S2 October 2017 (MA)

Q1a) $L \sim N(\mu, 0.2^2)$

$P(L < 200) = 0.05$

$P\left(Z < \frac{200-\mu}{0.2}\right) = 0.05$

$P\left(Z > \frac{\mu-200}{0.2}\right) = 0.05$

but $P(Z > 1.6449) = 0.05$

$\therefore \quad 1.6449 = \frac{\mu - 200}{0.2}$

$\Rightarrow \quad \mu = 0.2(1.6449) + 200$

$= \boxed{200.3}$

b) let X = no. of rods with length < 200.

$X \sim B[8, 0.05]$

$P(X \geq 3) = 1 - P(X \leq 2)$

$= 1 - 0.9942$

$= \boxed{0.0058}$

c) for 60 rods, $X_2 \sim B[60, 0.05]$

n large, p small $\therefore X_2 \approx \sim Po(3)$

$np = 60 \times 0.05 = 3$

$P(\text{required}) = P(X_2 > 5) = 1 - P(X_2 \leq 5)$

$= 1 - 0.9161 = \boxed{0.0839}$

(Q2a) $k\int_2^{10} (s-2)(10-s)\, ds = 1$

$k\int_2^{10} [10s - s^2 - 20 + 2s]\, ds = 1$

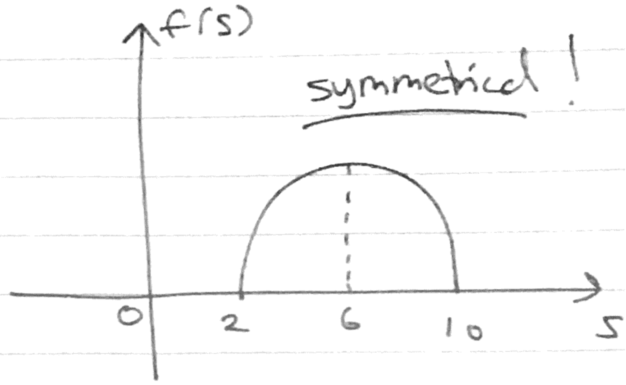
$k\int_2^{10} [-s^2 + 12s - 20]\, ds = 1$

$k\left[\frac{-s^3}{3} + 6s^2 - 20s\right]_2^{10} = 1$

$K\left[\frac{200}{3}\right] - k\left[\frac{-56}{3}\right] = 1$

$k\left[\frac{256}{3}\right] = 1 \qquad \therefore \boxed{k = \frac{3}{256}}$

b) $E(S) = 6$



c) $E(S^2) = \int_2^{10} s^2 f(s)\, ds$

$= \frac{3}{256}\int_2^{10} [12s^3 - 20s^2 - s^4]\, ds$

$= \frac{3}{256}\left[\frac{12s^4}{4} - \frac{20s^3}{3} - \frac{s^5}{5}\right]_2^{10}$

$= \frac{3}{256}\left[\frac{10000}{3}\right] - \frac{3}{256}\left[-\frac{176}{15}\right]$

$= 39.2$

$Var(S) = E(S^2) - [E(S)]^2$

$= 39.2 - (6)^2 = 3.2$

$\therefore$ s.d $= \sqrt{3.2} = 1.78885\ldots$

$= \boxed{£1790}$

d) $\frac{3}{256}\int_{7.1}^{10} [12s - 20 - s^2]\, ds = \frac{3}{256}\left[6s^2 - 20s - \frac{s^3}{3}\right]_{7.1}^{10}$

$= \frac{3}{256}\left[\frac{200}{3} - 41.1563\ldots\right] = \boxed{0.3}$

1dp.

e) $X \sim B[12, 0.3]$

where X = no. of weeks (in a quarter) in which weekly sales exceed £7100.

$P(X \leq 5) = 0.8822$

$P(X = 6) = \binom{12}{6}(0.3)^6(0.7)^6 = 0.07925$

$P(X \geq 7) = 1 - P(X \leq 6) = 1 - 0.9614$
$= 0.0386$

$\therefore$ Expected bonus $= 0.8822(0) + 0.07925(1000) + 0.0386(5000)$

$= \boxed{£272}$ to 3s.f

Q3a) $X \sim Po(8)$ where X = weekly demand for Birdscope Cameras.

