Write your name here

Surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number

Candidate Number

Core Mathematics C34

Advanced

Tuesday 19 June 2018 – Afternoon
Time: 2 hours 30 minutes

Paper Reference
WMA02/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

Pearson

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1. (i) Find

$$\int \frac{2x^2 + 5x + 1}{x^2} \, dx, \quad x > 0$$

(3)

(ii) Find

$$\int x \cos 2x \, dx$$

(3)

Leave blank

Question 1 continued

Q1

(Total 6 marks)

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2. A curve C has parametric equations

$$x = \frac{3}{2}t - 5, \quad y = 4 - \frac{6}{t} \qquad t \neq 0$$

(a) Find the value of $\frac{dy}{dx}$ at $t = 3$, giving your answer as a fraction in its simplest form.
(3)

(b) Show that a cartesian equation of C can be expressed in the form

$$y = \frac{ax + b}{x + 5} \qquad x \neq k$$

where a, b and k are integers to be found.
(4)

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Question 2 continued

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Question 2 continued

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Question 2 continued

Q2

(Total 7 marks)

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3. $$f(x) = 2^{x-1} - 4 + 1.5x \qquad x \in \mathbb{R}$$

(a) Show that the equation $f(x) = 0$ can be written as

$$x = \frac{1}{3}(8 - 2^x)$$

(2)

The equation $f(x) = 0$ has a root α, where $\alpha = 1.6$ to one decimal place.

(b) Starting with $x_0 = 1.6$, use the iteration formula

$$x_{n+1} = \frac{1}{3}(8 - 2^{x_n})$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 3 decimal places.

(3)

(c) By choosing a suitable interval, prove that $\alpha = 1.633$ to 3 decimal places.

(2)

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Question 3 continued

Q3

(Total 7 marks)

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4. (a) Find the binomial expansion of

$$(1 + px)^{-4}, \quad |px| < 1$$

in ascending powers of x, up to and including the term in x^3, giving each coefficient as simply as possible in terms of the constant p.

(3)

$$\mathrm{f}(x) = \frac{3 + 4x}{(1 + px)^4} \qquad |px| < 1$$

where p is a positive constant.

In the series expansion of $\mathrm{f}(x)$, the coefficient of x^2 is twice the coefficient of x.

(b) Find the value of p.

(5)

(c) Hence find the coefficient of x^3 in the series expansion of $\mathrm{f}(x)$, giving your answer as a simplified fraction.

(2)

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Question 4 continued

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Question 4 continued

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Question 4 continued

Q4

(Total 10 marks)

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5. (i) The functions f and g are defined by

$$\mathrm{f} : x \rightarrow \mathrm{e}^{2x} - 5, \qquad x \in \mathbb{R}$$

$$\mathrm{g} : x \rightarrow \ln(3x - 1), \qquad x \in \mathbb{R},\ x > \frac{1}{3}$$

(a) Find f^{-1} and state its domain.

(3)

(b) Find fg(3), giving your answer in its simplest form.

(2)

(ii) (a) Sketch the graph with equation

$$y = |4x - a|$$

where a is a positive constant. State the coordinates of each point where the graph cuts or meets the coordinate axes.

(2)

Given that

$$|4x - a| = 9a$$

where a is a positive constant,

(b) find the possible values of

$$|x - 6a| + 3|x|$$

giving your answers, in terms of a, in their simplest form.

(5)

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Question 5 continued

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Question 5 continued

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Question 5 continued

Q5

(Total 12 marks)

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6. (a) Express $\sqrt{5}\cos\theta - 2\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$

State the value of R and give the value of α to 4 significant figures.

(3)

(b) Solve, for $-\pi < \theta < \pi$,

$$\sqrt{5}\cos\theta - 2\sin\theta = 0.5$$

giving your answers to 3 significant figures.
[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(4)

$$f(x) = A\left(\sqrt{5}\cos\theta - 2\sin\theta\right) + B \qquad \theta \in \mathbb{R}$$

where A and B are constants.

Given that the range of f is

$$-15 \leqslant f(x) \leqslant 33$$

(c) find the value of B and the possible values of A.

(4)

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Question 6 continued

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Question 6 continued

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Question 6 continued

(Total 11 marks)

Q6

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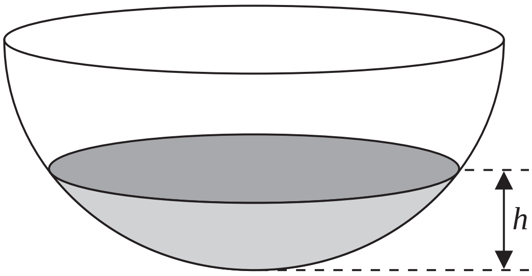

Figure 1

Figure 1 shows a hemispherical bowl.

