S2 June 2015 IAL (MA)

Q1a) $P(X>4) = 1 - F(4) = 1 - \frac{1}{20}(4^2 - 4)$

$= \boxed{\frac{2}{5}}$

b) $F(a) - F(3) = P(3 < X < a) = 0.642$

$F(a) = F(3) + 0.642$.

a must lie in the interval $4 < x \leq 5$ as $P(3 < X < a) = 0.642$ and $P(X < 4) = 0.6$

$F(a) = \frac{1}{5}(2a - 5) = \frac{1}{20}(3^2 - 4) + 0.642$

$2a - 5 = \frac{5}{4} + 3.21$

$a = \frac{\frac{5}{4} + 3.21 + 5}{2} = \boxed{4.73}$

c) $\frac{d}{dx}\left(\frac{1}{20}(x^2 - 4)\right) = \frac{x}{10}$

$\frac{d}{dx}\left(\frac{1}{5}(2x - 5)\right) = \frac{2}{5}$

$$\therefore f(x) = \begin{cases} \frac{x}{10}, & 2 \leq x \leq 4 \\ \frac{2}{5}, & 4 < x \leq 5 \\ 0, & \text{otherwise.} \end{cases}$$

Q2a) $X \sim Po(8)$ where X = no. of choc chips per biscuit

$P(X \neq 8) = 1 - P(X = 8)$

$= 1 - \frac{(e^{-8})(8^8)}{8!}$

$= \boxed{0.860}$

b) $P(X \geq 8) = 1 - P(X \leq 7) = 1 - 0.4530$

$= 0.5470$

$P(\text{required}) = (0.5470)^4 = \boxed{0.090}$

c) let Y = no. of choc chips in 9 biscuits,

$Y \sim Po(72)$

λ is large $\therefore Y \simeq \sim N[72, 72]$

$P(\text{required}) = P(Y > 75)$

(applying c.c) $= P(Y > 75.5)$

$= P\left(Z > \frac{75.5 - 72}{\sqrt{72}}\right)$

$= P(Z > 0.41)$

$= 1 - P(Z < 0.41) = \boxed{0.3409}$

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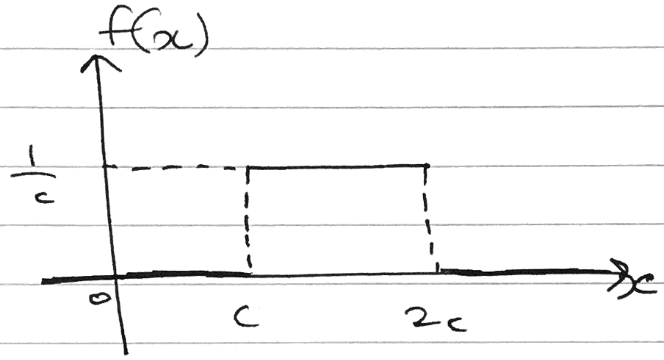
d) $H_0: \lambda = 1.5$ For 4 hours, $B \sim Po(6)$
$H_1: \lambda > 1.5$ B = no. of biscuits sold in 4 hrs.

$$P(B \geq 11) = 1 - P(B \leq 10)$$

$$= 1 - 0.9574 = 0.0426$$

$0.0426 < 0.05$
$\therefore$ Result is significant
Reject H_0.
Evidence suggests that the rate of sales of biscuits has increased.

Q3a) $U[c, 2c]$



b) $E(X) = \frac{c + 2c}{2} = \frac{3c}{2}$

$$E(X^2) = \int_c^{2c} x^2 f(x)\, dx = \frac{1}{c}\int_c^{2c} [x^2]\, dx$$

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$$= \frac{1}{c}\left[\frac{x^3}{3}\right]_c^{2c} = \frac{1}{c}\left[\frac{8c^3}{3} - \frac{c^3}{3}\right]$$

$$= \frac{7c^2}{3} = E(X^2)$$

$$\text{Var}(X) = E(X^2) - (E(X))^2$$

$$= \frac{7c^2}{3} - \left(\frac{3c}{2}\right)^2$$

$$= \boxed{\frac{c^2}{12}}$$

c) $P(\text{required}) = P(X > 2(2c - x)) = P(X > 4c - 2X)$

X = longer piece
$2c - X$ = shorter piece

$$= P(3X > 4c)$$

$$= P\left(X > \frac{4c}{3}\right)$$

$$= \left(2c - \frac{4c}{3}\right) \times \frac{1}{c}$$

$$= \frac{2c}{3} \times \frac{1}{c} = \boxed{\frac{2}{3}}$$

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Q4a) $H_0 : \lambda = k$ 2.5% each tail

$H_1 : \lambda \neq k$

<u>only</u> $P(X=0) < 0.025$ when $\lambda \leq 5.5$ and $\lambda \geq 4$

so λ could be 4 or 5.

now lets look at $(X \geq 9)$, the other part of the critical region.

$P(X \geq 9 \mid \lambda = 4) = 0.0214 < 0.025$.

$P(X \geq 9 \mid \lambda = 5) = 0.0681 > 0.025$.

