S2 January 2017 (MA) (IAL)

Q1a) $W \sim N(32, 4^2)$

$P(W = 0) = \boxed{0}$.

b) $X \sim B[20, 0.45]$

$P(X = 8) = \binom{20}{8}(0.45)^8(0.55)^{12} = \boxed{0.162}$

c) $np(1-p) = \text{variance} = 20(0.45)(0.55) = 4.95$

$\therefore \sigma = \sqrt{4.95} = 2.22\ldots$, Mean $= np = 9$

so $P(\text{required}) = P(9 - 2.22 < X < 9 + 2.22)$

$= P(6.78 < X < 11.22)$

$= P(X=7) + P(X=8) + P(X=9) + P(X=10) + P(X=11)$

$= P(7 \leq X \leq 11)$

$= P(X \leq 11) - P(X \leq 6)$

$= 0.8692 - 0.1299$

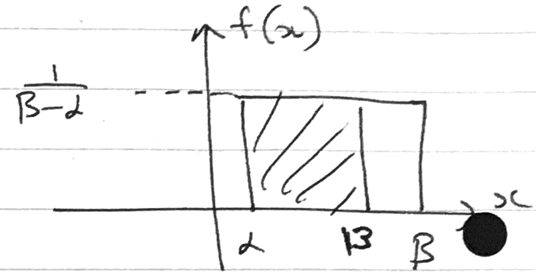
$= \boxed{0.7393}$

(Q2a) $X \sim U[\alpha, \beta]$

$E(X) = 8 \quad : \quad \dfrac{\alpha + \beta}{2} = 8$

$\therefore \boxed{\alpha + \beta = 16} \sim (1)$

b) $(13 - \alpha) \times \dfrac{1}{\beta - \alpha} = 0.7$



$\dfrac{13 - \alpha}{\beta - \alpha} = 0.7$

$13 - \alpha = 0.7\beta - 0.7\alpha$

$0.7\beta + 0.3\alpha = 13$

$\times 10 : \quad 7\beta + 3\alpha = 130$

(1) $\times 3 : \quad 3\alpha + 3\beta = 48$

$$\begin{array}{r} 7\beta + 3\alpha = 130 \\ -\,(3\beta + 3\alpha = 48) \\ \hline 4\beta + 0 = 82 \end{array}$$

$\therefore \beta = \dfrac{82}{4} = \boxed{20.5}$

and $\alpha = 16 - \beta = \boxed{-4.5}$

c) $Var(X) = \frac{(b-a)^2}{12} = \frac{(20.5 - -4.5)^2}{12} = \boxed{\frac{625}{12}}$

d) $P(5 \leq X \leq 35) = P(5 \leq X \leq 20.5)$

$= (20.5 - 5) \times \frac{1}{(20.5+4.5)} = \boxed{0.62}$

Q3a) λ is large (>10).

b) $X \sim Po(3.5)$

$P(X=3) = 0.2158$
$P(X=4) = 0.1888$

mean is 3.5 so consider $X=3$ and $X=4$.
$P(X=3) > P(X=4)$ so mode is $\boxed{3}$.

c) for 2 weeks, $X_2 \sim Po(7)$.

$P(X_2 > 5) = 1 - P(X_2 \leq 5) = 1 - 0.3007$

$= \boxed{0.6993}$

d) $P(4 \text{ in } 1^{st} \text{ week} \mid 6 \text{ in } 2 \text{ weeks}) = \frac{P(4 \text{ in } 1 \text{wk and } 6 \text{ in } 2 \text{wks})}{P(6 \text{ in } 2 \text{weeks})}$

$= \frac{(\overset{P(X=4)}{0.1888})(P(X=2))}{P(X_2=6)} = \frac{0.1888\left[\frac{(e^{-3.5})(3.5)^2}{2}\right]}{\left(\frac{(e^{-7})(7^6)}{6!}\right)}$

