S1 Jan 17 (IAL) MA

Q1a) Area of 2^{nd} bar $= (1)\times(5) = k \times$ freq.

$5 = 5k \quad \therefore k = 1$ $\therefore$ (Area = frequency)

for first bar: Area $= 2\times 2 = \boxed{4 = \text{frequency}}$

(Area $= k \times$ freq.)

b) $P(T > 3) = \dfrac{\text{no. of tomatoes with weight} > 3g}{\text{total no. of tomatoes}}$

$= \dfrac{100-5-4}{100} = \boxed{0.91}$

c) Area to the right of $(x = 6.25) = [(7-6.25)\times 16]$

$+ [25\times 1] + [10\times 1]$
$+ [8\times 1]$

$= 55$

$\therefore$ proportion $= \boxed{\dfrac{55}{100}}$

d) $0.55 > 0.50 \quad \therefore$ median > 6.25.

e) median $>$ mean for negative skew.
median $> 6.25 \quad \therefore$ $\boxed{\text{negative skew}}$

f) $P(5.5 < \text{weight} < 7) = \dfrac{(7-5.5)\times 16}{100} = \dfrac{24}{100}$

$\therefore$ P(one tomato is within 0.75g of mean) $= \dfrac{24}{100}$

but there is another tomato selected:

$$P(2^{nd} \text{ tomato is within } 0.75g \text{ of mean}) = \frac{23}{99}$$

$$\therefore P(\text{both are within } 0.75g) = \frac{23}{99} \times \frac{24}{100} = \boxed{0.056}$$

2a) The event that the integer is both a prime and ends in 3.

b) $$\frac{6+2+2+5}{50} = \boxed{\frac{3}{10}}$$

c) $$\frac{12}{50} = \boxed{\frac{6}{25}}$$ (A, B, C can't happen)

d) $$P(\text{prime} \mid >20) = P(A \mid C) = \frac{P(A \cap C)}{P(C)}.$$

$$P(A \cap C) = \frac{5+2}{50} = \frac{7}{50}$$

$$P(C) = \frac{5+2+1+22}{50} = \frac{30}{50}$$

$$P(A \mid C) = \frac{\frac{7}{50}}{\frac{30}{50}} = \boxed{\frac{7}{30}}$$

e) from (d), $P(A \mid C) = 7/30$.

from (b), $P(A) = 3/10$

these probabilities are different which means that whether or not C occurs has an effect on whether or not A occurs.

$\therefore$ they <u>aren't</u> independent.

f) $$P(B \mid A \cap C) = \frac{P(A \cap B \cap C)}{P(A \cap C)}$$

$$= \frac{2/50}{7/50} = \boxed{\frac{2}{7}}$$

3a) $$\text{mean} = \frac{\sum y}{12} = \frac{-27}{12} = \boxed{-2.25}$$

$$\text{variance} = \frac{\sum y^2}{n} - \left(\frac{\sum y}{n}\right)^2$$

$$= \frac{62.98}{12} - (-2.25)^2 = \boxed{0.186}$$

bi) $$S_{xy} = \sum xy - \frac{(\sum x)(\sum y)}{n}$$

$$= -1190.7 - \frac{(504)(-27)}{12} = -56.7$$

$$\therefore \text{PMCC} = \frac{-56.7}{\sqrt{1674 \times 2.23}} = \boxed{-0.928}$$

ii) PMCC $\simeq -1$ $\therefore$ a negative correlation is suggested between x and y.

