Pearson Edexcel

Mark Scheme (Results)

January 2021

Pearson Edexcel IAL Mathematics
Pure Mathematics P4
Paper WMA14/01

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January 2021
Publications Code WMA14_01_2021_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Pearson Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ or ft will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- d… or dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper or ag- answer given
- ⎣ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$$(x^2 + bx + c) = (x + p)(x + q), \text{ where } |pq| = |c|,$$ leading to *x* = ...

$$(ax^2 + bx + c) = (mx + p)(nx + q), \text{ where } |pq| = |c| \text{ and } |mn| = |a|,$$ leading to *x* = ...

2. Formula

Attempt to use correct formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c, \quad q \neq 0,$ leading to *x* = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Marks
1(a)	$\left(\frac{1}{4}-5x\right)^{\frac{1}{2}}=\frac{1}{2}(...)$	B1
	$=(1-20x)^{\frac{1}{2}}=1+\left(\frac{1}{2}\right)\times(-20x)+\frac{\left(\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)}{2!}\times(-20x)^2+\frac{\left(\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{3!}\times(-20x)^3...$	M1A1
	$=\frac{1}{2}-5x-25x^2-250x^3+...$	A1 A1
	Special case: If the final answer is left as $\frac{1}{2}\left(1-10x-50x^2-500x^3+...\right)$ Award SC B1M1A1A1A0	
		(5)
	Alternative by direct expansion	
	$\left(\frac{1}{4}-5x\right)^{\frac{1}{2}}=\left(\frac{1}{4}\right)^{\frac{1}{2}}+\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)^{-\frac{1}{2}}(-5x)^1+\frac{\frac{1}{2}\times-\frac{1}{2}}{2}\left(\frac{1}{4}\right)^{-\frac{3}{2}}(-5x)^2+\frac{\frac{1}{2}\times-\frac{1}{2}\times-\frac{3}{2}}{3!}\left(\frac{1}{4}\right)^{-\frac{5}{2}}(-5x)^3$	B1M1A1
	$=\frac{1}{2}-5x-25x^2-250x^3+...$	A1A1
(b)	$\left(\frac{1}{4}-\frac{5}{100}\right)^{\frac{1}{2}}=\left(\frac{1}{5}\right)^{\frac{1}{2}}=\frac{1}{2}-5\times\frac{1}{100}-25\left(\frac{1}{100}\right)^2-250\left(\frac{1}{100}\right)^3+...$ $\frac{\sqrt{5}}{5}\approx\frac{1789}{4000}$ or $\frac{1}{\sqrt{5}}\approx\frac{1789}{4000}$ $\Rightarrow\sqrt{5}\approx5\times\frac{1789}{4000}$ or $\sqrt{5}\approx1\div\frac{1789}{4000}$	M1
	$\sqrt{5}\approx\frac{1789}{800}$ or $\frac{4000}{1789}$	A1
		(2)
		(7 marks)

(a)

B1: For taking out a factor of $\left(\frac{1}{4}\right)^{\frac{1}{2}}$ or $\frac{1}{2}$ or 0.5 etc.

M1: Expands $(1+kx)^{\frac{1}{2}}$, $k\neq\pm1$ with the correct structure for the third or fourth term

e.g. $\pm\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2!}\times(kx)^2$ or $\pm\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}\times(kx)^3$ with or without the bracket around the *kx*

A1: For either term three or term four being correct in any form.

E.g. $\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}\times(20x)^2$ or $\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}\times(-20x)^2$ or $\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}\times(-20x)^3$ or $-\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}\times(20x)^3$

The brackets must be present unless they are implied by subsequent work. This mark is independent of the B mark.

A1: Two terms correct and simplified of $\frac{1}{2}-5x-25x^2-250x^3$. Allow if any of the '–' signs are written as "+–".

A1: All four terms correct and simplified of $\frac{1}{2}-5x-25x^2-250x^3$. Allow the terms to be listed.

Ignore any extra terms and apply isw if necessary. If any of the '–' signs are written as "+–" score A0.

Alternative:

B1: For a first term of $\left(\frac{1}{4}\right)^{\frac{1}{2}}$ or $\frac{1}{2}$ or 0.5 etc.

M1: For the correct structure for the third or fourth term. E.g. $\frac{\frac{1}{2}\times-\frac{1}{2}}{2}\left(\frac{1}{4}\right)^{-\frac{3}{2}}(kx)^2$ or $\frac{\frac{1}{2}\times-\frac{1}{2}\times-\frac{3}{2}}{3!}\left(\frac{1}{4}\right)^{-\frac{5}{2}}(kx)^3$

where $k \neq \pm 1$

A1: For either term three or term four being correct in any form.

$$\text{e.g. } \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}\times\left(\frac{1}{4}\right)^{-\frac{3}{2}}(-5x)^2 \text{ or } \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}\times\left(\frac{1}{4}\right)^{-\frac{5}{2}}(-5x)^3$$

The brackets must be present unless they are implied by subsequent work.

A1: Two terms correct and simplified of $\frac{1}{2}-5x-25x^2-250x^3$. Allow if any of the '–' signs are written as "+–".

A1: All four terms correct and simplified of $\frac{1}{2}-5x-25x^2-250x^3$. Allow the terms to be listed.

Ignore any extra terms and apply isw if necessary. If any of the '–' signs are written as "+–" score A0.

(b)

M1: Attempts to substitute $x=\frac{1}{100}$ into their part (a) and either multiplies by 5 or finds reciprocal.

A1: $\left(\sqrt{5}=\right)\frac{1789}{800}$ or $\frac{4000}{1789}$

Question Number	Scheme	Marks
2(a)	$\overrightarrow{BA}.\overrightarrow{BC} = -6\times2+2\times5-3\times8 = (-26)$	M1
	Uses $\overrightarrow{BA}.\overrightarrow{BC} = \lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BC}\rvert\cos\theta \Rightarrow -26 = \sqrt{49}\times\sqrt{93}\cos\theta \Rightarrow \theta = ...$	**d**M1
	$\theta = 112.65°$	A1
		(3)
(b)	Attempts to use $\lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BC}\rvert\sin\theta$ with their θ	M1
	Area = awrt 62.3	A1
		(2)
		(5 marks)

(a)

M1: Attempts the scalar product of $\pm\overrightarrow{AB}.\ \pm\overrightarrow{BC}$ condone slips as long as the intention is clear

Or attempts the vector product $\pm\overrightarrow{AB}\times\pm\overrightarrow{BC}$ **(see alternative 1)**

Or attempts vector AC **(see alternative 2)**

dM1: Attempts to use $\pm\overrightarrow{AB}.\overrightarrow{BC} = \lvert\overrightarrow{AB}\rvert\lvert\overrightarrow{BC}\rvert\cos\theta$ AND proceeds to a value for θ

Expect to see at least one correct attempted calculation for a modulus.