$= \boxed{0.234}$

e) For 40 wks, $X_3 \sim Po(140)$ $(40 \times 3.5 = 140)$

λ is large, $X_3 \approx\sim N(140, 140)$

$P(X_3 \geq 120) = P(X_3 > 119)$

(applying c.c) $= P(X_3 > 119.5)$

$= P\left(Z > \frac{119.5 - 140}{\sqrt{140}}\right)$

$= P(Z > -1.73)$

$= P(Z < 1.73) = \boxed{0.9582}$

Q4a) $E(X) = \int_0^4 x f(x)\,dx = \frac{3}{64}\int_0^4 [4x^3 - x^4]\,dx$

$= \frac{3}{64}\left[\frac{4x^4}{4} - \frac{x^5}{5}\right]_0^4$

$= \frac{3}{64}\left[256 - \frac{1024}{5}\right] = 2.4$

So mean # of hours $= \boxed{2400}$

b) $E(X^2) = \int_0^4 x^2 f(x)\,dx = \frac{3}{64}\int_0^4 [4x^4 - x^5]\,dx$

$$= \frac{3}{64}\left[\frac{4x^5}{5} - \frac{x^6}{6}\right]_0^4$$

$$= \frac{3}{64}\left[\frac{4096}{5} - \frac{2048}{3}\right]$$

$$= 6.4 = E(X^2)$$

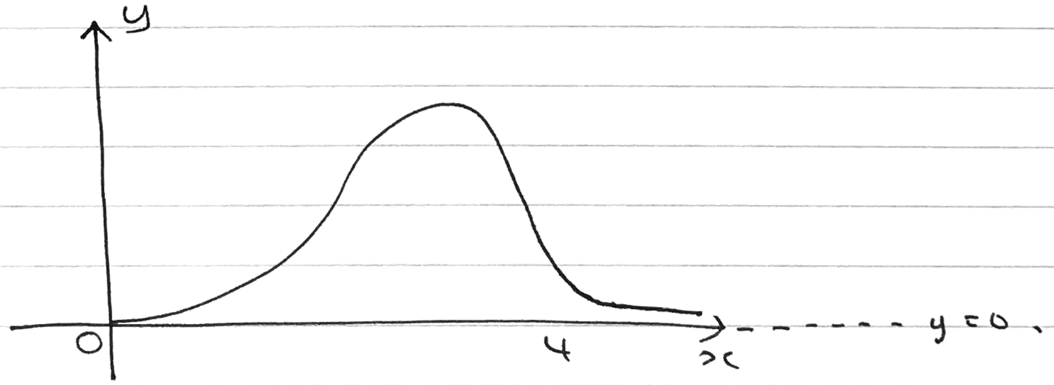
$$Var(X) = E(X^2) - \left[E(X)\right]^2$$

$$= 6.4 - (2.4)^2 = 0.64$$

$$\sigma_x = \sqrt{0.64} = \boxed{0.8}$$

c) The model suggests its impossible for a component to last longer than 4000 hours which may not necessarily be true.

d)



Q5a) $X \sim Po(6)$, where X = no. of defects in $15m^2$ of cloth.

$P(X \leq 7) = 0.7440$

so for 12 pieces... $Y \sim B[12, 0.7440]$

where Y = no. of pieces of cloth with at most 7 defects.

$P(\text{required}) = P(Y=6) = \binom{12}{6}(0.744)^6(1-0.744$

$= \boxed{0.044}$

bi) $H_0: \lambda = 10$
$H_1: \lambda \neq 10$

ii) $X_2 \sim Po(10)$
where X_2 = no. of defects in $25m^2$.

iii) the range of values for the number of defects that would result in the null hypothesis H_0 being rejected.

c) $\left.\begin{array}{l} P(X \leq 3) = 0.0103 \\ P(X \leq 4) = 0.0293 \end{array}\right\}$ 0.0103 is less than 0.025.

0.0143 is less than 0.025. $\left\{\begin{array}{l} P(X \geq 17) = 0.0270 \\ P(X \geq 18) = 0.0143 \end{array}\right.$

d) $0.0103 + 0.0143 = \boxed{0.0246}$

(2.46%)

Q6) $X \sim B[75, 0.96]$ where X = no. of seeds germinating.

$Y \sim B[75, 0.04]$ where Y = no. of seeds that don't germinate.

we use Y as p is small here, allowing us to use a poisson approximation.

$np = 75 \times 0.04 = 3$

9 beans didn't germinate.

