



Mark Scheme (Unused)

January 2022

Pearson Edexcel International Advanced Level
in Pure Mathematics P4 (WMA14)
Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.
3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

- bod – benefit of doubt
- ft – follow through
- the symbol \surd will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- d... or dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper or ag- answer given
- \square or d... The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.
6. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

| Question Number | Scheme | Notes | Marks |
|-----------------|--|---|----------------|
| 1(a) | $\frac{2}{\sqrt{9-2x}} = \frac{2}{3\sqrt{\left(1-\frac{2}{9}x\right)}}$ <p style="text-align: center;">or</p> $\frac{2}{\sqrt{9-2x}} = 2(9-2x)^{-\frac{1}{2}} = 2 \times \frac{1}{3} \left(1-\frac{2}{9}x\right)^{-\frac{1}{2}}$ | Obtains $\sqrt{9-2x} = 3\sqrt{(1-\dots)}$ | B1 |
| | $\left(1-\frac{2}{9}x\right)^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2}\right)\left(-\frac{2}{9}x\right) + \frac{-\frac{1}{2}\left(-\frac{1}{2}-1\right)}{2!}\left(-\frac{2}{9}x\right)^2 + \frac{-\frac{1}{2}\left(-\frac{1}{2}-1\right)\left(-\frac{1}{2}-2\right)}{3!}\left(-\frac{2}{9}x\right)^3 + \dots$ <p>M1: Attempts the binomial expansion of $(1+kx)^n$ to get the third and/or fourth term with an acceptable structure. The correct binomial coefficient must be combined with the correct power of x and the correct power of 2. A1: Correct simplified or unsimplified expansion (NB simplified is $= 1 + \frac{1}{9}x + \frac{1}{54}x^2 + \frac{5}{1458}x^3 + \dots$)</p> | | M1 A1 |
| | $\frac{2}{\sqrt{9-2x}} = \frac{2}{3} + \frac{2}{27}x + \frac{1}{81}x^2 + \frac{5}{2187}x^3 + \dots$ | 2 correct simplified terms | A1 |
| | | All correct | A1 |
| | | | (5) |
| (b) | $x=1 \Rightarrow \frac{2}{\sqrt{9-2}} = \frac{2}{3} + \frac{2}{27} + \frac{1}{81} + \frac{5}{2187} + \dots$ $\Rightarrow \sqrt{7} \approx 2 \div \frac{1652}{2187} \text{ " or } 2 \times \frac{2187}{1652} \text{ "}$ <p>Substitutes $x = 1$ and divides into 2 or equivalent</p> | | M1 |
| | = 2.6477 | Correct approximation | A1 |
| | | | (2) |
| | Alternative for (b): | | |
| | $x=1 \Rightarrow \frac{2}{\sqrt{9-2}} = \frac{2}{3} + \frac{2}{27} + \frac{1}{81} + \frac{5}{2187} + \dots$ $\frac{2}{\sqrt{7}} = \frac{2\sqrt{7}}{7} \Rightarrow \sqrt{7} \approx \frac{7}{2} \times \frac{1652}{2187} \text{ "}$ <p>Substitutes $x = 1$ and multiplies by $\frac{7}{2}$</p> | | M1 |
| | = 2.6438 | Correct approximation | A1 |
| | | | Total 7 |

| Question Number | Scheme | Notes | Marks |
|-----------------|--|--|----------------|
| 2(a) | $\frac{x}{y} = t$ | Cao | B1 |
| | | | (1) |
| (b) | $y = \frac{\left(\frac{x}{y}\right)^3}{2\left(\frac{x}{y}\right)+1}$ or $x = \frac{\left(\frac{x}{y}\right)^4}{2\left(\frac{x}{y}\right)+1}$ | Uses the y coordinate to obtain y in terms of x and y or uses the x coordinate to obtain x in terms of y and x | M1 |
| | $y = \frac{x^3}{2xy^2 + y^3} \Rightarrow y(2xy^2 + y^3) = x^3$ or $x = \frac{x^4}{2xy^3 + y^4} \Rightarrow x(2xy^3 + y^4) = x^4$ | Uses correct algebra to eliminate the fractions | M1 |
| | $x^3 - 2xy^3 - y^4 = 0^*$ | Cso | A1* |
| | | | (3) |
| | | | Total 4 |