For example $\sqrt{2^2+5^2+8^2}\left(=\sqrt{93}\right)$ or $\sqrt{6^2+2^2+3^2}\left(=7\right)$

Note that we condone poor notation such as: $\cos\theta = \frac{26}{7\sqrt{93}} = 67.35°$ **Depends on the first mark.**

<u>**Must be an attempt to find the correct angle.**</u>

A1: $\theta =$ awrt $112.65°$ Versions finishing with $\theta =$ awrt $67.35°$ will normally score M1 dM1 A0

Angles given in radians also score A0 (NB $\theta = 1.9661...$ or acute $1.1754...$)

Allow e.g. $\theta = 67.35° \Rightarrow \theta = 180-67.35° = 112.65$ and allow $\cos\theta = \frac{26}{7\sqrt{93}} \Rightarrow \theta = 112.65$

<u>**1. Alternative using the vector product:**</u>

M1: Attempts the vector product $\pm\overrightarrow{AB}\times\pm\overrightarrow{BC} = \pm\begin{pmatrix}6\\-2\\3\end{pmatrix}\times\pm\begin{pmatrix}2\\5\\8\end{pmatrix} = \pm\begin{pmatrix}-31\\-42\\34\end{pmatrix}$ condone slips as long as the intention is clear

dM1: Attempts to use $\pm\overrightarrow{AB}\times\overrightarrow{BC} = \lvert\overrightarrow{AB}\rvert\lvert\overrightarrow{BC}\rvert\sin\theta$ AND proceeds to a value for θ

Expect to see at least one correct attempted calculation for a modulus on rhs and attempt at the modulus of the vector product

For example $\sqrt{2^2+5^2+8^2}$ or $\sqrt{6^2+2^2+3^2}$ and $\sqrt{31^2+42^2+34^2}\left(=\sqrt{3881}\right)$

Note that we condone poor notation such as: $\sin\theta = \frac{\sqrt{3881}}{7\sqrt{93}} = 67.35°$ **Depends on the first mark.**

<u>Must be an attempt to find the correct angle.</u>

A1: $\theta = \text{awrt } 112.65°$ Versions finishing with $\theta = \text{awrt } 67.35°$ will normally score M1 dM1 A0

<u>2. Alternative using cosine rule:</u>

M1: Attempts $\pm\overrightarrow{AC} = \pm\left(\overrightarrow{AB} + \overrightarrow{BC}\right) = \pm(8\mathbf{i} + 3\mathbf{j} + 11\mathbf{k})$ condone slips and poor notation as long as the intention is clear e.g. allow $\begin{pmatrix} 8\mathbf{i} \\ 3\mathbf{j} \\ 11\mathbf{k} \end{pmatrix}$

dM1: Attempts to use $AC^2 = AB^2 + BC^2 - 2AB.BC\cos\theta$ AND proceeds to a value for θ

<u>Must be an attempt to find the correct angle.</u>

A1: $\theta = \text{awrt } 112.65°$

(b)

M1: Attempts to use $\left|\overrightarrow{AB}\right|\left|\overrightarrow{BC}\right|\sin\theta$ with their θ. You may see $\frac{1}{2}\left|\overrightarrow{AB}\right|\left|\overrightarrow{BC}\right|\sin\theta$ found first before it is doubled.

or attempts the magnitude of their vector product e.g. $\sqrt{3881}$

A1: Area = awrt 62.3. If this is achieved from an angle of $\theta = \text{awrt } 67.35°$ full marks can be scored

Note that there are other more convoluted methods for finding the area – score M1 for a complete and correct method using their values and send to review if necessary.

Question Number	Scheme	Marks
3	States the largest odd number and an odd number that is greater E.g. odd number n and $n + 2$	M1
	Fully correct proof including • the assumption: there exists a greatest odd number "n" • a correct statement that their second odd number is greater than their assumed greatest odd number • a minimal conclusion " this is a contradiction, hence proven" You can ignore any spurious information e.g. $n > 0$, $n + 2 > 0$ etc.	A1*
		(2)
		(2 marks)

M1: For starting the proof by **stating** an odd number and a larger odd number.
Examples of an allowable start are

- **odd number** "n" with "$n + 2$"
- **odd number** "n" with "n^2"
- "$2k + 1$" with "$2k + 3$"
- "$2k + 1$" with "$(2k + 1)^3$"
- "$2k + 1$" with "$2k + 1 + 2k$"

Note that stating $n = 2k$, even when accompanied by the statement that "n" is odd is M0

A1*: A fully correct proof using contradiction
This must consist of
1) An assumption E.g. "(Assume that) there exists a greatest odd number n"
"Let "$2k + 1$" be the greatest odd number"
2) A minimal statement showing their second number is greater than the first,
E.g. If "n" is odd and "$n + 2$" is greater than n

If "n" is odd and $n^2 > n$
$2k + 3 > 2k + 1$
$2k + 2k + 1 > 2k + 1$

Any algebra (e.g. expansions) must be correct. So $(2k+1)^2 = 4k^2 + 2k + 1$ would be A0

3) A minimal conclusion which could be
"hence there is no greatest odd number", "hence proven", or simply ✓

Question Number	Scheme	Marks
4(a)	$k = 2$ or $x > 2$	B1
	$t = \frac{1}{x-2} \Rightarrow y = \frac{1-\frac{2}{x-2}}{3+\frac{1}{x-2}}$	M1 A1
	$\frac{1-\frac{2}{x-2}}{3+\frac{1}{x-2}} = \frac{x-2-2}{\dots}$ or $\frac{\dots}{3(x-2)+1}$	A1 (M1 on EPEN)
	$y = \frac{x-4}{3x-5}$	A1
		(5)
(b)	$-2 < \mathrm{g} < \frac{1}{3}$	M1 A1
		(2)
		(7 marks)

(a)

B1: States that $k = 2$ or else states that the domain is $x > 2$. **Must be seen in part (a).**

M1: Attempts to find t in terms of x and substitutes into y.

