S2 June 2018 (IAL) (MA)

Q1ai) $X \sim B[10, 0.05]$

$$P(X=1) = \binom{10}{1}(0.05)^1(0.95)^9 = \boxed{0.315}$$

ii) $P(X \geq 3) = 1 - P(X \leq 2) = 1 - 0.9885$

$$= \boxed{0.0115}$$

b) $np = 3$
$p = 0.05 \quad \therefore \quad n = \frac{3}{0.05} = \boxed{60}$

c) $P(X_2 \geq 1) > 0.99$ where X_2 = no. of people sold insurance on Friday.

$$X_2 \sim B[n, 0.05]$$

$$1 - P(X_2 = 0) > 0.99$$

$$P(X_2 = 0) < 0.01$$

$$(1)(0.05)^0(0.95)^n < 0.01$$

$$\log(0.95^n) < \log(0.01)$$

$$n \log 0.95 < \log 0.01$$

$$n > \frac{\log 0.01}{\log 0.95} \longrightarrow n > 89.78\ldots$$

so $\boxed{n_{min} = 90}$

Q2a) $H_0: \lambda = 4$ for $100m^2$, # of faults $= X$

$H_1: \lambda > 4$

$X \sim Po(8)$

$$P(X \geq 14) = 1 - P(X \leq 13)$$

$$= 1 - 0.9658$$

$$= \boxed{0.0342} < 0.05$$

$\therefore$ Result is significant.
Reject H_0.
Evidence suggests that Emma's belief is correct.

b) let Y = no. of faults in one piece of cloth length,

$Y \sim Po\left(\frac{4L}{50}\right)$

why: from a, $\lambda = 4$ for $50m$.

so for $1m$, $\lambda = \frac{4}{50}$

so for Lm, $\lambda = \frac{4}{50} \times L$

$$P(Y=0) = 0.9$$

$$e^{-\frac{4L}{50}} = 0.9$$

$$\ln\left(e^{-\frac{4L}{50}}\right) = \ln(0.9)$$

$$-\frac{4L}{50} = \ln 0.9$$

$$L = \frac{-50\ln(0.9)}{4} = 1.317\ldots$$

$$= \boxed{1.3 \text{ to } 2\text{s.f}}$$

c) $P(Y=0) = e^{-\frac{4}{50}(1.3)} \simeq 0.90$

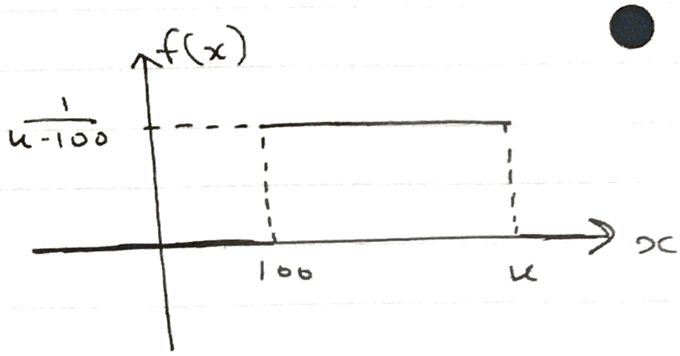
$P(Y \geq 1) = 1 - P(Y=0) \simeq 0.10$

$\therefore$ Expected profit $= [5000 \times 0.9] \times 2.5$

$$- [5000 \times 0.1] \times 0.5$$

$$= \boxed{£11000}$$

Q3a) $X \sim U[100, u]$



$$\frac{u-102}{u-100} = \frac{2}{3}$$

$$\frac{2}{3}u - \frac{200}{3} = u - 102$$

$$\frac{u}{3} = 102 - \frac{200}{3}$$

$$\therefore u = 3\left(102 - \frac{200}{3}\right) = \boxed{106}$$

b i) $P(X < 105) = \frac{5}{106-100} = \boxed{\frac{5}{6}}$

ii) 0.

c) $\frac{100+106}{2} = \boxed{103}$

d) $P(100 < X < a) = 0.15$

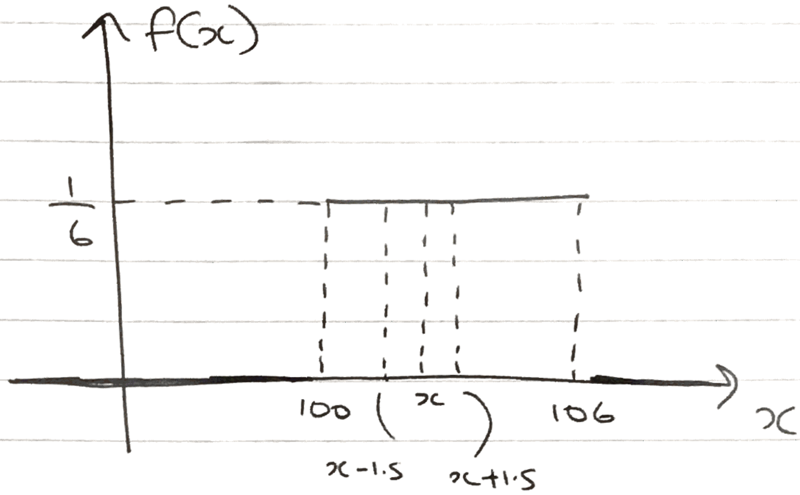
where a is the 15^{th} percentile.

$$\frac{a-100}{6} = 0.15$$

$$a - 100 = 6(0.15)$$

$$a = 100 + 6(0.15) = \boxed{100.9}$$

e) $P(X \geqslant x+1.5) = 3P(X \leqslant x-1.5)$



$$P(X \geqslant x+1.5) = \frac{106-(x+1.5)}{6} = \frac{104.5-x}{6}$$

$$3P(X \leqslant x-1.5) = 3\left[\frac{x-1.5-100}{6}\right] = \frac{x-101.5}{2}$$

$$\therefore \quad \frac{104.5-x}{6} = \frac{x-101.5}{2}$$

$\underline{\times 6}$: $104.5 - x = 3x - 304.5$

$$4x = 409$$

$$\therefore x = \frac{409}{4} = \boxed{102.25}$$

Q4a) Every possible sample has an equal chance of being selected.

b) Cartons of milk from the dairy (All of them)

c) $N(0, 1)$

d) The probability distribution of X.

ei) Just (II) isn't a statistic as it contains the population parameters μ and σ.

