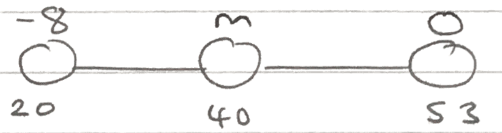
S1 October 2017 (MA)

Q1a) $25 - -38 = \boxed{63}$ (outlier counts)

b) $IQR = 10 - -8 = \boxed{18}$

c) $\frac{n}{2} = 40.$ 40th data point lies in interval $-8 < e \leq 0$

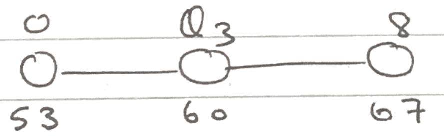


$$\frac{m - -8}{0 - -8} = \frac{40 - 20}{53 - 20}$$

$$m + 8 = 8\left(\frac{20}{33}\right)$$

$$\therefore m = \boxed{-3.15}$$

d) $\frac{3n}{4} = \frac{3}{4} \times 80 = 60$, 60th data point lies in interval $0 \leq e < 8$



$$\frac{Q_3}{8} = \frac{60 - 53}{67 - 53}$$

$$Q_3 = 8\left(\frac{60 - 53}{67 - 53}\right) = \boxed{4}$$

ei) $IQR = Q_3 - Q_1 = 4 - -8 = 12$

upper limit: $4 + 1.5(12) = 22$

lower limit: $-8 - 1.5(12) = -26$

So the only outliers are 23 and 28

ii) see mark scheme.

f) The IQR is lower and the median is closer to 0. This indicates their ability has improved.

Q2a) B and C $(P(B \cap C) = 0)$

b) $P(A) \times P(C) = P(A \cap C)$ if independent.

$0.6 \times 0.35 = P(A \cap C) = w = \boxed{0.21}$

c) $P(A) = 0.60 \longrightarrow 0.15 + y + w = 0.60$

$y = 0.6 - 0.15 - 0.21 = \boxed{0.24}$

$P(C) = w + x = 0.35 \longrightarrow x = 0.35 - 0.21 = \boxed{0.14}$

$z = 1 - 0.15 - 0.24 - 0.21 - 0.14 = \boxed{0.26}$

d) $P(A \text{ or } B \text{ or } C) = x + y = \boxed{0.38}$

(if you buy B then you must be buying A too.
so $P(\text{required}) = P(A \cap B' \cap C') + P(C \cap B' \cap A')$)

e) $P(B \cup C) = P(B) + P(C) - P(B \cap C)$

0

$= 0.15 + 0.35$

$= \boxed{0.50}$

f) $P(A \mid B \cup C) = \dfrac{P(A \cap B \cup C)}{P(B \cup C)} = \dfrac{0.15 + w}{0.50}$

$P(A \cap B \cup C) = P(A \cap B) + P(A \cap C)$

$= \dfrac{0.15 + 0.21}{0.50}$

$= \boxed{0.72}$

Q3a) $L \sim N(\mu, \sigma^2)$

We are told $P(L < 45) = 0.4$

$\Rightarrow P\left(Z < \dfrac{45 - \mu}{\sigma}\right) = 0.4$

$\Rightarrow P\left(Z > \dfrac{\mu - 45}{\sigma}\right) = 0.4$

but $P(Z > 0.2533) = 0.4$

$\therefore 0.2533 = \dfrac{\mu - 45}{\sigma}$ //

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hence $45 + 0.2533\sigma = \mu$ ①