Condone poor attempts but you should expect to see $t = \mathrm{f}(x)$ found from $x = \frac{1}{t} + 2$ substituted into $y = \frac{1-2t}{3+t}$ condoning slips.

A1: A correct unsimplified equation involving just x and y

A1(M1 on EPEN): Correct numerator or denominator with fraction removed (allow unsimplified)

A1: $y = \frac{x-4}{3x-5}$ or $\mathrm{g}(x) = \frac{x-4}{3x-5}$ (must be $y = \dots$ or $\mathrm{g}(x) = \dots$ but allow this mark as long as the $y = \dots$ or $\mathrm{g}(x) = \dots$ is present at some point)

<u>Alternative 1 for part (a)</u>

M1: Assume $\mathrm{g}(x) = \frac{ax+b}{cx+d}$ and substitute in $x = \frac{1}{t} + 2$

A1: $\mathrm{g}(x) = \frac{a+(b+2a)t}{c+(d+2c)t}$

A1(M1 on EPEN): Correct numerator or denominator

A1: $y = \frac{x-4}{3x-5}$ or $\mathrm{g}(x) = \frac{x-4}{3x-5}$ (must be $y = \dots$ or $\mathrm{g}(x) = \dots$ but allow this mark as long as the $y = \dots$ or $\mathrm{g}(x) = \dots$ is present at some point)

<u>Alternative 2 for part (a)</u>

M1: Attempts to find t in terms of y and substitutes into x.

Condone poor attempts but you should expect to see $t = \mathrm{f}(y)$ found from $y = \dfrac{1-2t}{3+t}$ substituted into $x = \dfrac{1}{t} + 2$ condoning slips. (NB $t = \dfrac{1-3y}{y+2} \Rightarrow x = \dfrac{y+2}{1-3y} + 2$)

A1: A correct unsimplified equation involving just x and y

A1(M1 on EPEN): Correct numerator or denominator

A1: $y = \dfrac{x-4}{3x-5}$ or $\mathrm{g}(x) = \dfrac{x-4}{3x-5}$ (must be $y = \ldots$ or $\mathrm{g}(x) = \ldots$ but allow this mark as long as the $y = \ldots$ or $\mathrm{g}(x) = \ldots$ is present at some point)

(b)

M1: For obtaining one of the 2 boundaries (just look for values) e.g. -2 or $\dfrac{1}{3}$ or for attempting g(2) for their g **or** for attempting $\dfrac{\text{their } a}{\text{their } c}$. Note that for this mark they must be attempting values of y (or $\mathrm{g}(x)$).

A1: Correct range: Allow $-2 < \mathrm{g} < \dfrac{1}{3}$, $-2 < \mathrm{g}(x) < \dfrac{1}{3}$, $-2 < y < \dfrac{1}{3}$, $\left(-2, \dfrac{1}{3}\right)$, $\mathrm{g} > -2$ **and** $\mathrm{g} < \dfrac{1}{3}$

Question Number	Scheme	Marks
5	$u=3+\sqrt{2x-1}\Rightarrow x=\dfrac{(u-3)^2+1}{2}\Rightarrow\dfrac{dx}{du}=u-3$ **or** $u=3+\sqrt{2x-1}\Rightarrow\dfrac{du}{dx}=\dfrac{1}{2}(2x-1)^{-\frac{1}{2}}\times 2=\dfrac{1}{\sqrt{2x-1}}=\dfrac{1}{u-3}$	M1 A1
	$\displaystyle\int\frac{4}{3+\sqrt{2x-1}}dx=\int\frac{4}{u}\times(u-3)du$	M1
	$\displaystyle\int\frac{4}{u}\times(u-3)\,du=\int\left(4-\frac{12}{u}\right)du$	**d**M1
	$\displaystyle\int\left(4-\frac{12}{u}\right)du=4u-12\ln u$ or $k(4u-12\ln u)$	**dd**M1 A1ft
	$\displaystyle\int_1^{13}\frac{4}{3+\sqrt{2x-1}}dx=\left[4u-12\ln u\right]_4^8=(4\times 8-12\ln 8)-(4\times 4-12\ln 4)$ **or** $\displaystyle\int_1^{13}\frac{4}{3+\sqrt{2x-1}}dx=\left[4\left(3+\sqrt{2x-1}\right)-12\ln\left(3+\sqrt{2x-1}\right)\right]_1^{13}=(4\times 8-12\ln 8)-(4\times 4-12\ln 4)$	M1
	$=16-12\ln 2$	A1
		(8 marks)

M1: Differentiates to get $\dfrac{du}{dx}$ in terms of x and then obtains $\dfrac{dx}{du}$ in terms of u

Need to see $\dfrac{du}{dx}=k(2x-1)^{-\frac{1}{2}}\rightarrow\dfrac{du}{dx}=\dfrac{1}{au+b}$ or $\dfrac{dx}{du}=au+b$

or

Attempts to change the subject of $u=3+\sqrt{2x-1}$ and differentiates to get $\dfrac{dx}{du}$ in terms of u

Need to see $x=\dfrac{(u\pm 3)^2\pm 1}{2}\rightarrow\dfrac{dx}{du}=au+b$

A1: $\dfrac{dx}{du}=u-3$ oe e.g. $\dfrac{du}{dx}=\dfrac{1}{u-3}$, $du=\dfrac{dx}{u-3}$, $dx=(u-3)du$

M1: Attempts to write the integral completely in terms of u.

Need to see $\displaystyle\int\frac{\dots}{u}\times\text{their}\frac{dx}{du}\,du$ with or without the "du" but **not** e.g. $\displaystyle\int\frac{\dots}{u}\times\frac{1}{\frac{dx}{du}}\,du$

dM1: Divides to reach an integral of the form $\displaystyle\int\left(A+B\times\frac{1}{u}\right)du$. **Depends on both previous M's**

dM1: Integrates to a form $Au+B\ln u$. **Depends on the previous M.**

An alternative for the previous 2 marks is to use integration by parts:

E.g. $\int \frac{4}{u}\times(u-3)\ \mathrm{d}u = 4(u-3)\ln u - \int 4\ln u\ \mathrm{d}u = 4u\ln u - 12\ln u - 4u\ln u + 4u = 4u - 12\ln u$

Score dM1 for $\int \frac{k}{u}\times(Au+B)\ \mathrm{d}u = k(Au+B)\ln u - \int k\ln u\ \mathrm{d}u$ and dM1 for integrating to a form $Au + B\ln u$.