Q5ai) in 10 min, $X \sim Po(6)$

$$P(X = 7) = \frac{(e^{-6})(6^7)}{7!} = \boxed{0.138}$$

ii) $P(X > 7) = 1 - P(X \leq 7) = 1 - 0.744$

$$= \boxed{0.256}$$

b) for 1 minute, $\lambda = 0.6$
for n minutes, $\lambda = 0.6n$ //

so let Y = no. of cars pulling in n minutes

$$Y \sim Po(0.6n)$$

$$Y \simeq \sim N(0.6n, 0.6n)$$

$P(Y > 40) = 0.2266$

$P(Y > 40.5) = 0.2266$ ← applying c.c

$P\left(Z > \frac{40.5 - 0.6n}{\sqrt{0.6n}}\right) = 0.2266$

$P\left(Z < \frac{40.5 - 0.6n}{\sqrt{0.6n}}\right) = 0.7734$

but $P(Z < 0.75) = 0.7734$

$\therefore \frac{40.5 - 0.6n}{\sqrt{0.6n}} = 0.75$

$40.5 - 0.6n = (0.75\sqrt{0.6})n^{\frac{1}{2}}$

$0.6n + (0.75\sqrt{0.6})n^{\frac{1}{2}} - 40.5 = 0$

By Quadratic formula: $n^{\frac{1}{2}} = 2\sqrt{15}$

$n^{\frac{1}{2}} = -8.71$

reject $\therefore n^{\frac{1}{2}} > 0$

$\therefore n = (2\sqrt{15})^2 = \boxed{60}$

Q6a) $E(X) = \frac{1}{4}\int_0^1 (x)\,dx + \frac{1}{5}\int_1^2 (x^4)\,dx$

$= \frac{1}{4}\left[\frac{x^2}{2}\right]_0^1 + \frac{1}{5}\left[\frac{x^5}{5}\right]_1^2$

$= \frac{1}{4}\left(\frac{1}{2}\right) + \frac{1}{5}\left[\frac{32}{5} - \frac{1}{5}\right]$

$= \frac{273}{200} = \boxed{1.37}$

b) $E(X^2) = \frac{1}{4}\int_0^1 (x^2)\,dx + \frac{1}{5}\int_1^2 (x^5)\,dx$

$= \frac{1}{4}\left[\frac{x^3}{3}\right]_0^1 + \frac{1}{5}\left[\frac{x^6}{6}\right]_1^2$

$= \frac{1}{4}\left[\frac{1}{3}\right] + \frac{1}{5}\left[\frac{32}{3} - \frac{1}{6}\right]$

$= \frac{131}{60}$

$Var(X) = E(X^2) - [E(X)]^2 = \frac{131}{60} - (1.37)^2$

$= \boxed{0.320}$

c) $\int_0^x \left(\frac{1}{4}\right) dx = \left[\frac{1}{4}x\right]_0^x = \frac{1}{4}x$ //

for $0 \le x < 1$

<u>for $1 \le x \le 2$:</u>

$$F(1) + \int_1^x \left(\frac{x^3}{5}\right) dx$$

$$= \frac{1}{4} + \left[\frac{x^4}{20}\right]_1^x = \frac{1}{4} + \frac{x^4}{20} - \frac{1}{20}$$

$$= \frac{1}{5} + \frac{x^4}{20} = \frac{1}{20}(x^4 + 4)$$ //

so... $F(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{4}x, & 0 \le x < 1 \\ \frac{1}{20}(x^4+4), & 1 \le x \le 2 \\ 1, & x > 2 \end{cases}$

d) $F(1) = 0.25$ so median lies in 2^{nd} interval.

$F(m) = 0.50$

$\frac{1}{20}(m^4 + 4) = \frac{1}{2}$.

$m^4 + 4 = 10$

$m^4 = 6$

$m = \sqrt[4]{6} = \boxed{1.57}$

e) Mean < median
1.37 < 1.57

$\therefore$ Negative skew

Q7 a) $X \sim B[25, 0.40]$

$\left.\begin{array}{l} P(X \leq 3) = 0.0024 \\ P(X \leq 4) = 0.0095 \end{array}\right\}$ 0.0024 is closer to 0.0025

0.0043 is closer to 0.0050 $\left\{\begin{array}{l} P(X \geq 17) = 0.0043 \\ P(X \geq 18) = 0.0132 \end{array}\right.$

so C.R is $\boxed{X \geq 17}$ and $\boxed{X \leq 3}$

b) $0.0043 + 0.0024 = \boxed{0.0067}$

(0.67%)

c) $\underline{H_0}: p = 0.4 \quad \underline{H_1}: p < 0.4$

$R \sim B[50, 0.4]$

Where R = no. of red sweets (150).
(two - packets)

$P(R \leq 8) = 0.0002$ //

$0.0002 < 0.01$

∴ Result is significant
Reject H_0.
Evidence suggests the changes were successful