b) also, $P(L < 35) = 0.15$

$$P\left(Z < \frac{35-\mu}{\sigma}\right) = 0.15$$

$$P\left(Z > \frac{\mu-35}{\sigma}\right) = 0.15$$

but $P(Z > 1.0364) = 0.15$

hence $\dfrac{\mu-35}{\sigma} = 1.0364$

$\Rightarrow \mu - 35 = 1.0364\sigma$

$\Rightarrow \mu = 35 + 1.0364\sigma$ ②

c) ① = ② : $35 + 1.0364\sigma = 45 + 0.2533\sigma$

$$\sigma(1.0364 - 0.2533) = 10$$

so $\sigma = \dfrac{10}{1.0364+0.2533} = \boxed{12.8}$

and $\mu = 35 + 1.0364(12.8)$

$$\boxed{\mu = 48.2}$$

d i) $P(L > 35 \mid L < 45) = \dfrac{P(35 < L < 45)}{P(L < 45)}$

recall that $P(L > 45) = 0.60$
$P(L < 35) = 0.15$

$= \dfrac{P(L < 45) - P(L < 35)}{P(L < 45)}$

$= \dfrac{(1 - 0.60) - (0.15)}{(1 - 0.60)}$

$= \boxed{\dfrac{5}{8}}$

ii) $P(L < 45 \mid L > 35) = ~~P(\quad < L) <~~$

$= \dfrac{P(L < 45 \cap L > 35)}{P(L > 35)} = \dfrac{P(35 < L < 45)}{P(L > 35)}$

$= \dfrac{(1 - 0.60) - (0.15)}{(1 - 0.15)}$

$= \boxed{\dfrac{5}{17}}$

e) 0.60 - red
0.15 - blue
0.25 - yellow

$P(L > 35 \mid L < 45) = P(\text{stick is yellow} \mid \text{its blue or yellow})$

$= \dfrac{5}{8}$ //

$P(L < 45 \mid L > 35) = P(\text{stick is yellow} \mid \text{its red or yellow})$

$= \dfrac{5}{17}$ //

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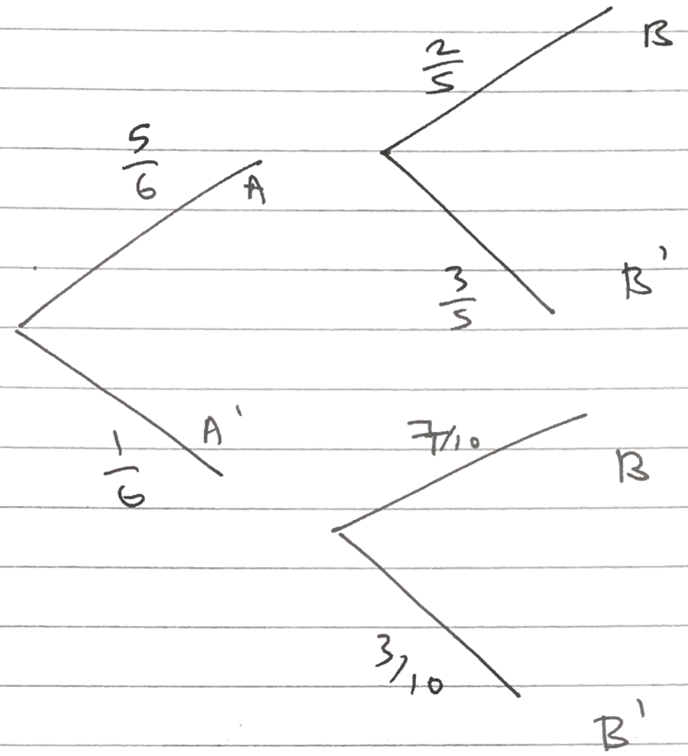
$\frac{5}{8} > \frac{5}{17}$ hence the stick is more likely to be from Heil.

Q4a)



$$P(B) = P(A) \times \frac{2}{5} + P(A') \times \frac{7}{10}$$

$$\frac{9}{20} = \frac{2}{5}P(A) + (1 - P(A)) \times \frac{7}{10}$$

$$\frac{9}{20} = \frac{2}{5}P(A) + \frac{7}{10} - \frac{7}{10}P(A)$$

$$\therefore P(A) \times \frac{3}{10} = \frac{1}{4}$$

$$P(A) = \frac{\frac{1}{4}}{\frac{3}{10}} = \boxed{\frac{5}{6}}$$

b) $P(A' | B') = \dfrac{P(A' \cap B')}{P(B')}$

$= \dfrac{\frac{1}{6} \times \frac{3}{10}}{\frac{1}{6}\left(\frac{3}{10}\right) + \frac{5}{6}\left(\frac{3}{5}\right)} = \boxed{\frac{1}{11}}$