A1ft: $4u-12\ln u$ or $k(4u-12\ln u)$ following through on $\frac{\mathrm{d}x}{\mathrm{d}u} = k(u-3)$ only.

M1: Substitutes 8 and 4 into their $4u-12\ln u$ and subtracts **or** substitutes 13 and 1 into their $4u-12\ln u$ with $u = 3+\sqrt{2x-1}$ and subtracts. This mark depends on there having been an attempt to integrate, however poor.

A1: $16-12\ln 2$

Question Number	Scheme	Marks
6(a)	$4y^2+3x=6y\mathrm{e}^{-2x}$	
	$4y^2+3x \to 8y\dfrac{\mathrm{d}y}{\mathrm{d}x}+3$	B1
	$6y\mathrm{e}^{-2x} \to -12y\mathrm{e}^{-2x}+6\mathrm{e}^{-2x}\dfrac{\mathrm{d}y}{\mathrm{d}x}$	M1 A1
	$8y\dfrac{\mathrm{d}y}{\mathrm{d}x}+3=-12y\mathrm{e}^{-2x}+6\mathrm{e}^{-2x}\dfrac{\mathrm{d}y}{\mathrm{d}x} \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{12y\mathrm{e}^{-2x}+3}{6\mathrm{e}^{-2x}-8y}$ oe	M1 A1
		(5)
(b)	Sets $x=0$ in $4y^2+3x=6y\mathrm{e}^{-2x} \Rightarrow y=\dfrac{3}{2}$ oe	B1
	Substitutes $\left(0,\dfrac{3}{2}\right)$ in their $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{12y\mathrm{e}^{-2x}+3}{6\mathrm{e}^{-2x}-8y}=\left(\dfrac{7}{-2}\right)$	M1
	$m_N=-1\div"\dfrac{7}{-2}" \Rightarrow y="\dfrac{2}{7}"x+"\dfrac{3}{2}"$	**d**M1
	$y=\dfrac{2}{7}x+\dfrac{3}{2}$ oe e.g. $y=\dfrac{6}{21}x+\dfrac{3}{2}$	A1
		(4)
		(9 marks)

(a)

B1: Differentiates $4y^2+3x$ to obtain $8y\dfrac{\mathrm{d}y}{\mathrm{d}x}+3$. Allow unsimplified forms such as $4\times 2y\dfrac{\mathrm{d}y}{\mathrm{d}x}+3$

M1: Uses the product rule on $6y\mathrm{e}^{-2x}$ to obtain an expression of the form $Ay\mathrm{e}^{-2x}+B\mathrm{e}^{-2x}\dfrac{\mathrm{d}y}{\mathrm{d}x}$

A1: Differentiates $6y\mathrm{e}^{-2x}$ to obtain $-12y\mathrm{e}^{-2x}+6\mathrm{e}^{-2x}\dfrac{\mathrm{d}y}{\mathrm{d}x}$

M1: Collects two terms in $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ (one from attempting to differentiate $4y^2$ and one from attempting to differentiate $6y\mathrm{e}^{-2x}$) and proceeds to make $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ the subject.

A1: $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{12y\mathrm{e}^{-2x}+3}{6\mathrm{e}^{-2x}-8y}$ or equivalent e.g. $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{2\mathrm{e}^{-2x}\times 6y+3}{6\mathrm{e}^{-2x}-8y}$ or $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{12y+3\mathrm{e}^{2x}}{6-8y\mathrm{e}^{2x}}$

You can ignore any spurious $"\dfrac{\mathrm{d}y}{\mathrm{d}x}="$ at the start and allow y' for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

(b)

B1: Uses $x = 0$ to obtain $y=\dfrac{3}{2}$ oe e.g. $\dfrac{6}{4}$ (ignore any reference to $y = 0$)

M1: Substitutes $x = 0$ and their y at $x = 0$ which has come from substituting $x = 0$ into the original equation into their $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{12y\mathrm{e}^{-2x}+3}{6\mathrm{e}^{-2x}-8y}$ to find a numerical value. Working is normally shown here but you may need to check for evidence. Use of $x = 0$ and $y = 0$ is M0.

dM1: Uses the negative reciprocal of $"\frac{7}{-2}"$ for the gradient of the normal and uses this and their value of y at $x = 0$ to form the equation of the normal. **Depends on the previous M.**

A1: $y = \frac{2}{7}x + \frac{3}{2}$ oe e.g. $y = \frac{6}{21}x + \frac{6}{4}$

Note that the use of (0, 0) for P will generally lose the final 3 marks in (b)

Question Number	Scheme	Marks
7(a) Way 1	$\int e^{2x}\sin x\,dx = \frac{1}{2}e^{2x}\sin x - \int \frac{1}{2}e^{2x}\cos x\,dx$	M1
	$= ... - \frac{1}{4}e^{2x}\cos x - \int \frac{1}{4}e^{2x}\sin x\,dx$	**d**M1
	$\int e^{2x}\sin x\,dx = \frac{1}{2}e^{2x}\sin x - \frac{1}{4}e^{2x}\cos x - \int \frac{1}{4}e^{2x}\sin x\,dx$	A1
	$\frac{5}{4}\int e^{2x}\sin x\,dx = \frac{1}{2}e^{2x}\sin x - \frac{1}{4}e^{2x}\cos x \Rightarrow \int e^{2x}\sin x\,dx = ...$	**dd**M1
	$= \frac{2}{5}e^{2x}\sin x - \frac{1}{5}e^{2x}\cos x + c$	A1
		(5)
7(a) Way 2	$\int e^{2x}\sin x\,dx = -e^{2x}\cos x + \int 2e^{2x}\cos x\,dx$	M1
	$= ... + 2e^{2x}\sin x - \int 4e^{2x}\sin x\,dx$	**d**M1
	$\int e^{2x}\sin x\,dx = -e^{2x}\cos x + 2e^{2x}\sin x - \int 4e^{2x}\sin x\,dx$	A1
	$5\int e^{2x}\sin x\,dx = -e^{2x}\cos x + 2e^{2x}\sin x \Rightarrow \int e^{2x}\sin x\,dx = ...$	**dd**M1
	$= \frac{2}{5}e^{2x}\sin x - \frac{1}{5}e^{2x}\cos x + c$	A1
		(5)
(b)	$\left(\frac{2}{5}e^{2\pi}\sin\pi - \frac{1}{5}e^{2\pi}\cos\pi\right) - \left(\frac{2}{5}e^{0}\sin 0 - \frac{1}{5}e^{0}\cos 0\right) = ...$	M1
	$= \frac{1}{5}e^{2\pi} + \frac{1}{5} = \frac{e^{2\pi}+1}{5}$ *	A1*
		(2)
		(7 marks)

Note that you can condone the omission of the d*x*'s throughout.