$\therefore Y \simeq \sim Po(3)$

$P(Y \geq 9) = 1 - P(Y \leq 8) = 1 - 0.9962$

$= \boxed{0.0038}$

$0.0038 < 0.01$

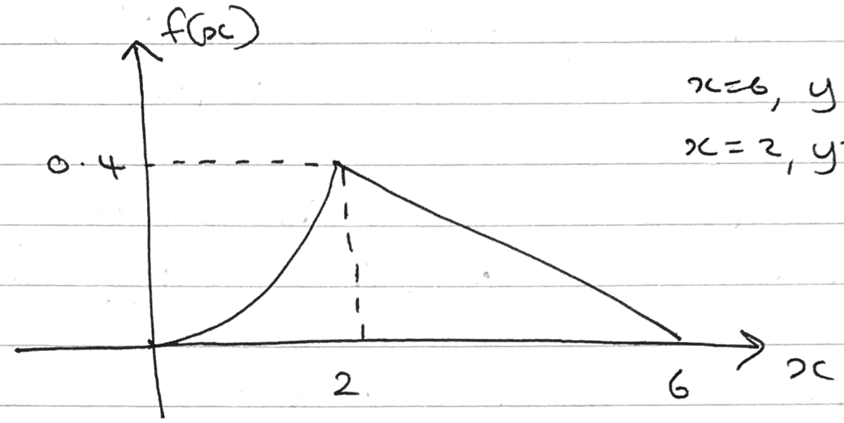
$\therefore$ Result is significant.

Reject H_0.

Evidence suggests the producer is overstating the probability.

Q7a)

$$f(x) = \begin{cases} \frac{1}{20}x^3, & 0 \le x \le 2 \\ \frac{1}{10}(6-x), & 2 < x \le 6 \\ 0, & \text{otherwise.} \end{cases}$$



$x=6,\ y=0$
$x=2,\ y=0.4$

$\frac{1}{10}(6-x)$ is a straight line.

b) mode => highest point.

so $(x=2)$ = mode

c) $P(X>2) = \frac{1}{10}\int_2^6 [6-x]\,dx = \frac{1}{10}\left[6x - \frac{x^2}{2}\right]_2^6$

$= \frac{1}{10}\left[36 - \frac{36}{2}\right] - \frac{1}{10}\left[12-2\right]$

$= \frac{1}{10}[8] = \boxed{0.8}$ //

alt: use area of Δ: $\frac{1}{2} \times (6-2) \times 0.4 = \boxed{0.8}$

For $0 \leq x \leq 2$:

d) $\frac{1}{20}\int_0^x (x^3)\, dx = \frac{1}{20}\left[\frac{x^4}{4}\right] = \frac{1}{80}x^4$ //

For $2 < x \leq 6$:

$F(2) + \frac{1}{10}\int_2^x (6-x)\, dx = \frac{2^4}{80} + \frac{1}{10}\left[6x - \frac{x^2}{2}\right]_2^x$

$= 0.2 + \frac{1}{10}\left[6x - \frac{x^2}{2}\right] - \frac{1}{10}\left[10\right]$

$= \frac{1}{5} + \frac{6x}{10} - \frac{x^2}{20} - 1$

$= \frac{3x}{5} - \frac{x^2}{20} - \frac{4}{5} = \frac{1}{20}\left[12x - x^2 - 16\right]$ //

so.... $F(x) = \begin{cases} 0, & x < 0 \\ \frac{x^4}{80}, & 0 \leq x \leq 2 \\ \frac{1}{20}\left[12x - x^2 - 16\right], & 2 < x \leq 6 \\ 1, & x > 6 \end{cases}$

e) $P(X < a \mid X > 2) = \frac{5}{8}$

$$P(X < a \mid X > 2) = \frac{P(X < a \cap X > 2)}{P(X > 2)}$$

$$\therefore \quad \frac{P(2 < X < a)}{P(X > 2)} = \frac{5}{8}$$

but $P(X > 2) = 0.8$ (from c)

$$\therefore \quad P(2 < X < a) = \frac{5}{8}(0.8) = \frac{1}{2}$$

$$P(X < a) - P(X < 2) = \frac{1}{2}$$

$$P(X < a) = \frac{1}{2} + P(X < 2)$$

$$P(X < a) = \frac{1}{2} + 0.2 = 0.7$$

$$F(a) = \boxed{0.7} = P(X < a)$$

f) $F(a) = \frac{1}{20}(12a - a^2 - 16) = \frac{7}{10} \quad (a > 2)$

$\times 20$: $\quad 12a - a^2 - 16 = 14$

By Quadratic Formula

$$a^2 - 12a + 30 = 0 \qquad \therefore \boxed{a = 3.55}$$

reject $a = 8.45$, $(a \leq 6)$