Water is flowing into the bowl at a constant rate of $180\,\text{cm}^3\,\text{s}^{-1}$.

When the height of the water is h cm, the volume of water $V\,\text{cm}^3$ is given by

$$V = \frac{1}{3}\pi h^2(90 - h), \qquad 0 \leqslant h \leqslant 30$$

Find the rate of change of the height of the water, in $\text{cm}\,\text{s}^{-1}$, when $h = 15$
Give your answer to 2 significant figures.

(5)

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Question 7 continued

Q7

(Total 5 marks)

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8. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \qquad l_2 : \mathbf{r} = \begin{pmatrix} 6 \\ 4 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

where λ and μ are scalar parameters.

(a) Show that l_1 and l_2 do not meet.

(4)

The point P is on l_1 where $\lambda = 0$, and the point Q is on l_2 where $\mu = -1$

(b) Find the acute angle between the line segment PQ and l_1, giving your answer in degrees to 2 decimal places.

(5)

(c) Find the shortest distance from the point Q to the line l_1, giving your answer to 3 significant figures.

(2)

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Question 8 continued

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Question 8 continued

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Question 8 continued

(Total 11 marks)

Q8

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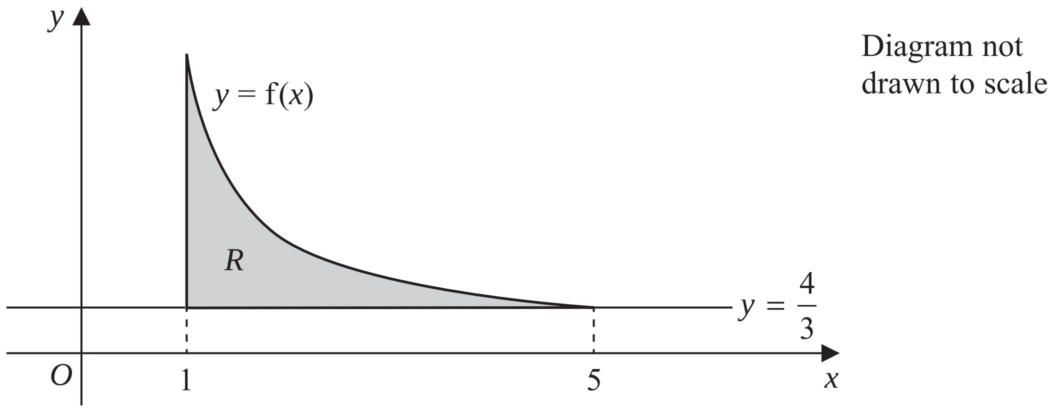

Figure 2

(a) Find

$$\int \frac{1}{(2x-1)^2}\,dx$$

(2)

Figure 2 shows a sketch of the curve with equation $y = f(x)$ where

$$f(x) = \frac{12}{(2x-1)} \qquad 1 \leqslant x \leqslant 5$$

The finite region R, shown shaded in Figure 2, is bounded by the line with equation $x = 1$, the curve with equation $y = f(x)$ and the line with equation $y = \frac{4}{3}$.

The region R is rotated through 2π radians about the x-axis to form a solid of revolution.

(b) Find the exact value of the volume of the solid generated, giving your answer in its simplest form.

(6)

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Question 9 continued

Q9

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10. The curve C satisfies the equation

$$x\mathrm{e}^{5-2y} - y = 0 \qquad x > 0, \quad y > 0$$

The point P with coordinates $(2\mathrm{e}^{-1}, 2)$ lies on C.

The tangent to C at P cuts the x-axis at the point A and cuts the y-axis at the point B.

Given that O is the origin, find the exact area of triangle OAB, giving your answer in its simplest form.

(7)

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Question 10 continued

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Question 10 continued

Q10

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11.

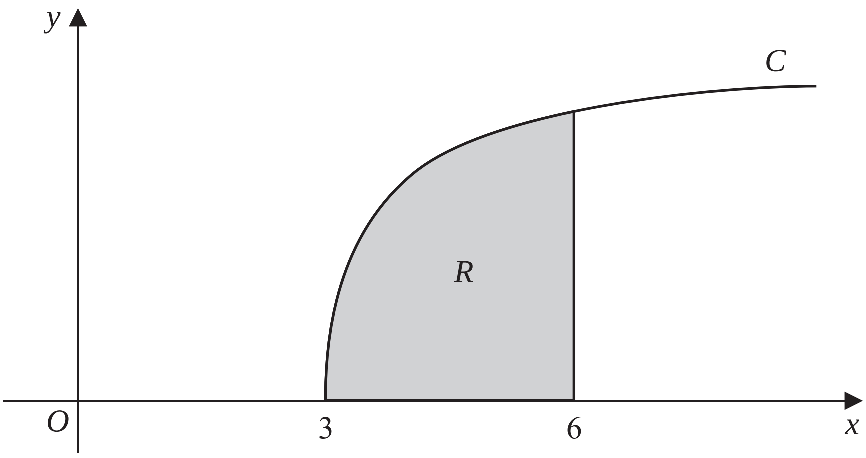

Figure 3

(a) By writing $\sec\theta$ as $\dfrac{1}{\cos\theta}$, show that when $x = 3\sec\theta$,

$$\frac{\mathrm{d}x}{\mathrm{d}\theta} = 3\sec\theta\tan\theta$$

(2)