P(demand can't be met) $= P(X \geq 10) = 1 - P(X \leq 9)$

$= 1 - 0.7166$
$= \boxed{0.2834}$

b) Expected no. $= 0.2834 \times 50 = 14.2$

so $\boxed{14 \text{ weeks}}$

c) (ie) $p \times 50 < 2$ $\qquad (X \sim Po(8))$

$p < \frac{2}{50} \rightarrow p < 0.04$

so $P(X \geqslant n) < 0.04$

$P(B \geqslant 13) = 0.0638$
and $P(B \geqslant 14) = 0.0342$

so if they stock $\boxed{13}$ cameras then the probability that the demand can't be met on any week is 0.0342.

Hence in 50 weeks the estimated no. of weeks where demand can't be met will be less than 2.

$(0.0342 < 0.04)$

Answer = $\boxed{13}$

d) $H_0: \lambda = 80$ $\qquad$ for 10 weeks, $Y \sim Po(80)$.
$H_1: \lambda > 80$

λ is large so use normal approx.

$Y \simeq \sim N(80, 80)$

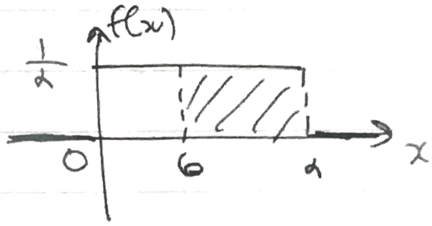
(applying c.c) $P(Y \geqslant 95) = P(Y > 94) = P(Y > 94.5)$

$= P\left(Z > \frac{94.5 - 80}{\sqrt{80}}\right) = P(Z > 1.62)$

$= 1 - P(Z < 1.62) = 0.0526$

$0.0526 > 0.05$

$\therefore$ Result is insignificant.
Accept H_0.
Evidence suggests that the demand for Birdscope cameras has not increased.



Q4a) $X \sim U[0, \alpha]$

$P(X > 6) = 0.6$

$$(\alpha - 6) \times \frac{1}{\alpha} = 0.6$$

$$\frac{\alpha - 6}{\alpha} = 0.6$$

$$1 - \frac{6}{\alpha} = 0.6$$

$$\frac{6}{\alpha} = 0.4$$

$$\therefore \alpha = \frac{6}{0.4} = \boxed{15}$$

b) $P(4 < X < 10) = \dfrac{(10-4)}{15} = \boxed{\dfrac{2}{5}}$

c) $Y \sim U[0, 20]$

$$E(Y) = \frac{0+20}{2} = \boxed{10}$$

$$Var(y) = \frac{(20-0)^2}{12} = \frac{100}{3}$$

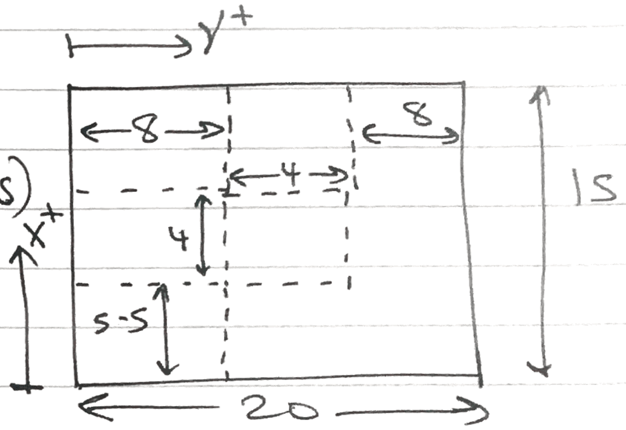
$$\therefore \sigma_y = \sqrt{\frac{100}{3}} = \frac{10}{\sqrt{3}} = \boxed{5.77}$$

d) $P(|y-4| < 2) = P(-2 < y-4 < 2)$

$$= P(2 < y < 6) = \frac{6-2}{20} = \boxed{\frac{1}{5}}$$

ei) P(required) . . . .