(a) Way 1

M1: Attempts integration by parts with $u = \sin x$ and $v' = e^{2x}$ to obtain

$$\int e^{2x}\sin x\,dx = Ae^{2x}\sin x \pm B\int e^{2x}\cos x\,dx \quad A > 0$$

dM1: Attempts integration by parts again with $u = \cos x$ and $v' = e^{2x}$ on $B\int e^{2x}\cos x\,dx$ to obtain

$$B\int e^{2x}\cos x\,dx = \pm Ce^{2x}\cos x \pm D\int e^{2x}\sin x\,dx$$

Depends on the previous mark.

A1: For $\int e^{2x}\sin x\,dx = \frac{1}{2}e^{2x}\sin x - \frac{1}{4}e^{2x}\cos x - \int \frac{1}{4}e^{2x}\sin x\,dx$

Allow unsimplified e.g. $\int e^{2x}\sin x\,dx = \frac{1}{2}e^{2x}\sin x - \left\{\frac{1}{4}e^{2x}\cos x + \int \frac{1}{4}e^{2x}\sin x\,dx\right\}$

ddM1: **Dependent upon having scored both M's.**

It is for collecting $\int e^{2x}\sin x\,dx$ terms together and making it the subject of the formula

A1: $\int e^{2x}\sin x\,dx = \frac{2}{5}e^{2x}\sin x - \frac{1}{5}e^{2x}\cos x + c$ (allow with or without "+ c")

(a) **Way 2**

M1: Attempts integration by parts with $u = e^{2x}$ and $v' = \sin x$ to obtain

$$\int e^{2x}\sin x\,dx = \pm Ae^{2x}\cos x \pm B\int e^{2x}\cos x\,dx$$

dM1: Attempts integration by parts again with $u = e^{2x}$ and $v' = \cos x$ on $B\int e^{2x}\cos x\,dx$ to obtain

$$B\int e^{2x}\cos x\,dx = \pm Ce^{2x}\sin x \pm D\int e^{2x}\sin x\,dx$$

Depends on the previous mark.

A1: For $\int e^{2x}\sin x\,dx = -e^{2x}\cos x + 2e^{2x}\sin x - \int 4e^{2x}\sin x\,dx$

Allow unsimplified e.g. $\int e^{2x}\sin x\,dx = -e^{2x}\cos x - \left\{-2e^{2x}\sin x - \int -4e^{2x}\sin x\,dx\right\}$

ddM1: **Dependent upon having scored both M's.**

It is for collecting $\int e^{2x}\sin x\,dx$ terms together and making it the subject of the formula

A1: $\int e^{2x}\sin x\,dx = \frac{2}{5}e^{2x}\sin x - \frac{1}{5}e^{2x}\cos x + c$ (allow with or without "+ c")

(a) **Way 3**

M1: Attempts integration by parts with $u = \sin x$ and $v' = e^{2x}$ to obtain

$\int e^{2x}\sin x\,dx = Ae^{2x}\sin x \pm B\int e^{2x}\cos x\,dx \quad A > 0$

or attempts integration by parts with $u = e^{2x}$ and $v' = \sin x$ to obtain

$\int e^{2x}\sin x\,dx = \pm Ae^{2x}\cos x \pm B\int e^{2x}\cos x\,dx$

dM1: Attempts integration by parts with $u = \sin x$ and $v' = e^{2x}$ to obtain

$\int e^{2x}\sin x\,dx = \pm Ae^{2x}\sin x \pm B\int e^{2x}\cos x\,dx$

and attempts integration by parts with $u = e^{2x}$ and $v' = \sin x$ to obtain

$\int e^{2x}\sin x\,dx = \pm Ae^{2x}\cos x \pm B\int e^{2x}\cos x\,dx$

A1: $I_1 = \int e^{2x}\sin x\,dx = \frac{1}{2}e^{2x}\sin x - \int \frac{1}{2}e^{2x}\cos x\,dx$ **AND** $I_2 = \int e^{2x}\sin x\,dx = -e^{2x}\cos x + \int 2e^{2x}\cos x\,dx$

ddM1: E.g. $4I_1 + I_2 = 2e^{2x}\sin x - e^{2x}\cos x = 5I \Rightarrow I = ...$ Correct attempt to eliminate $\int e^{2x}\cos x\,dx$ term.

A1: $\int e^{2x}\sin x\,dx = \frac{2}{5}e^{2x}\sin x - \frac{1}{5}e^{2x}\cos x + c$ (allow with or without "+ c")

(b)

M1: For applying the limits 0 and π to an expression containing at least one term of the form $Ae^{2x}\sin x$ and at least one term of the form $Be^{2x}\cos x$. **There must be some evidence that <u>both</u> limits have been used.**

A1*: $\frac{e^{2\pi}+1}{5}$ found correctly **<u>from the correct answer in part (a)</u>** via at least one intermediate line which could be $\frac{e^{2\pi}}{5}+\frac{1}{5}$

Note a correct answer in (a) and evidence of use of the limits 0 and pi followed by $\frac{e^{2\pi}+1}{5}$ with no intermediate line scores M1A0

