$
$\therefore$ can't reject H_0 as not enough evidence.

$n = 4$
$X \sim B(8, 0.25)$

$P(X \geq 4) = 1 - P(X \leq 3)$

$= 0.1138 > 0$
$\therefore$ not enough evidence to reject H_0.

$n = 5$.
$X \sim B(10, 0.25)$

$P(X \geq 5) = 1 - P(X \leq 4)$

$= 0.0781 < 0.1$

$\therefore$ Reject H_0.

$\therefore n = 5$

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7. A multiple choice examination paper has n questions where $n > 30$
Each question has 5 answers of which only 1 is correct. A pass on the paper is obtained by answering 30 or more questions correctly.

The probability of obtaining a pass by randomly guessing the answer to each question should not exceed 0.0228

Use a normal approximation to work out the greatest number of questions that could be used.

(8)

$n > 30$. $1/5 \to q$ correct

$4/5 \to q$ incorrect.

pass - 30 or more q correct.

$Y \sim N\left(\frac{n}{5}, \frac{4n}{25}\right)$ $np = n \times \frac{1}{5}$

$np(1-p) = \frac{4n}{25}$.

$P(Y \geq 30) = P(Y > 29.5)$

$P\left(Z \geq \frac{29.5 - n/5}{\sqrt{4n/25}}\right) \leq 0.0228$.

$1 - P\left(Z \leq \frac{29.5 - n/5}{\sqrt{\frac{4n}{25}}}\right) \geq 0.9772$

$\frac{29.5 - n/5}{\sqrt{\frac{4n}{25}}} \geq 2$.

$\sqrt{\frac{4n}{25}} = \frac{2}{5}\sqrt{n}$.

$29.5 - \frac{n}{5} = \frac{4}{5}\sqrt{n}$.

$147.5 - n = 4\sqrt{n}$.

$n + 4\sqrt{n} - 147.5 = 0$.

let $n = a^2$.

$a^2 + 4a - 147.5 = 0$.

$a = 10.31$ $a = -14.31$ ✗

$(10.31)^2 = 106.25$

$(-14.31)^2 = 204.78$.

$\underline{\underline{n = 106}}$