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S1 Specimen (IAL) (MA)

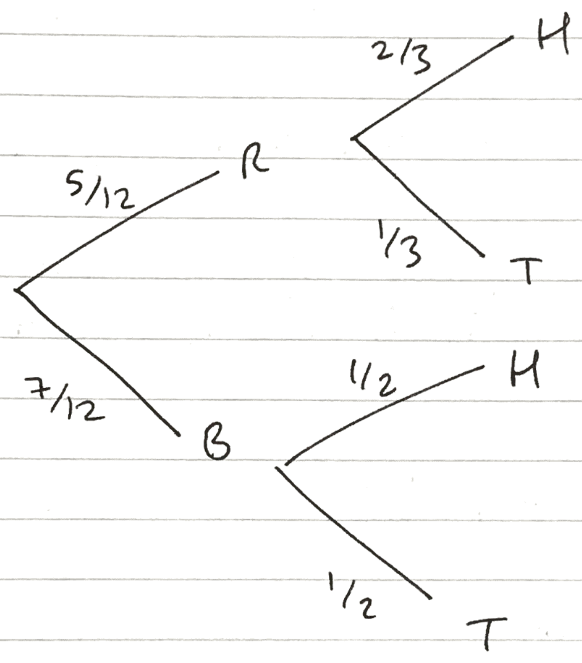
Q1a) $r = \dfrac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \dfrac{8825}{\sqrt{1022500 \times 130.9}} = \boxed{0.763}$

b) $r \simeq 1$ $\therefore$ a strong positive correlation is seen.

[Teams with greater attendance score more goals]

c) 0.763 (PMCC unaffected by coding).

Q2a)



b) $P(\text{head}) = P(RH) + P(BH) = \left(\frac{5}{12}\right)\left(\frac{2}{3}\right) + \left(\frac{7}{12}\right)\left(\frac{1}{2}\right)$

$= \boxed{\dfrac{41}{72}}$

c) $P(\text{red ball} \mid \text{head}) = \dfrac{P(\text{red} \cap \text{head})}{P(\text{head})} = \dfrac{\frac{5}{12} \times \frac{2}{3}}{\frac{41}{72}}$

$= \boxed{\dfrac{20}{41}}$

d) P(both pick red) + P(both pick blue)

$$= \left(\frac{5}{12}\right)^2 + \left(\frac{7}{12}\right)^2 = \boxed{\frac{37}{72}}$$

Q3a) $\sum P(X=x) = 1$ $\therefore$ $\frac{1}{5} + a + \frac{1}{10} + a + \frac{1}{5} = 1$

$$2a + \frac{1}{2} = 1$$

$$2a = \frac{1}{2} \quad \therefore \boxed{a = \frac{1}{4}}$$

b) $E(X) = \sum x\,p(X=x) = -\frac{1}{5} + \frac{1}{10} + 2\left(\frac{1}{4}\right) + \frac{3}{5} = \boxed{1}$

This can also be spotted without any working as the distribution is symmetrical about $x=1$.

c) $E(X^2) = \frac{1}{5} + \frac{1}{10} + 4\left(\frac{1}{4}\right) + \frac{9}{5} = \frac{31}{10}$ //

$\text{Var}(X) = E(X^2) - [E(X)]^2$

$\text{Var}(X) = \frac{31}{10} - (1)^2 = \boxed{2.1}$

d) $\text{Var}(Y) = \text{Var}(6-2X) = 4\text{Var}(X) = 4 \times 2.1$
$= \boxed{8.4}$

e) $P(X \geq Y) = P(X \geq 6-2X) = P(3X \geq 6)$

$= P(X \geq 2) = a + \frac{1}{5} = \boxed{0.45}$

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add up all intersections

Q4a) $P(\text{required}) = \dfrac{2+3}{10+4+2+5+3+6} = \dfrac{5}{30} = \boxed{\dfrac{1}{6}}$

total students

b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \dfrac{6}{30} + \dfrac{10}{30} - \dfrac{2}{30} = \dfrac{14}{30} = \boxed{\dfrac{7}{15}}$

c) 0 (mutually exclusive)

d) $P(C \mid \text{reads at least } 1) = \dfrac{P(\text{reads } C \cap \text{reads at least } 1)}{P(\text{reads at least } 1)}$

9 read C

$= \dfrac{6+3}{20} = \boxed{\dfrac{9}{20}}$

20 people read at least 1

e) $P(B) = \dfrac{5+2+3}{30} = \dfrac{1}{3}$ $\quad P(C) = \dfrac{9}{30} = \dfrac{3}{10}$

$P(B \cap C) = \dfrac{3}{30} = \dfrac{1}{10}$

for independence, $P(B) \times P(C) = P(B \cap C)$

$P(B) \times P(C) = \dfrac{1}{3} \times \dfrac{3}{10} = \dfrac{1}{10} = P(B \cap C)$

Hence B and C are statistically independent.

