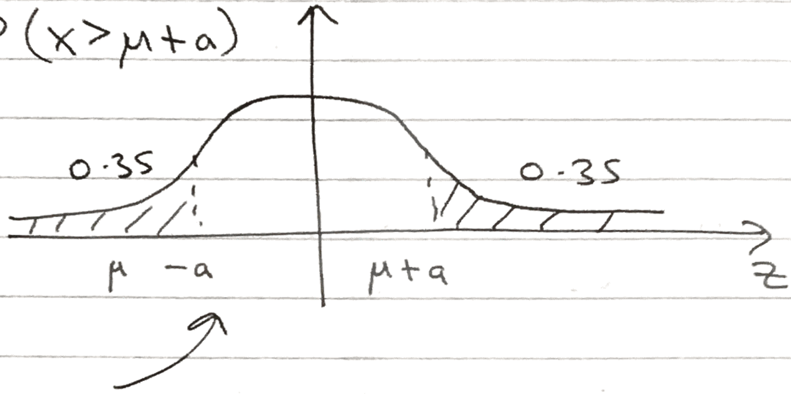
S1 October 2016 (MA)

Q1a) $X \sim N(\mu, \sigma^2)$ $\qquad P(X > \mu + a) = 0.35$

$P(X > \mu - a) = 1 - P(X > \mu + a)$

$= 1 - 0.35$

$= \boxed{0.65}$



b) $P(\text{required}) = \text{unshaded area} = 1 - 0.35 - 0.35$

$= \boxed{0.30}$

c) $P(X < \mu + a \mid X > \mu - a) = \dfrac{P(\mu - a < X < \mu + a)}{P(X > \mu - a)}$

$= \dfrac{0.30}{0.65} = \boxed{\dfrac{6}{13}}$

Q2a) $b + a + a + b + \frac{1}{5} = 1$

$\boxed{2b + 2a = 0.8}$

b) $E(X) = \sum x P(X = x) = -2b - a + a + 2b + \frac{3}{5}$

$= \boxed{\dfrac{3}{5}}$

c) i) $E(X^2) = \sum x^2 P(X=x) = (-2)^2 b + (1)^2 a + (1)^2 a + (2)^2 b + (3)^2\left(\frac{1}{5}\right)$

$E(X^2) = 4b + a + a + 4b + \frac{9}{5} = 3.5$

$\therefore \quad 8b + 2a = 1.7$ //

ii) $\left[\text{from } (a)\right] - \left[2b + 2a = 0.8\right]$

$6b + 0 = 0.9$

$\therefore b = \frac{0.9}{6} = \boxed{0.15}$

so $\quad 2a + 2(0.15) = 0.8$

$\therefore a = \frac{0.8 - 2(0.15)}{2} = \boxed{0.25}$

d) $Var(X) = E(X^2) - \left[E(X)\right]^2$

$E(X) = 0.6, \quad E(X^2) = 3.5$

so $Var(X) = 3.5 - (0.6)^2 = \boxed{3.14}$

e) $P(Y > 0) = P(5 - 3X > 0) = P\left(\frac{5}{3} > X\right)$

$= P\left(X < \frac{5}{3}\right) = P(X=-2) + P(X=-1) + P(X=1)$

$= b + a + a = 0.15 + 0.50$

$= \boxed{0.65}$

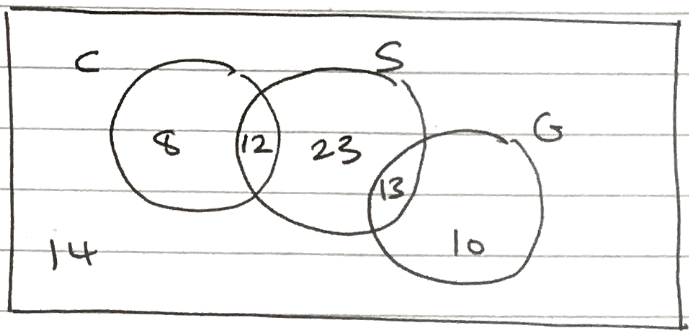
f i) $E(Y) = E(5-3X) = -3E(X) + 5$

$= -3(0.6) + 5 = \boxed{3.2}$

ii) $Var(Y) = Var(5-3X) = 9\,Var(X) = 9 \times 3.14$

$= \boxed{28.3}$

Q3a)



b i) $P(S) = \dfrac{12+23+13}{80} = \boxed{0.6}$

ii) $P(S \mid \text{coat}) = \dfrac{P(S \cap C)}{P(C)} = \dfrac{12}{8+12} = \boxed{0.6}$

iii) $P(S) = P(S \mid C)$ so S and C are independent

$0.6 = 0.6$

c) $P(S \mid G) = \dfrac{P(S \cap G)}{P(G)} = \dfrac{13}{13+10} = \dfrac{13}{23} = 0.57$ //