$\therefore$ Priya's belief is incorrect.

c) $b = \frac{S_{xy}}{S_{xx}} = \frac{-56.7}{1674} = -0.03387..$ (-0.03387)

$a = \bar{y} - b\bar{x} = -\frac{27}{12} + 0.3387\left(\frac{504}{12}\right) = -0.827$

$\therefore \; y = -0.827 - 0.0339x$

d) $x = 32$: $y = -0.827 - 0.0339(32) = -1.91°C$

e) $\frac{w-32}{1.8} = y$: $\frac{w-32}{1.8} = -0.827 - 0.0339x$

$w - 32 = -1.4886 - 0.06102x$

$w = 30.5 - 0.061x$

fi) $Var(w) = Var(1.8y + 32) = 1.8^2 Var(y)$

$= 1.8^2 \times 0.186 \simeq \boxed{0.602}$

ii) PMCC not affected by coding

$\therefore \; r_{new} = \boxed{-0.928}$

4a) $E(X) = 5(0.13) + 6(0.21) + 7(0.29) + 8(0.37)$
$= \boxed{6.9}$

b) $E(X^2) = 25(0.13) + 36(0.21) + 49(0.29) + 64(0.37)$
$= 48.7$

$Var(X) = 48.7 - (6.9^2) = \boxed{1.09}$

c) $Var(3-2X) = Var(-2X+3) = 4\,Var(X)$
$= 4 \times 1.09 = \boxed{4.36}$

d) $E(Y) = \frac{1}{4}(5) + \frac{1}{4}(6) + \frac{1}{4}(7) + \frac{1}{4}(8) = \boxed{\frac{13}{2}}$

alt: uniform distribution $\therefore$ mean $= \frac{5+8}{2}$.

e) $P(X=Y) = P(X=5 \cap Y=5) + P(X=6 \cap Y=6) + \ldots$

$= \frac{1}{4}(0.13) + \frac{1}{4}(0.21) + \frac{1}{4}(0.29) + \frac{1}{4}(0.37)$

X and Y are independent!

$\boxed{= \frac{1}{4}}$

f) $P(X>Y)$: possible ways this could happen:

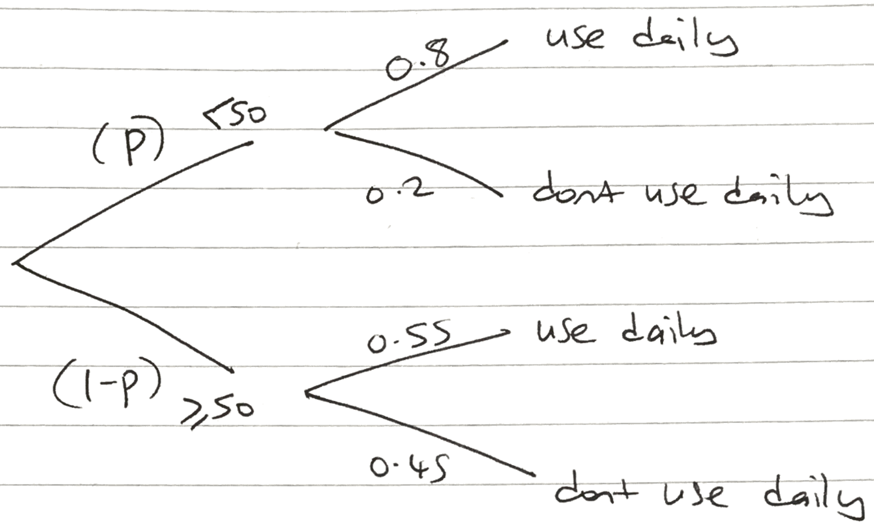
x	y
5	no possibilities.
6	5
7	5, 6
8	5, 6, 7

$\therefore P(X>Y) = P(X=6 \cap Y=5) + P(X=7 \cap Y=5 \text{ or } 6)$
$+ P(X=8 \cap Y=5 \text{ or } 6 \text{ or } 7)$

$= \frac{1}{4}(0.21) + 0.29\left(\frac{1}{4}+\frac{1}{4}\right) + 0.37\left(\frac{1}{4}+\frac{1}{4}+\frac{1}{4}\right)$

$$= \frac{21}{400} + \frac{29}{200} + \frac{111}{400} = \boxed{\frac{19}{40}}$$

5a)



b) P(a random person uses computer daily) $= 0.7$

this is what we are told.