Question Number	Scheme	Marks
8	$\begin{pmatrix}-1\\5\\4\end{pmatrix}+\lambda\begin{pmatrix}2\\-1\\5\end{pmatrix}=\begin{pmatrix}2\\-2\\-5\end{pmatrix}+\mu\begin{pmatrix}4\\-3\\b\end{pmatrix}\Rightarrow \begin{matrix}-1+2\lambda=2+4\mu\ (1)\\5-\lambda=-2-3\mu\ (2)\\4+5\lambda=-5+\mu b\ (3)\end{matrix}$	
	Uses equations (1) and (2) to find either λ **or** μ e.g. (1) + 2(2) $\Rightarrow \mu=...$ or 3(1) + 4(2) $\Rightarrow \lambda=...$	M1
	Uses equations (1) and (2) to find both λ and μ	**d**M1
	$\mu=-\frac{11}{2}$ and $\lambda=-\frac{19}{2}$	A1
	$4+5\lambda=-5+\mu b\Rightarrow 4+5\times-\frac{19}{2}=-5-\frac{11}{2}b$ or $4+5\lambda=-5+7\mu\Rightarrow 4+5\times-\frac{19}{2}=-5-\frac{11}{2}\times 7$	**dd**M1
	$\Rightarrow 11b=77\Rightarrow b=7$ or obtains $-\frac{87}{2}=-\frac{87}{2}$	A1
	States that when $b=7$, lines intersect or when $b\neq 7$, lines do not intersect Lines are not parallel so when $b\neq 7$ lines are skew. *	A1 **Cso**
		(6)
	Alternative assuming b = 7:	
	$\begin{pmatrix}-1\\5\\4\end{pmatrix}+\lambda\begin{pmatrix}2\\-1\\5\end{pmatrix}=\begin{pmatrix}2\\-2\\-5\end{pmatrix}+\mu\begin{pmatrix}4\\-3\\7\end{pmatrix}\Rightarrow \begin{matrix}-1+2\lambda=2+4\mu\ (1)\\5-\lambda=-2-3\mu\ (2)\\4+5\lambda=-5+7b\ (3)\end{matrix}$	
	Uses any 2 equations to find either λ **or** μ	M1
	Uses any 2 equations to find both λ and μ	**d**M1
	$\mu=-\frac{11}{2}$ and $\lambda=-\frac{19}{2}$	A1
	Checks in the 3rd equation e.g. equation 3: $4+5\left(-\frac{19}{2}\right)=-5+7\left(-\frac{11}{2}\right)=...$ equation 1: $-1+2\left(-\frac{19}{2}\right)=2+4\left(-\frac{11}{2}\right)=...$ equation 2: $5-\left(-\frac{19}{2}\right)=-2-3\left(-\frac{11}{2}\right)=...$	**dd**M1
	Equation 3: $-\frac{87}{2}$ Equation 1: -20 Equation 2: $\frac{29}{2}$	A1
	States that when $b=7$, lines intersect or when $b\neq 7$, lines do not intersect Lines are not parallel so when $b\neq 7$ lines are skew. *	A1 **Cso**
		(6 marks)

M1: For attempting to solve equations (1) and (2) to find **either** λ **or** μ

dM1: For attempting to solve equations (1) and (2) to find **both** λ **and** μ **Depends on the first M.**

A1: $\mu = -\frac{11}{2}$ and $\lambda = -\frac{19}{2}$

ddM1: Attempts to solve $4 + 5\lambda = -5 + \mu b$ for their values of λ and μ. Or uses $b = 7$ with their λ and μ in an attempt to show equality. **Depends on both previous M's.**

A1: Achieves (without errors) that they will intersect when $b = 7$

Note that the previous 3 marks may be scored without explicitly seeing the values of both parameters e.g.

$\mu = -\frac{11}{2}$, (2) → $\lambda = 3\mu + 7$ → $4 + 5(3\mu + 7) = -5 + \mu b$ → $b = 7$

A1*:**Cso** States that when $b = 7$, lines intersect and since lines are not parallel it shows that when $b \neq 7$ lines are skew.

Alternative:

M1: Uses $b = 7$ and attempts to solve 2 equations to find **either** λ **or** μ

dM1: For attempting to solve 2 equations to find **both** λ **and** μ **Depends on the first M.**

A1: $\mu = -\frac{11}{2}$ and $\lambda = -\frac{19}{2}$

ddM1: Attempts to show that the 3rd equation is true for their values of λ and μ

Depends on both previous M's.

A1: Achieves (without errors) that the 3rd equation gives the same values for (or equivalent)

A1*: **Cso** States that when $b = 7$, lines intersect and since lines are not parallel it shows that when $b \neq 7$ lines are skew.

To score the final mark there must be some statement that the lines intersect (or equivalent e.g. meet at a point, cross, etc.) when $b = 7$ or that they do not intersect if $b \neq 7$ **and** that the lines are not parallel which may appear anywhere (reason not needed but may be present) so lines are skew when $b \neq 7$.

Ignore any work attempting to show that the lines are perpendicular or not.

Question Number	Scheme	Marks
9(a)	$\tan\theta=\sqrt{3}\Rightarrow k=\frac{\pi}{3}\left(\text{or } 60°\right)$ (Allow $x=\sqrt{3}\Rightarrow\theta=\frac{\pi}{3}\left(\text{or } 60°\right)$)	B1
	$V=(\pi)\int y^2\,\mathrm{d}x=(\pi)\int\left(2\sin 2\theta\right)^2\sec^2\theta\,\mathrm{d}\theta$ oe	M1A1
	$4(\pi)\int\sin^2 2\theta\sec^2\theta\ \mathrm{d}\theta=4(\pi)\int 4\sin^2\theta\cancel{\cos^2\theta}\times\frac{1}{\cancel{\cos^2\theta}}\ \mathrm{d}\theta$	**d**M1
	$=16(\pi)\int\sin^2\theta\,\mathrm{d}\theta$ oe e.g. $16(\pi)\int\left(1-\cos^2\theta\right)\mathrm{d}\theta$	A1
	$\sin^2\theta=\frac{1-\cos 2\theta}{2}\Rightarrow 16(\pi)\int\sin^2\theta\ \mathrm{d}\theta=16(\pi)\int\frac{1-\cos 2\theta}{2}\ \mathrm{d}\theta$	**d**M1
	Volume = $\int_0^{\frac{\pi}{3}}8\pi\left(1-\cos 2\theta\right)\mathrm{d}\theta$	A1 **Cso**
		(7)
(b)	$\int\left(1-\cos 2\theta\right)\mathrm{d}\theta\to\theta-\frac{\sin 2\theta}{2}$	B1
	Volume = $\int_0^{\frac{\pi}{3}}8\pi\left(1-\cos 2\theta\right)\mathrm{d}\theta=\left[8\pi\theta-4\pi\sin 2\theta\right]_0^{\frac{\pi}{3}}=\frac{8}{3}\pi^2-2\sqrt{3}\pi$	M1 A1
		(3)
		(10 marks)

(a)

B1: States or uses $\tan\theta=\sqrt{3}\Rightarrow k=\frac{\pi}{3}$ (Allow 60° here). May be implied by their integral.