$P(S \mid C) = 0.6$ //

$0.57 < 0.60$

so the assistant selling coats is better //

Q4a) $\sum f = 456$

$\sum f^2 = 26072$

$n = 8$

$S_{ff} = 26072 - \frac{(456)^2}{8}$

$= \boxed{80}$

b) $S_{hh} = 610463 - \frac{(2209)^2}{8} = \boxed{503}$ (3s.f)

c) $PMCC = \frac{S_{fh}}{\sqrt{S_{ff} \times S_{hh}}} = \frac{182}{\sqrt{80 \times 503}} = \boxed{0.907}$

d) $r \approx 1$ so strong +ve correlation between f and h. Hence belief is supported.

e) $b = \frac{S_{fh}}{S_{ff}} = \frac{182}{80} = 2.275$

$a = \bar{y} - b\bar{x} = \bar{h} - b\bar{f} = \frac{2209}{8} - 2.275\left(\frac{456}{8}\right)$

$= 146.45$

So $\boxed{h = 146 + 2.28f}$ $(h = a + bf)$

f) P(required) $= P(-3 < E < 3)$

$= P\left(\frac{-3-0}{4} < Z < \frac{3-0}{4}\right) = P(-0.75 < Z < 0.75)$

$= P(Z < 0.75) - [1 - P(Z < 0.75)] =$

$= 0.7734 - [1 - 0.7734] = \boxed{0.547}$

(Q5a) $A \sim N(\mu, \sigma^2)$

$P(A < 388) = 0.001$

$P\left(Z < \frac{388-\mu}{\sigma}\right) = 0.001$

$\therefore P\left(Z > \frac{\mu - 388}{\sigma}\right) = 0.001$

but $P(Z > 3.0902) = 0.001$

hence $\frac{\mu - 388}{\sigma} = 3.0902$

① ~ $\mu - 388 = 3.0902\,\sigma$

also, $P(A > 410) = 0.0197$

$P\left(Z > \frac{410-\mu}{\sigma}\right) = 0.0197$

$P\left(Z < \frac{410-\mu}{\sigma}\right) = 1 - 0.0197 = 0.9803$

but $P(Z < 2.06) = 0.9803$

so $\frac{410-\mu}{\sigma} = 2.06$

so $410 - \mu = 2.06\sigma$ ~ ②

① + ② : $410 - 388 = (2.06 + 3.0902)\sigma$

so $\sigma = \frac{22}{5.1502} = \boxed{4.27}$

and so $\mu = 388 + 3.0902(4.27)$

$\boxed{\mu = 401}$ (3sf)

b) $P(A < 388) = 0.001$

$P(A > 410) = 0.0197$

$P(388 < A < 410) = 1 - 0.001 - 0.0197 = 0.9793$

so... Expected profit will be

$$\begin{array}{rl} & 0.001 \times (-100) \\ + & 0.0197 \times (-0.3) \\ + & 0.9793 \times (0.25) \\ \hline & 0.14 \end{array}$$

so expected profit will be 14p.

$\Rightarrow \boxed{£0.14}$

Q6a) $\frac{19}{4} = 4.75 \rightarrow 5^{th}$ term $= \boxed{Q_1 = 117}$

$\frac{19}{2} = 9.5 \rightarrow 10^{th}$ term $= \boxed{\text{median} = 122}$

$\frac{3 \times 19}{4} = 14.25 \rightarrow 15^{th}$ term $= \boxed{Q_3 = 125}$

b) $125 - 117 = \boxed{8} = IQR.$

c) Lower limit $= Q_1 - 1.5(Q_3 - Q_1)$

$= 117 - 1.5(8) = 105$

Upper limit $= Q_3 + 1.5(Q_3 - Q_1)$

$= 125 + 1.5(8) = 137$

so outliers are: 101, 102, 139

d) see mark scheme.

e) mean $= \frac{\sum x}{n} = \frac{2299}{19} = \boxed{121}$

variance $= \frac{\sum x^2}{n} - \left[\frac{(\sum x)}{(\sum n)}\right]^2 = \frac{279709}{19} - (121)^2$

so s.d $= \sigma = \sqrt{\frac{279709}{19} - (121)^2}$

$= \boxed{8.97}$

f) $\bar{x} \pm 2.7\sigma$

$\bar{x} + 2.7\sigma = \boxed{145}$

$\bar{x} - 2.7\sigma = \boxed{96.8}$

g) from a, $Q_3 - Q_2 < Q_2 - Q_1$

$3 < 5$

so data is skewed.

$\therefore$ N.D not suitable