P(a random person uses comp. daily) $= p(0.80) + (1-p)(0.55)$

$$\therefore \quad p(0.8) + (1-p)(0.55) = 0.70$$

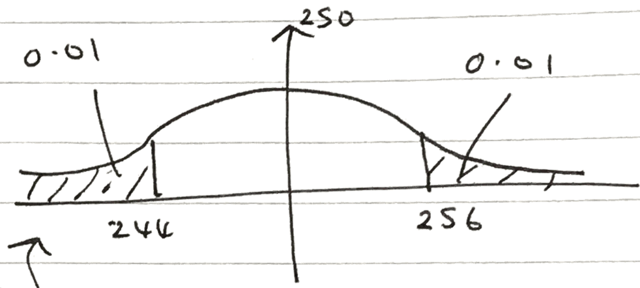
$$0.8p + 0.55 - 0.55p = 0.70$$

$$0.25p = 0.15 \qquad \therefore \boxed{p = 0.6}$$

c) $P(<50 \mid \text{daily use}) = \dfrac{P(\text{under 50} \cap \text{uses comp. daily})}{P(\text{uses comp. daily})}$

$$= \frac{0.8 \times 0.6}{0.70} = \boxed{\frac{48}{70}}$$

6a) $W \sim N(250, \sigma^2)$



$1 \cdot 00 - 0 \cdot 01 - 0 \cdot 01$

$= \boxed{0 \cdot 98} = 98\%$

(By symmetry)

b)

$P(W > 256) = 0 \cdot 01$

$P\left(Z > \frac{256-250}{\sigma}\right) = 0 \cdot 01$

↳ from tables, $2 \cdot 3263 = \frac{6}{\sigma}$

$\therefore \sigma = \frac{6}{2 \cdot 3263} = \boxed{2 \cdot 58}$

c) For the machine to shut down we need 2 rice bags selected to both be more than 4g from mean.

$P(\text{one bag} > 4\text{g from mean}) = P(W > 254) + P(W < 246)$

$= 2P(W > 254)$ [Due to symmetry]

$= 2P\left(Z > \frac{254-250}{2 \cdot 58}\right)$

$= 2P(Z > 1 \cdot 55)$

$= 2[1 - P(Z < 1 \cdot 55)]$

$= 2[1 - 0 \cdot 9394] = 0 \cdot 1212$

$\therefore P(\text{both bags} > 4\text{g from mean}) = (0 \cdot 1212)^2 = \boxed{0 \cdot 0147}$

7a)

X	1	2	3	4
$P(X=x)$	$\frac{a+b}{60}$	$\frac{2a+b}{60}$	$\frac{3a+b}{60}$	$\frac{4a+b}{60}$

$$\sum_{x=1}^{4} P(X=x) = 1.$$

$$\Rightarrow \frac{a+b+2a+b+3a+b+4a+b}{60} = 1$$

$$\Rightarrow 10a + 4b = 60$$

$\div 2$: $5a + 2b = 30$ —— 1

b) $F(3) = P(X=1) + P(X=2) + P(X=3) = \frac{13}{20}$

$$\Rightarrow \frac{a+b+2a+b+3a+b}{60} = \frac{13}{20}$$

$$\Rightarrow 6a + 3b = 39 \text{ —— } 2$$

solving 1 and 2 simultaneously:

2 $\div 3$: $2a + b = 13$

$\therefore b = 13 - 2a$

$\hookrightarrow$ 1 : $5a + 2(13-2a) = 30$

$5a + 26 - 4a = 30$

$a + 26 = 30$

$\boxed{a = 4}$ $\therefore b = 13 - 8 = \boxed{5 = b}$

c)

y	1	4	9	16
$P(Y=y)$	$\frac{9}{60}$	$\frac{13}{60}$	$\frac{17}{60}$	$\frac{21}{60}$

So for cumulative ...

y	1	4	9	16
$F(y)$	$\frac{9}{60}$	$\frac{22}{60}$	$\frac{39}{60}$	1

$F(1) = P(y=1)$

$F(4) = P(y=1) + P(y=4) = \frac{9}{60} + \frac{22}{60}$

$F(9) = P(y=1) + P(y=4) + P(y=9) = \frac{9+13+17}{60}$