| Question Number | Scheme | Notes | Marks |
|-----------------|--|---|-----------------|
| 3(a) | $3y^2 - 11x^2 + 11xy = 20y - 36x + 28$ $\Rightarrow 6y \frac{dy}{dx} - 22x + 11x \frac{dy}{dx} + 11y = 20 \frac{dy}{dx} - 36$ <p style="text-align: center;">M1: $y^2 \rightarrow Ay \frac{dy}{dx}$</p> <p style="text-align: center;">M1: $11xy \rightarrow px \frac{dy}{dx} + qy$</p> <p style="text-align: center;">A1: All correct</p> | | M1M1A1 |
| | $(6y + 11x - 20) \frac{dy}{dx} = 22x - 11y - 36 \Rightarrow \frac{dy}{dx} = \dots$ <p>Collects terms in $\frac{dy}{dx}$ (must be 3 and from the appropriate terms) and makes $\frac{dy}{dx}$ the subject</p> | | M1 |
| | $\frac{dy}{dx} = \frac{22x - 11y - 36}{6y + 11x - 20}$ | Correct expression or correct equivalent | A1 |
| | | | (5) |
| (b) | $x = 4 \Rightarrow 3y^2 - 176 + 44y = 20y - 144 + 28$ | Substitutes $x = 4$ into C to obtain a 3TQ in y | M1 |
| | $3y^2 + 24y - 60 = 0 \Rightarrow y = \dots$ | Solves for y | M1 |
| | $y = -10 \text{ (}, 2)$ | Correct value | A1 |
| | $(4, -10) \rightarrow \frac{dy}{dx} = \frac{88 + 110 - 36}{-60 + 44 - 20}$ | Substitutes $x = 4$ and their negative y into their $\frac{dy}{dx}$ | M1 |
| | $\frac{dy}{dx} = -\frac{9}{2}$ | Correct value | A1 |
| | | | (5) |
| | | | Total 10 |

| Question Number | Scheme | Notes | Marks |
|-----------------|---|---|----------------|
| 4(a) | $\frac{4-4x}{x(x-2)^2} \equiv \frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$ | Correct form for the partial fractions | B1 |
| | $4-4x = A(x-2)^2 + Bx(x-2) + Cx$ $\Rightarrow A = \dots \text{ or } B = \dots \text{ or } C = \dots$ | Uses a correct strategy to find at least one of their constants | M1 |
| | $\frac{4-4x}{x(x-2)^2} \equiv \frac{1}{x} - \frac{1}{x-2} - \frac{2}{(x-2)^2}$ | 2 correct constants | A1 |
| | | All correct | A1 |
| | | | (4) |
| (b) | $\int \left(\frac{1}{x} - \frac{1}{x-2} - \frac{2}{(x-2)^2} \right) dx = \ln x - \ln(x-2) + \frac{2}{x-2} (+c)$ | | M1 |
| | M1 for $\int \frac{\alpha}{x} dx = \beta \ln x$ or $\int \frac{\alpha}{x-2} dx = \beta \ln(x-2)$ | | M1 |
| | M1 for $\int \frac{\alpha}{(x-2)^2} dx = \frac{\beta}{x-2}$ | | A1 |
| | A1: All correct | | |
| | | | (3) |
| (c) | $\left[\ln x - \ln(x-2) + \frac{2}{x-2} \right]_3^5 = \left(\ln 5 - \ln 3 + \frac{2}{3} \right) - (\ln 3 - \ln 1 + 2)$ $= \ln \frac{5}{9} - \frac{4}{3}$ | | M1 |
| | M1: Correct use of limits and reaches the required form using log rules | | A1 |
| | A1: Correct answer | | |
| | | | (2) |
| | | | Total 9 |