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Q5a)

21–25	31–40
11	8
$\boxed{23}$	$\boxed{35.5}$

b) width of 11–20 = 4

$\therefore$ width of 26–30 = 2

Area = $k \times f$

$6 \times 4 = 15k$

$k = \frac{8}{5}$

for 26–30

$\therefore \quad 2 \times h = \frac{8}{5}(13)$

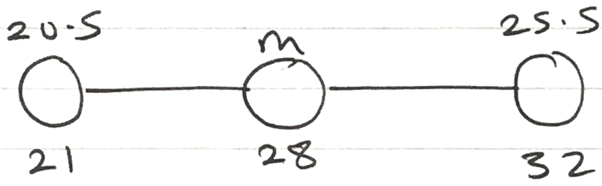
$\boxed{h = 10.4\text{cm}} = \frac{8(13)}{5(2)}$

c) $\sum fx = 1316.5 \quad \therefore \bar{x} = \frac{1316.5}{56} = \boxed{23.5}$

$\sum fx^2 = 37378.25$.

$\therefore \quad \sigma = \sqrt{\frac{37378.25}{56} - \left(\frac{1316.5}{56}\right)^2} = 10.7$

d)



$\frac{n}{2} = 28$

$$\frac{m - 20.5}{25.5 - 20.5} = \frac{28 - 21}{32 - 21}$$

$$\frac{m - 20.5}{5} = \frac{7}{11}$$

$$m = \frac{5(7)}{11} + 20.5 = \boxed{23.7} \text{ min.}$$

e) $\left.\begin{array}{l} Q_3 - Q_2 = 5.6 \\ Q_2 - Q_1 = 7.9 \end{array}\right\}$ $7.9 > 5.6$
$\therefore$ negative skew

$[Q_2 - Q_1 > Q_3 - Q_2]$

Q6a) [see mark-scheme]

b) The plotted scatter diagram should show points lying in (approximately) a straight line

c) $\Sigma d = 27.7 \quad \Sigma f = 146$

$S_{dd} = 152.09 - \frac{(27.7)^2}{6} = \boxed{24.21}$

$S_{fd} = 723.1 - \frac{27.7(146)}{6} = \boxed{49.06}$

d) $b = \frac{S_{fd}}{S_{dd}} = 2.026$

$a = \bar{y} - b\bar{x} = \frac{146}{6} - b\left(\frac{27.7}{6}\right) = 14.97.. \simeq 15.0$

$\Rightarrow \boxed{f = 15.0 + 2.03d}$

e) For every extra 100 km, the flight costs £2.03 more.

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5p per km $\rightarrow$ 500p per 100km.

f) $15 + 2.03d < 5d$

$\therefore d > \dfrac{15}{(5-2.03)} \rightarrow d > 5.05$

d is no of 100km, t is in km.

$\therefore \boxed{t > 505}$

Q7a) $D \sim N(30, 8^2)$

$P(D > 20) = P\left(Z > \dfrac{20-30}{8}\right) = P(Z > -1.25)$

$= P(Z < 1.25) = \boxed{0.8944}$

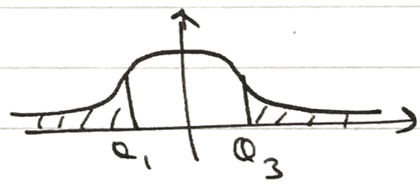
b) $P(D < Q_3) = 0.75$

from tables, $\dfrac{Q_3 - 30}{8} = 0.67$

$\therefore Q_3 = 30 + 8(0.67) = \boxed{35.4}$

c) Normal distribution is symmetrical.

$Q_3 - \mu = 35.4 - 30 = 5.4$



$\therefore \mu - 5.4 = Q_1 = 30 - 5.4 = \boxed{24.6}$

d) $h = 24.6 - 1.5(35.4 - 24.6) = \boxed{8.4}$

$u = 35.4 + 1.5(35.4 - 24.6) = \boxed{51.6}$

e) $\underbrace{P(D > 51.6) + P(D < 8.4)}_{\text{they are equal.}} = 2P(D > 51.6)$

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$$= 2\left[P\left(Z > \frac{51.6 - 30}{8}\right)\right] = 2\left[P(Z > 2.7)\right]$$

$$= 2\left[1 - P(Z < 2.7)\right] = 2\left[1 - 0.9965\right]$$

$$\simeq \boxed{0.007}$$