Allow if seen anywhere in the question either stated or used as their upper limit.

M1: Attempts volume = $(A\pi)\int y^2\mathrm{d}x=(A\pi)\int\left(2\sin 2\theta\right)^2\sec^2\theta\,\mathrm{d}\theta$ with or without π or "dθ".

Condone bracketing errors

A1: For a volume of $(A\pi)\int\left(2\sin 2\theta\right)^2\sec^2\theta\,\mathrm{d}\theta$ with or without π or "dθ". The brackets must be present but may be implied by subsequent work.

dM1: Uses $\sin 2\theta=2\sin\theta\cos\theta$ and proceeds to Volume = $B\int\sin^2\theta\ \mathrm{d}\theta$ with or without "dθ". (No requirement for limits yet). Note that if $\left(2\sin 2\theta\right)^2$ becomes $2\sin^2\theta\cos^2\theta$ with no evidence of a correct identity then score dM0

<u>**Depends on the first M.**</u>

A1: Volume $=\left(A\pi\right)\int 16\sin^2\theta\ \mathrm{d}\theta$ oe e.g. $\left(A\pi\right)\int 16\left(1-\cos^2\theta\right)\mathrm{d}\theta$ with or without π or "dθ". (No requirement for limits yet)

dM1: Attempts to use $\sin^2\theta=\frac{1-\cos 2\theta}{2}$ or $\cos^2\theta=\frac{1+\cos 2\theta}{2}$ and obtains Volume = $\int\left(P\pm Q\cos 2\theta\right)\ \mathrm{d}\theta$

Depends on the first M.

A1: **CSO** $\int_0^{\frac{\pi}{3}} 8\pi(1-\cos 2\theta)\,d\theta$. Fully correct integral with both limits and the "dθ" but the 8 and/or the π can be either side of the integral sign.

Note this alternative solution for part (a):

$$V=(A\pi)\int y^2dx=(A\pi)\int(2\sin 2\theta)^2\sec^2\theta\,d\theta=(A\pi)\int\frac{4\sin^2 2\theta}{\cos^2\theta}\,d\theta$$ **M1 A1** as above

$$=(A\pi)\int\frac{4\sin^2 2\theta}{\frac{1}{2}(1+\cos 2\theta)}\,d\theta$$

dM1: uses $\cos^2\theta=\frac{1+\cos 2\theta}{2}$ in the denominator. **A1:** Correct integral

$$=8(A\pi)\int\frac{1-\cos^2 2\theta}{1+\cos 2\theta}\,d\theta=8(A\pi)\int\frac{(1+\cos 2\theta)(1-\cos 2\theta)}{1+\cos 2\theta}\,d\theta$$

dM1: Uses $\sin^2 2\theta=1-\cos^2 2\theta$ and the difference of 2 squares in the numerator and cancels

Volume = $\int_0^{\frac{\pi}{3}} 8\pi(1-\cos 2\theta)\,d\theta$ **A1 CSO**

Note that a Cartesian approach in part (a) essentially follows the main scheme e.g.

$$V=(\pi)\int y^2dx=(A\pi)\int(4x\cos^2\theta)^2\sec^2\theta\,d\theta=4(A\pi)\int 4\sin^2\theta\,\cancel{\cos^2\theta}\times\frac{1}{\cancel{\cos^2\theta}}\,d\theta \text{ etc.}$$

If in doubt whether such attempts deserve credit send to review.

(b)

B1: States or uses $\int(1-\cos 2\theta)d\theta\to\theta-\frac{\sin 2\theta}{2}$

M1: Volume = $\int_0^{\frac{\pi}{3}} p(1-\cos 2\theta)\,d\theta=[p\theta\pm kp\sin 2\theta]_0^{\frac{\pi}{3}}$ and uses the limit $\frac{\pi}{3}$ (not 60°).

(The limit of 0 may not be seen)

A1: $\frac{8}{3}\pi^2-2\sqrt{3}\pi$ oe e.g. $8\pi\left(\frac{\pi}{3}-\frac{\sqrt{3}}{4}\right)\frac{8}{3}$, $\pi^2-2\sqrt{3}\pi$, $\frac{2\pi}{3}(4\pi-3\sqrt{3})$, $\frac{8\pi^2-6\sqrt{3}\pi}{3}$