| Question Number | Scheme | Notes | Marks |
|-----------------|--|--|-----------------|
| 5(a) | $4 + 2\lambda = 13 + 5\mu$ $4 - 3\lambda = -1 + \mu$ $-5 + 6\lambda = 4 - 3\mu$ | For writing down any 2 of these equations. | M1 |
| | E.g. $4 + 2\lambda = 13 + 5\mu$ $4 - 3\lambda = -1 + \mu$ $\Rightarrow \lambda = \dots \text{ or } \mu = \dots$ | Full method for finding λ or μ | M1 |
| | $\lambda = 2, \mu = -1$ | Both correct values | A1 |
| | $-5 + 6\lambda = -5 + 12 = 7$ $4 - 3\mu = 4 + 3 = 7$ So lines intersect | Shows that the parameters satisfy the third equation and makes a conclusion. | B1 |
| | $\lambda = 2 \rightarrow (4 + 4)\mathbf{i} + (4 - 6)\mathbf{j} + (-5 + 12)\mathbf{k}$ <p style="text-align: center;">or</p> $\mu = -1 \rightarrow (13 - 5)\mathbf{i} + (-1 - 1)\mathbf{j} + (4 + 3)\mathbf{k}$ | Uses their λ or μ to find A. | M1 |
| | $8\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}$ | Correct vector or coordinates | A1 |
| | | | (6) |
| (b) | $\begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 1 \\ -3 \end{pmatrix} = 10 - 3 - 18 = \sqrt{2^2 + 3^2 + 6^2} \sqrt{5^2 + 1^2 + 3^2} \cos \theta$ Full attempt at the scalar product between the direction vectors | | M1 |
| | $\cos \theta = \pm \frac{11}{7\sqrt{35}}$ | Correct magnitude for $\cos \theta$ (may be implied by e.g. $\theta = 105.4\dots$ or $74.6\dots$) | A1 |
| | $\theta = 74.6^\circ$ | Awrt 74.6 | A1 |
| | | | (3) |
| (c) | $ 2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k} = \sqrt{2^2 + 3^2 + 6^2} = 7$ | Finds the magnitude of the direction of l_1 | M1 |
| | $35 \div 7 = 5 \Rightarrow \lambda = 5$ $8\mathbf{i} - 2\mathbf{j} + 7\mathbf{k} \pm 5(2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k})$ | Correct strategy for one of the points | M1 |
| | $(18, -17, 37) \text{ or } (-2, 13, -23)$ | One correct point (ignore labels) | A1 |
| | $P(18, -17, 37) \text{ and } Q(-2, 13, -23)$ | Correct points with correct labels | A1 |
| | | | (4) |
| | | | Total 13 |

| Question Number | Scheme | Notes | Marks |
|--------------------------|---|-------|----------------|
| 6 Way 1 | $\int e^{2x} \cos 3x \, dx = \frac{1}{3} e^{2x} \sin 3x - \frac{2}{3} \int e^{2x} \sin 3x \, dx (+c)$ <p>M1: For applying parts to obtain $\alpha e^{2x} \sin 3x \pm \beta \int e^{2x} \sin 3x \, dx (+c)$</p> <p>A1: Correct expression</p> | | M1A1 |
| | $\int e^{2x} \cos 3x \, dx = \frac{1}{3} e^{2x} \sin 3x - \frac{2}{3} \left\{ -\frac{1}{3} e^{2x} \cos 3x + \frac{2}{3} \int e^{2x} \cos 3x \, dx \right\} (+c)$ <p>Applies parts again to $\int e^{2x} \sin 3x \, dx$ and obtains $\alpha e^{2x} \cos 3x \pm \beta \int e^{2x} \cos 3x \, dx$</p> | | M1 |
| | $\int e^{2x} \cos 3x \, dx = \frac{1}{3} e^{2x} \sin 3x + \frac{2}{9} e^{2x} \cos 3x - \frac{4}{9} \int e^{2x} \cos 3x \, dx (+c)$ <p>Fully correct application of parts twice</p> | | A1 |
| | $\int e^{2x} \cos 3x \, dx + \frac{4}{9} \int e^{2x} \cos 3x \, dx = \frac{1}{3} e^{2x} \sin 3x + \frac{2}{9} e^{2x} \cos 3x (+c)$ $\Rightarrow \frac{13}{9} \int e^{2x} \cos 3x \, dx = \frac{1}{3} e^{2x} \sin 3x + \frac{2}{9} e^{2x} \cos 3x (+c) \Rightarrow \int e^{2x} \cos 3x \, dx = \dots$ <p>Fully correct strategy for finding $\int e^{2x} \cos 3x \, dx$</p> | | M1 |
| | $= \frac{3}{13} e^{2x} \sin 3x + \frac{2}{13} e^{2x} \cos 3x + k$ | Cao | A1 |
| | | | (6) |
| Way 2 | $\int e^{2x} \cos 3x \, dx = \frac{1}{2} e^{2x} \cos 3x + \frac{3}{2} \int e^{2x} \sin 3x \, dx (+c)$ <p>M1: For applying parts to obtain $\alpha e^{2x} \cos 3x \pm \beta \int e^{2x} \sin 3x \, dx (+c)$</p> <p>A1: Correct expression</p> | | M1A1 |
| | $\int e^{2x} \cos 3x \, dx = \frac{1}{2} e^{2x} \cos 3x + \frac{3}{2} \left\{ \frac{1}{2} e^{2x} \sin 3x - \frac{3}{2} \int e^{2x} \cos 3x \, dx \right\} (+c)$ <p>Applies parts again to $\int e^{2x} \sin 3x \, dx$ and obtains $\alpha e^{2x} \sin 3x \pm \beta \int e^{2x} \cos 3x \, dx$</p> | | M1 |
| | $\int e^{2x} \cos 3x \, dx = \frac{1}{2} e^{2x} \cos 3x + \frac{3}{4} e^{2x} \sin 3x - \frac{9}{4} \int e^{2x} \cos 3x \, dx (+c)$ <p>Fully correct application of parts twice</p> | | A1 |
| | $\int e^{2x} \cos 3x \, dx + \frac{9}{4} \int e^{2x} \cos 3x \, dx = \frac{1}{2} e^{2x} \cos 3x + \frac{3}{4} e^{2x} \sin 3x (+c)$ $\Rightarrow \frac{13}{4} \int e^{2x} \cos 3x \, dx = \frac{1}{2} e^{2x} \cos 3x + \frac{3}{4} e^{2x} \sin 3x (+c) \Rightarrow \int e^{2x} \cos 3x \, dx = \dots$ <p>Fully correct strategy for finding $\int e^{2x} \cos 3x \, dx$</p> | | M1 |
| | $= \frac{3}{13} e^{2x} \sin 3x + \frac{2}{13} e^{2x} \cos 3x + k$ | Cao | A1 |
| | | | Total 6 |

