Write your name here

Surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number

Candidate Number

Core Mathematics C34

Advanced

Tuesday 16 January 2018 – Morning
Time: 2 hours 30 minutes

Paper Reference
WMA02/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

1/1/1/1/1/1/

Pearson

Leave blank

1. A curve C has equation

$$3^x + xy = x + y^2, \qquad y > 1$$

The point P with coordinates (4, 11) lies on C.

Find the exact value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point P.

Give your answer in the form $a + b\ln 3$, where a and b are rational numbers.

(6)

Leave blank

Question 1 continued

Q1

(Total 6 marks)

Leave blank

2. $$\mathrm{f}(x) = (125 - 5x)^{\frac{2}{3}} \qquad |x| < 25$$

(a) Find the binomial expansion of $\mathrm{f}(x)$, in ascending powers of x, up to and including the term in x^2, giving the coefficient of x and the coefficient of x^2 as simplified fractions.
(4)

(b) Use your expansion to find an approximate value for $120^{\frac{2}{3}}$, stating the value of x which you have used and showing your working. Give your answer to 5 decimal places.
(3)

Leave blank

Question 2 continued

(Total 7 marks)

Q2

Leave blank

3. $$\mathrm{f}(x) = \frac{x^2}{4} + \ln(2x), \qquad x > 0$$

(a) Show that the equation $\mathrm{f}(x) = 0$ can be rewritten as

$$x = \frac{1}{2}\mathrm{e}^{-\frac{1}{4}x^2}$$

(2)

The equation $\mathrm{f}(x) = 0$ has a root near 0.5

(b) Starting with $x_1 = 0.5$ use the iterative formula

$$x_{n+1} = \frac{1}{2}\mathrm{e}^{-\frac{1}{4}x_n^{\,2}}$$

to calculate the values of x_2, x_3 and x_4, giving your answers to 4 decimal places.

(3)

(c) Using a suitable interval, show that 0.473 is a root of $\mathrm{f}(x) = 0$ correct to 3 decimal places.

(2)

Leave blank

Question 3 continued

Q3

(Total 7 marks)

P 5 1 4 0 3 A 0 7 4 8

Leave blank

4.

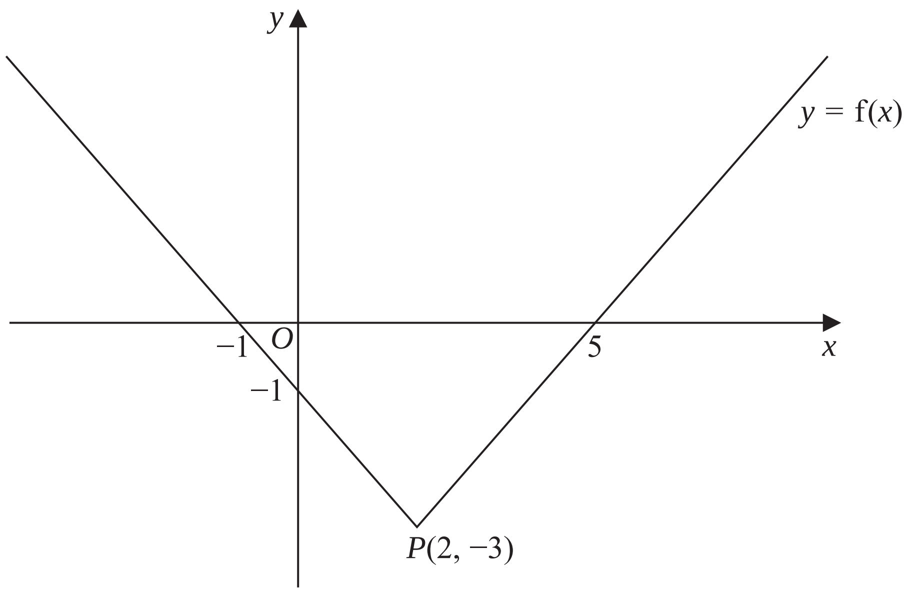

Figure 1

Figure 1 shows a sketch of part of the graph with equation $y = \mathrm{f}(x), \quad x \in \mathbb{R}$

The graph consists of two half lines that meet at the point P (2, −3), the vertex of the graph.

The graph cuts the y-axis at the point (0, −1) and the x-axis at the points (−1, 0) and (5, 0).

Sketch, on separate diagrams, the graph of

(a) $y = \mathrm{f}(|x|)$,

(3)

(b) $y = 2\mathrm{f}(x + 5)$.

(3)

In each case, give the coordinates of the points where the graph crosses or meets the coordinate axes.

Also give the coordinates of any vertices corresponding to the point P.

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Q4

(Total 6 marks)

Leave blank

5. (a) Express $\dfrac{9(4+x)}{16-9x^2}$ in partial fractions.

(3)

Given that

$$f(x) = \frac{9(4+x)}{16-9x^2}, \qquad x \in \mathbb{R}, \qquad -\frac{4}{3} < x < \frac{4}{3}$$

(b) express $\int f(x)\,dx$ in the form $\ln(g(x))$, where $g(x)$ is a rational function.

(4)

Leave blank

Question 5 continued

Q5

(Total 7 marks)

Leave blank

6.

