JUNE 2017 IAL MA S2

1 a) $X \sim Po\left(\frac{1}{4}\right)$

$P(X=0) = e^{-\frac{1}{4}} = 0.7788$

b) $P(X \geq 1) = 1 - P(X=0)$

$1 - 0.7788$

$= 0.221199\ldots$

on 3 days in a row

$= (0.221199\ldots)^3 = \underline{0.0108}$

c) p of no failures $= 0.7788$ per da

$Y \sim B(7, 0.7788)$

$P(X=5)$

$= {}^{7}_{5}C \times (0.7788)^5 \times (1-0.7788)^{7-5}$

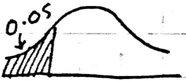
$= 0.294387\ldots$

$= \underline{0.2944}$

d) $H_0 : \lambda = 0.25$

$H_1 : \lambda < 0.25$

e) $0.25 \times 32 = 8$



$\therefore X \sim Po(8)$

$P(X \leq 3) = 0.0424$

CR : $X \leq 3$ $\therefore$ largest possible $= 3$

2ai) $X \sim B(6, 0.25)$

ii) * Prizes are randomly placed in packets

* Each packet has a 25% chance of having a prize.

* Each packet contains a prize indepen-dently to the others

b) $P(X=1) = {}^6C_1 \times 0.25^1 \times (1-0.25)^5$

$= 0.355957...$

$P(X=0) = 1 - P(X=1) = 1 - 0.355957...$

$= 0.64402...$

in 2 boxes $= 2 \times P(X=0) \times P(X=1)$

$2 \times (0.64402) \times (0.355957$

$= \underline{0.4584}$

c) $P(X \geq 2) = 1 - P(X \leq 1)$

$1 - 0.5339$

$= \underline{0.4661}$

d) $X \sim B(80, 0.4661)$

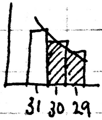
0.4661 = probability of 1 box ≥ 2 prizes

$P(X \leq 30)$

continued d) $Y \sim N(0.4661 \times 80,\ 0.4661 \times (1-0.4661) \times 80)$

$Y \sim N(37.3,\ 19.9)$

$P(X \leq 30)$



$= P(Y < 30.5)$

$= P\left(Z < \dfrac{30.5 - 37.3}{\sqrt{19.9}}\right)$

$= \phi(-1.524...)$

$1 - \phi(1.524..)$

$= 1 - 0.9357 = \underline{\underline{0.0643}}$

3ai) $f(m) = \frac{1}{2}$

$\therefore \frac{3}{2} - \frac{1}{4}m = \frac{1}{2}$

$-\frac{1}{4}m = -1$

$m = -1 \times -4 = \underline{4}$

b) area under $= 1$, as $4 =$ median
area under $1 < x < 4 = \frac{1}{2}$

$\frac{1}{2} \times b \times h = \frac{1}{2}$ $\quad \frac{1}{2} \times (4-1) \times (4a + b)$

$6a + \frac{3}{2}b = \frac{1}{2} \quad \ldots (1)$

$f(1) = 0$

$\therefore a(1) + b = 0$

$-a = b \quad \ldots (2)$

sub (2) into (1)

$6a - \frac{3}{2}a = \frac{1}{2}$

$\frac{9}{2}a = \frac{1}{2}$

$\left(\underline{a = \frac{1}{9}}\right) \quad b = -a \left(\underline{b = -\frac{1}{9}}\right)$

c)

$$F(x) = \begin{cases} 0 & x < 1 \\ \frac{1}{18}x^2 - \frac{1}{9}x + c_1 & 1 \le x < 4 \\ \frac{3}{2}x - \frac{1}{8}x^2 + c_2 & 4 \le x \le 6 \\ 1 & x > 6 \end{cases}$$

when $x = 1$

$$\frac{1}{18}(1)^2 - \frac{1}{9}(1) + c_1 = 0$$

$$\therefore \quad c_1 = \frac{1}{18}$$

when $x = 6$

$$\frac{3}{2}(6) - \frac{1}{8}(6)^2 + c_2 = 1$$

$$\therefore \quad c_2 = -\frac{7}{2}$$

$$F(x) = \begin{cases} 0 & x < 1 \\ \frac{1}{18}x^2 - \frac{1}{9}x + \frac{1}{18} & 1 \le x < 4 \\ \frac{3}{2}x - \frac{1}{8}x^2 - \frac{7}{2} & 4 \le x \le 6 \\ 1 & x > 6 \end{cases}$$