| Question Number | Scheme | Notes | Marks |
|------------------|---|---|-----------------|
| 7(a) | $\frac{dV}{dt} = 300 - kV \Rightarrow \int \frac{dV}{300 - kV} = \int dt$ | Correct separation of variables | B1 |
| | $\int \frac{dV}{300 - kV} = -\frac{1}{k} \ln(300 - kV)$ | $\int \frac{dV}{300 - kV} = \alpha \ln(300 - kV)$ | M1 |
| | $-\frac{1}{k} \ln(300 - kV) = t + c$ | Correct equation including a constant of integration | A1 |
| | $-\frac{1}{k} \ln(300 - kV) = t + c \Rightarrow \ln(300 - kV) = -kt + d$ $\Rightarrow 300 - kV = e^{-kt+d}$ Correct processing to remove the "ln" | | M1 |
| | $kV = 300 - e^{-kt+d} \Rightarrow V = \frac{300}{k} - Be^{-kt}$ $V = \frac{300}{k} + Ae^{-kt} *$ | Correct proof | A1* |
| | | | (5) |
| (b) | $V = 0, t = 0 \Rightarrow 0 = \frac{300}{k} + A \Rightarrow A = -\frac{300}{k}$ | Uses $V = 0$ when $t = 0$ to find A in terms of k | M1 |
| | $V = \frac{300}{k} - \frac{300}{k} e^{-kt} \Rightarrow \frac{dV}{dt} = 300e^{-kt}$ | $\frac{dV}{dt} = \alpha e^{-kt}$ | M1 |
| | $300e^{-10k} = 200 \Rightarrow e^{-10k} = \frac{2}{3} \Rightarrow k = \dots$ | Uses $\frac{dV}{dt} = 200$ when $t = 10$ and correct processing to find k | M1 |
| | $k = -\frac{1}{10} \ln \frac{2}{3}$ | Oe e.g. $\frac{1}{10} \ln \frac{3}{2}$ | A1 |
| | | | (4) |
| (b) Way 2 | $V = 0, t = 0 \Rightarrow 0 = \frac{300}{k} + A \Rightarrow A = -\frac{300}{k}$ | Uses $V = 0$ when $t = 0$ to find A in terms of k | M1 |
| | $\frac{dV}{dt} = 200, t = 10 \Rightarrow 200 = 300 - kV$ $\Rightarrow kV = 100$ | Uses $\frac{dV}{dt} = 200$ when $t = 10$ to find a value for kV | M1 |
| | $V = \frac{300}{k} + Ae^{-kt} \Rightarrow kV = 300 - 300e^{-10k}$ $\Rightarrow 100 = 300 - 300e^{-10k} \Rightarrow e^{-10k} = \frac{2}{3} \Rightarrow k = \dots$ | Substitutes for kV, kA and $t = 10$ and uses correct processing to find k | M1 |
| | $k = -\frac{1}{10} \ln \frac{2}{3}$ | Oe e.g. $\frac{1}{10} \ln \frac{3}{2}$ | A1 |
| (c) | $6000 = \frac{3000}{\ln 1.5} - \frac{3000}{\ln 1.5} e^{-\frac{t}{10} \ln 1.5}$ $\Rightarrow e^{-\frac{t}{10} \ln 1.5} = 1 - 2 \ln 1.5$ $\Rightarrow -\frac{t}{10} \ln 1.5 = \ln(1 - 2 \ln 1.5)$ | Correct strategy using $V = 6000$ to reach $at = \dots$ | M1 |
| | $t = 41$ | Correct value | A1 |
| | | | (2) |
| | | | Total 11 |