Q5a) $\left.\begin{array}{l} \sum x = 283 \\ \sum x^2 = 9011 \\ n = 10 \end{array}\right\}$ $\bar{x} = \dfrac{283}{10} = \boxed{28.3}$

$\sigma^2 = \dfrac{9011}{10} - (28.3)^2$

$= 100.21 = \boxed{100}$

b) $\bar{y} = \dfrac{306.1}{10} = \boxed{30.61}$ or 30.6 to 3sf

$S_{yy} = \sum y^2 - \dfrac{(\sum y)^2}{n}$

variance $= \dfrac{\sum y^2}{n} - \dfrac{(\sum y)^2}{(n)^2}$

hence Variance $= \dfrac{S_{yy}}{n} = \dfrac{546.3}{10} = \boxed{54.63}$

c) $b = 0.659 = \dfrac{S_{xy}}{S_{xx}}$ //

$S_{xx} = 10 \times Var(x) = 1002.1$ //

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So $S_{xy} = 0.659 \times 1002.1 = 660.4$

hence $PMCC = \dfrac{660.4}{\sqrt{1002.1 \times 546.3}} = \boxed{0.892}$

d) $r \approx 1$ so strong +ve correlation suggested. So yes the PMCC supports the use of a regression line.

e) $y = 12 + 0.659(35) = 35.065$

so salary will be $\boxed{£35,065}$

Q6a) Discrete Uniform.

b) $P(3 \text{ points}) = P(D=3) + [P(D=1) \times P(D=2)]$

$= \frac{1}{4} + \left(\frac{1}{4}\right)\left(\frac{1}{4}\right) = \boxed{\frac{5}{16}}$

c) $P(0) = [P(D=1)]^2 = \boxed{\frac{1}{16}}$

d)

x	0	2	3	4	5
$P(X=x)$	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{5}{16}$	$\frac{1}{16}$

$E(X) = \sum x P(X=x) = 2\left(\frac{1}{4}\right) + 3\left(\frac{5}{16}\right) + 4\left(\frac{5}{16}\right) + 5\left(\frac{1}{16}\right)$

$= \boxed{3}$

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$$= \sum x^2 P(X = x)$$

e) $E(X^2) = 2^2\left(\frac{1}{4}\right) + 3^2\left(\frac{5}{16}\right) + 4^2\left(\frac{5}{16}\right) + 5^2\left(\frac{1}{16}\right)$

$= \frac{83}{8}$ //

$\mathrm{Var}(X) = E(X^2) - [E(X)]^2$

$= \frac{83}{8} - (3)^2 = \boxed{\frac{11}{8}}$

f) r can only be 1 or 2.

$P(r=1) = P(d = 2, 3 \text{ or } 4) = \frac{3}{4}$ //

$\therefore P(r=2) = 1 - \frac{3}{4} = 1/4$ //

r	1	2
$P(R=r)$	$\frac{3}{4}$	$\frac{1}{4}$

//

g) $Y = 2R + 0.5$

$E(Y) = 2E(R) + 0.5$

$E(R) = 1\left(\frac{3}{4}\right) + 2\left(\frac{1}{4}\right) = \frac{5}{4}$

so $E(Y) = 2\left(\frac{5}{4}\right) + 0.5 = 2.5 + 0.5$

$= 3$ // $= E(X)$

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b) $y = 2.5$ if $r = 1$
$y = 4.5$ if $r = 2$

$$P(X > Y) = P(X = 3 \cap R = 1) + P(X = 4 \cap R = 1) + P(X = 5)$$

$$= \frac{1}{4} + \frac{1}{4} + \frac{1}{16}$$

$$= \boxed{\frac{9}{16}}$$

<u>note</u>: if $r = 1$ then x can be 3, 4, 5 for $x > y$ to be true.

if $r = 2$ then x can only be 5 for $x > y$ to be true.

so $P(x > y) = \frac{1}{4} + \frac{1}{4} + \frac{1}{16}$

$P(X = 3 \cap R = 1) = P(D = 3)$

$P(X = 4 \cap R = 1) = P(D = 4)$

$P(X = 5 \cap R = 1) + P(X = 5 \cap R = 2) = P(X = 5)$ $\left(= \frac{1}{16}\right)$