4) $p = \frac{1}{200}$

ai) mean (μ) $= np = \frac{1}{200} \times 700 = 3.5$

aii) standard deviation (σ) $= \sqrt{Var(x)}$

$= \sqrt{np(1-p)} = \sqrt{700 \times \frac{1}{200}(1-\frac{1}{200})}$

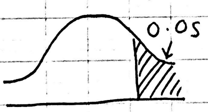
$= 1.8661...$

$= 1.866$ (3dp)

b) $X \sim B(500, \frac{1}{200})$

$H_0 : p = \frac{1}{200}$

$H_1 : p > \frac{1}{200}$



ii) $Y \sim Po(2.5)$

$H_0 : \lambda = 2.5$

$H_1 : \lambda > 2.5$

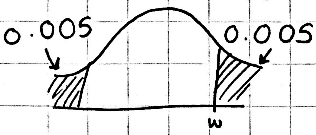
$P(X \leq 5) = 0.9580$

$1 - P(X \leq 5) = 0.0420$

$1 - P(X \leq 5) = P(X \geq 6)$

CR: $X \geq 6$ 5 is not ≥ 6 $\therefore$ insufficient evidence to reject H_0, $\therefore$ accept H_0.
there is no indication that the proportion of adults with the allergy is higher.

c) $X \sim B(n, 0.3)$



CR: $Y = 0 \quad = 0.005$

or $Y \geq W = 0.995$

$$P(Y=0) = {}^nC_0\, 0.3^0 (1-0.3)^{n-0}$$

$$0.7^n \leq 0.005$$

$$\log_{0.7} 0.005 = n \quad > 14.85$$

$\underline{\underline{n = 15}}$ (as whole number as people).

$\therefore X \sim B(15, 0.3)$

$P(Y \leq 9) = 0.9963$

$1 - P(Y \leq 9) = 0.0037$

$1 - P(Y \leq 9) = P(Y \geq 10)$

$= P(Y \geq 10) = CR.$

$\therefore \underline{\underline{W = 10}}$

5a) > 4 mins

$= 1 - F(4)$

$= 1 - (0.3(4) - 0.004(4)^3) = \underline{0.056}$

b) $\dfrac{P(T>4)}{P(T>2)} = \dfrac{0.056}{1-F(2)}$

$= \dfrac{0.056}{1-(0.3(2)-0.004(2)^3)} = \dfrac{7}{54}$

c) upper quartile $F(Q) = 0.75$

$F(2.7) = 0.731\ldots$

$F(2.8) = 0.7521\ldots$

$\therefore F(2.7) < 0.75 < F(2.8)$

d) $\mu = \int t f(t)\, dt$ of PDF / $f(t)$

$f(t) = \dfrac{d}{dt} F(t)$

$= 0.3 - 0.012t^2$

$\mu = \int_0^5 0.3t - 0.012t^3\, dt$

5d continued)

$$\mu = \left[\frac{0.3}{2}t^2 - \frac{0.012}{4}t^4\right]_0^5$$

$$= \frac{15}{8} = \underline{1.875}$$

6 a) 3,3,3   443
334   433
344   434
444   343

= 333
334 × 3
344 × 3
444

b) all possible

combos	maxM
333	3
334 × 3	4
335 × 3	5
444	4
443 × 3	4
445 × 3	5
555	5
553 × 3	5
554 × 3	5
345 × 6	5

$P(M=3) = (0.5)^3 = \frac{1}{8}$

$P(M=4) = [3(0.5)^2(0.3)] + [(0.3)^3] + [3(0.3)^2(0.5)]$

$= \frac{387}{1000}$

$P(M=5) = [3(0.3)^2(0.2)] + [3(0.5)^2(0.2)] + (0.2)^3 + [(0.2)^2(0.5)3] + [3(0.2)^2(0.3)] + [6(0.2)(0.3)(0.5)]$

$= \frac{61}{125}$

	3	4	5
$P(M=m)$	$\frac{1}{8}$	$\frac{387}{1000}$	$\frac{61}{125}$
	0.125	0.387	0.488

c) Mode $S_1 = 3$ mode $m = 5$

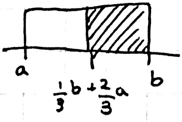
7a) $E(3-2X)$ $E(X) = \frac{1}{2}(a+b)$

$= 3 - 2E(X)$

$= 3 - 2\left(\frac{a+b}{2}\right)$

$= 3 - a - b$

b) $P(X > \frac{1}{3}b + \frac{2}{3}a)$



$= \dfrac{b - (\frac{1}{3}b + \frac{2}{3}a)}{b-a}$

$= \dfrac{b - \frac{1}{3}b - \frac{2}{3}a}{b-a}$

$= \dfrac{\frac{2}{3}(b-a)}{\cancel{(b-a)}} = \dfrac{2}{3}$

c) $E(X) = 0 \quad \therefore \quad \frac{1}{2}(a+b) = 0 \quad \therefore \quad a = -b$

$E(3X^2)$

$Var(X) = E(X^2) - [E(X)]^2$

$\frac{1}{12}(b-a)^2 = E(X^2) - 0$

$E(3X^2) = \frac{3}{12}(b-a)^2$

$a = -b$

$E(3X^2) = \frac{3}{12}(b--b)^2 = \frac{3}{12} \times 4b^2 = \underline{\underline{b^2}}$

d) range $= 18$

$b - a = \text{range} = 18$

$b - -b = 2b = 18$

$b = 9$

$Var(X) = \frac{1}{12}(9 - -9)^2 = \underline{27}$