| Question Number | Scheme | Notes | Marks |
|-----------------|---|---|----------------|
| 8 | Assume that there exist positive real numbers x and y such $\frac{9x}{y} + \frac{y}{x} < 6$ | Starts the proof by contradicting the given statement | B1 |
| | $\frac{9x}{y} + \frac{y}{x} < 6 \Rightarrow 9x^2 + y^2 < 6xy$ as x and y are both positive | Multiplies through by xy | M1 |
| | $\Rightarrow 9x^2 + y^2 - 6xy < 0$ $\Rightarrow (3x - y)^2 < 0$ | Reaches a correct contradictory statement | A1 |
| | As x and y are positive real numbers, this is a contradiction and so $\frac{9x}{y} + \frac{y}{x} < 6$ must be incorrect and so $\frac{9x}{y} + \frac{y}{x} \dots 6^*$ | Makes a suitable conclusion | A1* |
| | | | (4) |
| | | | Total 4 |

| Question Number | Scheme | Notes | Marks |
|-----------------|---|---|-----------------|
| 9(a) | $V = \pi \int y^2 dx = \pi \int y^2 \frac{dx}{d\theta} d\theta$ $= \pi \int (3 \sin \theta - \sin 2\theta)^2 (-5 \sin \theta) d\theta$ | Applies $V = \pi \int y^2 \frac{dx}{d\theta} d\theta$ with or without the π | M1 |
| | $= \pi \int (3 \sin \theta - 2 \sin \theta \cos \theta)^2 (-5 \sin \theta) d\theta$ | Applies $\sin 2\theta = 2 \sin \theta \cos \theta$ | M1 |
| | $= \pi \int \sin^2 \theta (3 - 2 \cos \theta)^2 (-5 \sin \theta) d\theta$ $V = -5\pi \int \sin^3 \theta (3 - 2 \cos \theta)^2 d\theta$ $V = -5\pi \int_{\pi}^0 \sin^3 \theta (3 - 2 \cos \theta)^2 d\theta$ $V = 5\pi \int_0^{\pi} \sin^3 \theta (3 - 2 \cos \theta)^2 d\theta^*$ | Fully correct integral in terms of $\sin \theta$ and $\cos \theta$ only (π not needed) | A1 |
| | $= \pi \int \sin^2 \theta (3 - 2 \cos \theta)^2 (-5 \sin \theta) d\theta$ $V = -5\pi \int \sin^3 \theta (3 - 2 \cos \theta)^2 d\theta$ $V = -5\pi \int_{\pi}^0 \sin^3 \theta (3 - 2 \cos \theta)^2 d\theta$ $V = 5\pi \int_0^{\pi} \sin^3 \theta (3 - 2 \cos \theta)^2 d\theta^*$ | Completes correctly with correct limits and no incorrect statements previously. The factor of π must be present throughout. | A1* |
| | | | (4) |
| (b) | $u = \cos \theta \Rightarrow V = 5\pi \int \sin^3 \theta (3 - 2u)^2 \frac{du}{-\sin \theta}$ | Applies the substitution correctly | M1 |
| | $\theta = 0 \Rightarrow u = 1, \theta = \pi \Rightarrow u = -1$ | Attempts to change θ limits to u limits | M1 |
| | $V = -5\pi \int \sin^2 \theta (3 - 2u)^2 du = -5\pi \int (1 - u^2)(3 - 2u)^2 du$ <p style="text-align: center;">Correct integral in terms of u only</p> | | A1 |
| | $(1 - u^2)(3 - 2u)^2 = (1 - u^2)(9 - 12u + 4u^2)$ $= 9 - 12u - 5u^2 + 12u^3 - 4u^4$ | Attempt to expand | M1 |
| | | Correct expansion | A1 |
| | $V = 5\pi \int_{-1}^1 (9 - 12u - 5u^2 + 12u^3 - 4u^4) du$ $= 5\pi \left[9u - 6u^2 - \frac{5u^3}{3} + 3u^4 - \frac{4u^5}{5} \right]_{-1}^1 = \dots$ <p style="text-align: center;">Integrates and applies their u limits</p> | | M1 |
| | $= \frac{196}{3} \pi$ | Cao | A1 |
| | | | (7) |
| | | | Total 11 |