$$= P(8 < y < 12) \times P(5.5 < x < 9.5)$$

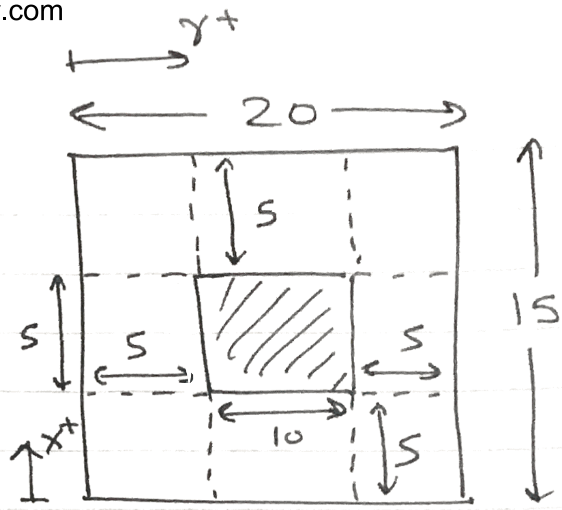


$$P(8 < y < 12) = \frac{12-8}{20} = \frac{4}{20} = \frac{2}{10} = \frac{1}{5}$$

$$P(5.5 < x < 9.5) = \frac{9.5 - 5.5}{15} = \frac{4}{15}$$

$$\therefore P(\text{required}) = \frac{4}{15} \times \frac{1}{5} = \boxed{\frac{4}{75}}$$

ii) Let shaded region represent a rectangle that is 5 cm from each side and top and bottom.



P(required) = 1 − P(centre of ship is in shaded area).

P(ship in shaded area) $= P(5 < x < 10) \times P(5 < y < 15)$

$$P(5 < x < 10) = \frac{(10-5)}{15} = \frac{1}{3}$$

$$P(5 < y < 15) = \frac{(15-5)}{20} = \frac{1}{2}$$

$\therefore$ P(ship in shaded area) $= \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$ //

hence P(required) $= 1 - \frac{1}{6} = \boxed{\frac{5}{6}}$ //

(Q5a) at $y = \alpha$, $k(y^2 - 2y - 3) = 4k(2y - 7)$ //

so.... $k(\alpha^2 - 2\alpha - 3) = 4k(2\alpha - 7)$

$\div k$: $\alpha^2 - 2\alpha - 3 = 8\alpha - 28$

$\Rightarrow$ $\alpha^2 - 10\alpha + 25 = 0$ //

By Quadratic formula

$\alpha = 5$ (R.R)

(you can also factorise to give $(\alpha-5)^2=0$)

now that we know α, we must find k...

$F(6) = 1$

$4k(2(6)-7) = 1$

$20k = 1 \quad \therefore k = \frac{1}{20}$

$P(\text{required}) = P(4.5 < Y \leq 5.5)$

$= F(5.5) - F(4.5)$

$= \frac{4}{20}(2(5.5)-7) - \frac{1}{20}((4.5)^2 - 2(4.5) - 3)$

$= \boxed{\frac{31}{80}}$

b) $\frac{d}{dy}\left(\frac{1}{20}(y^2 - 2y - 3)\right) = \frac{1}{20}(2y-2)$

$\frac{d}{dy}\left(\frac{4}{20}(2y-7)\right) = \frac{2}{5}$

$$\therefore f(y) = \begin{cases} \frac{1}{20}(2y-2), & 3 \leq y \leq 4.5 \\ \frac{2}{5}, & 4.5 < y \leq 6 \\ 0, & \text{otherwise.} \end{cases}$$

Q6) $X \sim B[n, \frac{1}{6}]$ where X = no. of sixes.

$$P(X < 50) = 0.0082.$$

$$np = \frac{n}{6}$$

$$np(1-p) = \frac{n}{6} \times \frac{5}{6} = \frac{5n}{36}$$

$$\therefore X \simeq \sim N\left(\frac{n}{6}, \frac{5n}{36}\right)$$

(applying c.c) $P(X < 50) = P(X < 49.5)$

$$= P\left(Z < \frac{49.5 - \frac{n}{6}}{\sqrt{\frac{5n}{36}}}\right) = 0.0082 \text{ //}$$

$$\therefore P\left(Z < \frac{\frac{n}{6} - 49.5}{\sqrt{\frac{5n}{36}}}\right) = 0.9918 \text{ //}$$

but $P(Z < 2.40) = 0.9918$ //

$$\therefore \quad \frac{\frac{n}{6} - 49.5}{\sqrt{\frac{5n}{36}}} = 2.40$$

$$\frac{n}{6} - 49.5 = \left(2.4\sqrt{\frac{5}{36}}\right) n^{\frac{1}{2}}$$

$$\underline{\times 6}: \quad n - 297 = \frac{12\sqrt{5}}{5} n^{\frac{1}{2}}$$

$$n - \frac{12\sqrt{5}}{5} n^{\frac{1}{2}} - 297 = 0$$

$$n^{\frac{1}{2}} = 9\sqrt{5}, \quad \underbrace{n^{\frac{1}{2}} = -\frac{33\sqrt{5}}{5}}_{\text{reject as } n^{\frac{1}{2}} > 0}$$

$$\therefore \quad n = (9\sqrt{5})^2$$

$$= \boxed{405}$$