Figure 3 shows a sketch of part of the curve C with equation

$$y = \frac{\sqrt{x^2 - 9}}{x} \qquad x \geqslant 3$$

The finite region R, shown shaded in Figure 3, is bounded by the curve C, the x-axis and the line with equation $x = 6$

(b) Use the substitution $x = 3\sec\theta$ to find the exact value of the area of R.
[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(7)

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Question 11 continued

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Question 11 continued

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Question 11 continued

Q11

(Total 9 marks)

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12. (a) Show that

$$\cot x - \tan x \equiv 2\cot 2x, \qquad x \neq 90n°,\ n \in \mathbb{Z}$$

(4)

(b) Hence, or otherwise, solve, for $0 \leqslant \theta < 180°$

$$5 + \cot(\theta - 15°) - \tan(\theta - 15°) = 0$$

giving your answers to one decimal place.
[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(5)

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Question 12 continued

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Question 12 continued

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Question 12 continued

(Total 9 marks)

Q12

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13. (a) Express $\dfrac{1}{(4-x)(2-x)}$ in partial fractions.

(2)

The mass, x grams, of a substance at time t seconds after a chemical reaction starts is modelled by the differential equation

$$\frac{dx}{dt} = k(4-x)(2-x), \qquad t \geqslant 0, \ 0 \leqslant x < 2$$

where k is a constant.

Given that when $t = 0$, $x = 0$

(b) solve the differential equation and show that the solution can be written as

$$x = \frac{4 - 4e^{2kt}}{1 - 2e^{2kt}}$$

(7)

Given that $k = 0.1$

(c) find the value of t when $x = 1$, giving your answer, in seconds, to 3 significant figures.

(2)

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Question 13 continued

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Question 13 continued

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Question 13 continued

Q13

(Total 11 marks)

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14. Given that

$$y = \frac{(x^2 - 4)^{\frac{1}{2}}}{x^3} \qquad x > 2$$

(a) show that

$$\frac{dy}{dx} = \frac{Ax^2 + 12}{x^4(x^2 - 4)^{\frac{1}{2}}} \qquad x > 2$$

where A is a constant to be found.

(6)

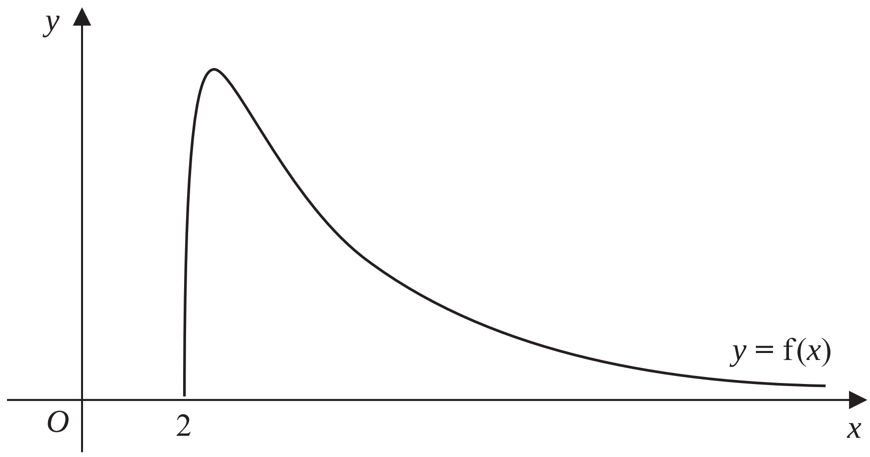

Figure 4

Figure 4 shows a sketch of part of the curve with equation $y = f(x)$ where

$$f(x) = \frac{24(x^2 - 4)^{\frac{1}{2}}}{x^3} \qquad x > 2$$

(b) Use your answer to part (a) to find the range of f.

(5)

(c) State a reason why f^{-1} does not exist.

(1)

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Question 14 continued

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Question 14 continued

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Question 14 continued

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Question 14 continued

Q14

(Total 12 marks)

TOTAL FOR PAPER: 125 MARKS

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