So the only possible value λ can take where the critical region is $(X=0) \cup (X \geq 9)$ is $\boxed{4}$

<u>hence $k = 4$</u>

b) asl $= 0.0214 + 0.0183 = 0.0397$

(3.97%)

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Q5a)

$P(X=x)$	x
0.35	6
0.65	9

$$Y_1 = \frac{2(6)+9}{3} = \boxed{7}$$

$$Y_2 = \frac{2(6)+6}{3} = \boxed{6}$$

$$Y_3 = \frac{2(9)+6}{3} = \boxed{8}$$

$$Y_4 = \frac{2(9)+9}{3} = \boxed{9}$$

b) $P(6,9) = P(9,6) = 0.35 \times 0.65 = 0.2275$
$= P(Y=7) = P(Y=8)$

$P(6,6) = 0.35^2 = 0.1225 = P(Y=6)$

$P(9,9) = 0.65^2 = 0.4225 = P(Y=9)$

y	6	7	8	9
$P(Y=y)$	0.1225	0.2275	0.2275	0.4225

c) $E(Y) = \sum yP(Y=y) = 6(0.1225) + 7(0.2275)$
$+ 8(0.2275) + 9(0.4225)$
$= \boxed{7.95}$

(Q6a) $X \sim B[30,\ 0.40]$

where X = no. of customer's buying insurance when purchasing a product.

b) Customer's buy insurance Independently of each other.

c) $P(X < r) < 0.05$

$P(X \leq 8) = 0.0940$

$P(X \leq 7) = 0.0435$

$\therefore \boxed{r_{max} = 8}$

d) For 100 customers, $X_2 \sim B[100, 0.40]$

n is large, $p \simeq 0.50$ so use normal.

$\therefore X_2 \simeq \sim N[40,\ 24]$

$\left[\begin{array}{l} np = 40 \\ np(1-p) = 24 \end{array}\right]$

$P(X_2 \geq t) = 0.938$

$P(X_2 > t-1) = 0.938$

$P(X_2 > t-0.5) = 0.938$

$\Rightarrow P\left(Z > \frac{t-0.5-40}{\sqrt{24}}\right) = 0.938$

$$P\left(Z > \frac{t-40.5}{\sqrt{24}}\right) = 0.938$$

$$P\left(Z < \frac{40.5-t}{\sqrt{24}}\right) = 0.938$$

$$P(Z < 1.54) = 0.9382$$

$$\therefore \quad 1.54 \simeq \frac{40.5-t}{\sqrt{24}}$$

$$t \simeq -1.54\sqrt{24} + 40.5$$

$$t \simeq 32.9556\ldots$$

$$\text{so } \boxed{t = 33}$$

e) $\left.\begin{array}{l} H_0 : p = 0.4 \\ H_1 : p < 0.4 \end{array}\right\}$ new sample . . . $X_3 \sim B[25, 0.4]$

$$P(X_3 \leq 6) = 0.0736$$

$$0.0736 < 0.10$$

$\therefore$ Result is significant.
Reject H_0.
Evidence suggests proportion of customers buying insurance has decreased.

Q7a) Total area = 1.

$$\int_0^k \left(\frac{2x}{15}\right) dx + \int_k^5 \left(\frac{1}{5}(5-x)\right) dx = 1$$

$$\left[\frac{2x^2}{30}\right]_0^k + \frac{1}{5}\left[5x - \frac{x^2}{2}\right]_k^5 = 1$$

$$\left[\frac{2k^2}{30}\right] + \frac{1}{5}\left[25 - \frac{25}{2}\right] - \frac{1}{5}\left[5k - \frac{k^2}{2}\right] = 1$$

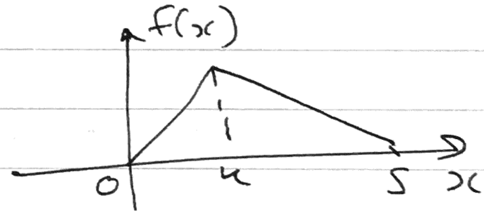
$$\frac{k^2}{15} + 5 - \frac{5}{2} - k + \frac{k^2}{10} = 1$$

$$\frac{k^2}{6} - k + \frac{3}{2} = 0$$

By Quadratic formula,

$$\boxed{k = 3} \quad (\text{R.R})$$

b) $\boxed{3}$



c) $P(X \leq \frac{u}{2} \mid X \leq u)$

$= P(X \leq 1.5 \mid X \leq 3)$

$= \dfrac{P(X \leq 1.5 \cap X \leq 3)}{P(X \leq 3)} \rightsquigarrow = P(X \leq 1.5)$

$= \dfrac{P(X \leq 1.5)}{P(X \leq 3)} = \dfrac{F(1.5)}{F(3)}$

$= \dfrac{\int_0^{1.5} \left[\frac{2x}{15}\right] dx}{\int_0^{3} \left[\frac{2x}{15}\right] dx} = \dfrac{\left[\frac{2x^2}{30}\right]_0^{1.5}}{\left[\frac{2x^2}{30}\right]_0^{3}}$

$= \dfrac{\left[\frac{2}{30}(1.5^2)\right]}{\left[\frac{2}{30}(3^2)\right]} = \boxed{0.25}$