Question Number	Scheme	Marks
10(a)	$\frac{1}{(H-5)(H+3)}=\frac{A}{H-5}+\frac{B}{H+3}\Rightarrow A=\ldots$ or $B=\ldots$	M1
	$A=\frac{1}{8}$ or $B=-\frac{1}{8}$	A1
	$\frac{1}{(H-5)(H+3)}=\frac{1}{8(H-5)}-\frac{1}{8(H+3)}$ or $\frac{\frac{1}{8}}{(H-5)}-\frac{\frac{1}{8}}{(H+3)}$ or $\frac{\frac{1}{8}}{(H-5)}+\frac{-\frac{1}{8}}{(H+3)}$ or $\frac{1}{8H-40}-\frac{1}{8H+24}$	A1
		(3)
(b)	$\frac{\mathrm{d}H}{\mathrm{d}t}=-\frac{(H-5)(H+3)}{40}$ $\int\frac{40}{(H-5)(H+3)}\mathrm{d}H=\int -1\,\mathrm{d}t$ or e.g. $\int\frac{1}{(H-5)(H+3)}\mathrm{d}H=\int-\frac{1}{40}\,\mathrm{d}t$ $\int\frac{5}{(H-5)}-\frac{5}{(H+3)}\mathrm{d}H=\int -1\,\mathrm{d}t$ or e.g. $\frac{1}{8}\int\frac{1}{(H-5)}-\frac{1}{(H+3)}\mathrm{d}H=\int-\frac{1}{40}\,\mathrm{d}t$	M1
	$5\ln\|H-5\|-5\ln\|H+3\|=-t(+c)$ oe e.g. $\frac{1}{8}\ln\|H-5\|-\frac{1}{8}\ln\|H+3\|=-\frac{1}{40}t(+c)$ Or e.g. $5\ln(8H-40)-5\ln(8H+24)=-t(+c)$ etc.	M1 A1ft
	Substitutes $t=0, H=13\Rightarrow c=\ldots$ Note that this may happen at a later stage e.g. may attempt to remove logs and then substitute to find the constant	M1
	$5\ln\|H-5\|-5\ln\|H+3\|=-t+5\ln\left(\frac{1}{2}\right)$ oe e.g. $\frac{1}{8}\ln\|H-5\|-\frac{1}{8}\ln\|H+3\|=-\frac{1}{40}t+\frac{1}{8}\ln\left(\frac{1}{2}\right)$	A1
	$5\ln\left(2\left\|\frac{H-5}{H+3}\right\|\right)=-t\Rightarrow\frac{H-5}{H+3}=\frac{1}{2}\mathrm{e}^{-0.2t}\Rightarrow H=\ldots$	**ddd**M1
	$H=\frac{10+3\mathrm{e}^{-0.2t}}{2-\mathrm{e}^{-0.2t}}$ *	A1*
		(7)
(c)	Sets $\frac{10+3\mathrm{e}^{-0.2t}}{2-\mathrm{e}^{-0.2t}}=8\Rightarrow\mathrm{e}^{-0.2t}=\left(\frac{6}{11}\right)$	M1
	$\Rightarrow t=-5\ln\left(\frac{6}{11}\right)=$ awrt 3.03 days	**d**M1 A1
		(3)
(d)	$k=5$	B1
		(1)
		(14 marks)

(a)

M1: Attempts any correct method to find either constant. It is implied by one correct constant

A1: One correct constant

A1: Correct partial fractions: $\frac{1}{8(H-5)}-\frac{1}{8(H+3)}$. Note that this mark is not just for the correct constants, it is for the correctly stated fractions either in part (a) **or used in part (b).** Allow 0.125 for 1/8.

(b)

M1: Separates the variables and uses part (a) to reach: $\int \frac{P}{(H-5)}+\frac{Q}{(H+3)}\mathrm{d}H=\int \pm k\ \mathrm{d}t$ with or without the integral signs

M1: Attempts to integrate both sides to reach: $\alpha\ln|H-5|+\beta\ln|H+3|=kt$

or e.g. $\alpha\ln|8H-40|+\beta\ln|8H+24|=kt$ Condone $|\ |\leftrightarrow(\ )$ and condone the omission of brackets e.g. allow $\alpha\ln H-5+\beta\ln H+3=kt$ or e.g. $\alpha\ln 8H-40+\beta\ln 8H+24=kt$

A1ft: Correct integration of both sides following through on their PF in (a). Condone $|\ |\leftrightarrow(\ )$ and condone the omission of + c but brackets must be present unless they are implied by subsequent work.

Also follow through on a MR of $\frac{\mathrm{d}H}{\mathrm{d}t}=\frac{(H-5)(H+3)}{40}$ for $\frac{\mathrm{d}H}{\mathrm{d}t}=-\frac{(H-5)(H+3)}{40}$

E.g. obtains $\frac{1}{8}\ln|H-5|-\frac{1}{8}\ln|H+3|=\frac{1}{40}t(+c)$

M1: Substitutes $t=0, H=13\Rightarrow c=...$ For this to be scored there must have been a + c and depends on some attempt at integration of both sides however poor.

Alternatively attempts $\int_{13}^{H}\frac{1}{(H-5)}-\frac{1}{(H+3)}\mathrm{d}H=\int_{0}^{t}-\frac{1}{5}\mathrm{d}t\Rightarrow\left[\ln\frac{H-5}{H+3}\right]_{13}^{H}=\left[-\frac{1}{5}\right]_{0}^{t}\Rightarrow\ln\frac{H-5}{H+3}-\ln\frac{1}{2}=-\frac{1}{5}t$

A1: For a correct equation in H and t. Condone $|\ |\leftrightarrow(\ )$ but brackets must be present unless they are implied by subsequent work.

dddM1: A correct attempt to make H the subject of the formula. **<u>All previous M's in (b) must have been scored.</u>**

A1*: $H=\frac{10+3\mathrm{e}^{-0.2t}}{2-\mathrm{e}^{-0.2t}}$ cso with sufficient working shown and no errors.

Note that marks in (b) may need to be awarded retrospectively:

E.g. First 3 marks gained to reach $\ln|H-5|-\ln|H+3|=-\frac{1}{5}t+c$ and then:

$$\ln\frac{H-5}{H+3}=-\frac{1}{5}t+c\Rightarrow\frac{H-5}{H+3}=A\mathrm{e}^{-0.2t}\Rightarrow H=\frac{5+3A\mathrm{e}^{-0.2t}}{1-A\mathrm{e}^{-0.2t}}$$

$$H=13,\ t=0\Rightarrow 13=\frac{5+3A}{1-A}\Rightarrow A=\frac{1}{2}\Rightarrow H=\frac{5+\frac{3}{2}\mathrm{e}^{-0.2t}}{1-\frac{1}{2}\mathrm{e}^{-0.2t}}=\frac{10+3\mathrm{e}^{-0.2t}}{2-\mathrm{e}^{-0.2t}}*$$

The M3 can be awarded when they attempt to find "A", the dddM4 can be awarded for a correct attempt to make H the subject and then A2 and A3 can be awarded together at the end.

(c)

M1: Sets $\frac{10+3\mathrm{e}^{-0.2t}}{2-\mathrm{e}^{-0.2t}}=8$ or possibly an earlier version of H or possibly their t in terms of H and reaches $A\mathrm{e}^{\pm 0.2t}=p,\quad p>0$

dM1: Correct processing of an equation of the form $A\mathrm{e}^{\pm 0.2t}=p$ with correct log work leading to $t=...$

<u>Depends on the first M.</u>

A1: $t = -5\ln\left(\frac{6}{11}\right)$ or $t = 5\ln\left(\frac{11}{6}\right)$ or awrt 3.03 (days)

(d)

B1: $k = 5$ (Allow H = 5 or just "5")

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