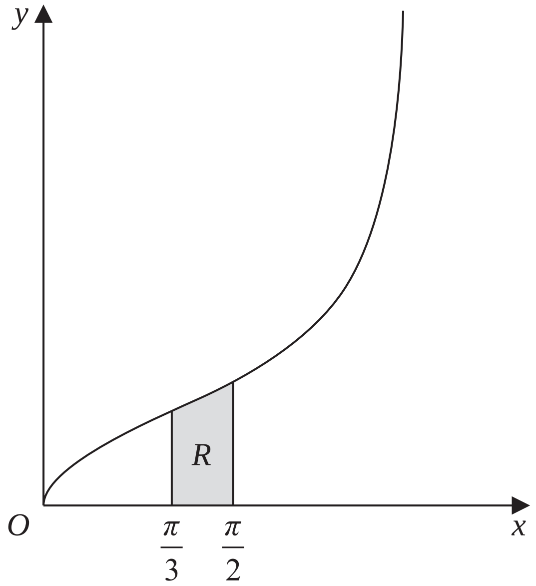

Figure 2

The curve shown in Figure 2 has equation

$$y^2 = 3\tan\left(\frac{x}{2}\right), \qquad 0 < x < \pi, \qquad y > 0$$

The finite region R, shown shaded in Figure 2, is bounded by the curve, the line with equation $x = \frac{\pi}{3}$ the x-axis and the line with equation $x = \frac{\pi}{2}$

The region R is rotated through 360° about the x-axis to generate a solid of revolution.

Show that the exact value of the volume of the solid generated may be written as $A\ln\left(\frac{3}{2}\right)$, where A is a constant to be found.

(5)

Leave blank

Question 6 continued

P 5 1 4 0 3 A 0 1 5 4 8

Turn over

Leave blank

Question 6 continued

Leave blank

Question 6 continued

(Total 5 marks)

Q6

P 5 1 4 0 3 A 0 1 7 4 8

Turn over

Leave blank

7. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = (13\mathbf{i} + 15\mathbf{j} - 8\mathbf{k}) + \lambda(3\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$$

$$l_2 : \mathbf{r} = (7\mathbf{i} - 6\mathbf{j} + 14\mathbf{k}) + \mu(2\mathbf{i} - 3\mathbf{j} + 2\mathbf{k})$$

where λ and μ are scalar parameters.

(a) Show that l_1 and l_2 meet and find the position vector of their point of intersection, B.
(6)

(b) Find the acute angle between the lines l_1 and l_2
(3)

The point A has position vector $-5\mathbf{i} - 3\mathbf{j} + 16\mathbf{k}$

(c) Show that A lies on l_1
(1)

The point C lies on the line l_1 where $\overrightarrow{AB} = \overrightarrow{BC}$

(d) Find the position vector of C.
(3)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Leave blank

Question 7 continued

P 5 1 4 0 3 A 0 1 9 4 8

Turn over

Leave blank

Question 7 continued

Leave blank

Question 7 continued

(Total 13 marks)

Q7

Leave blank

8. Given that

$$y = 8\tan(2x), \quad -\frac{\pi}{4} < x < \frac{\pi}{4}$$

show that

$$\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{A}{B + y^2}$$

where A and B are integers to be found.

(4)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Leave blank

Question 8 continued

(Total 4 marks)

Q8

P51403A02348

Leave blank

9. (a) Show that

$$\frac{\cot^2 x}{1+\cot^2 x} \equiv \cos^2 x$$

(3)

(b) Hence solve, for $0 \leqslant x < 360°$,

$$\frac{\cot^2 x}{1+\cot^2 x} = 8\cos 2x + 2\cos x$$

Give each solution in degrees to one decimal place.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Q9

(Total 9 marks)

Leave blank

10. It is given that

$$f(x) = e^{-2x} \qquad x \in \mathbb{R}$$

$$g(x) = \frac{x}{x-3} \qquad x > 3$$

(a) Sketch the graph of $y = f(x)$, showing the coordinates of any points where the graph crosses the axes.

(2)

(b) Find the range of g

(2)

(c) Find $g^{-1}(x)$, stating the domain of g^{-1}

(4)

(d) Using algebra, find the exact value of x for which $fg(x) = 3$

(4)

Leave blank

Question 10 continued

Leave blank

Question 10 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Leave blank

Question 10 continued

Q10

(Total 12 marks)

Leave blank

11.

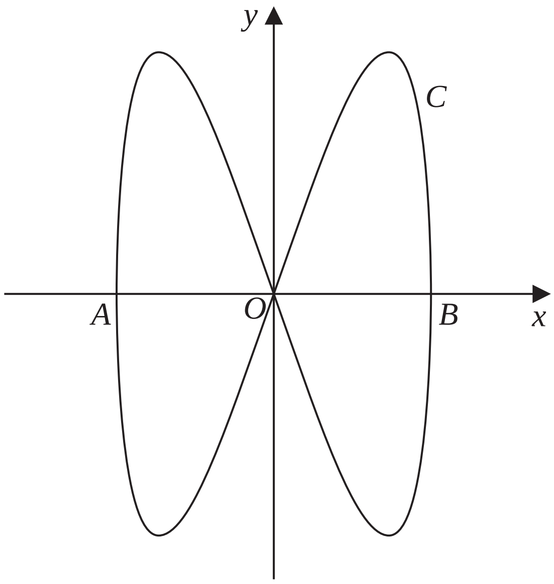

Figure 3

The curve C shown in Figure 3 has parametric equations

$$x = 3\cos t, \qquad y = 9\sin 2t, \qquad 0 \leqslant t \leqslant 2\pi$$

The curve C meets the x-axis at the origin and at the points A and B, as shown in Figure 3.

(a) Write down the coordinates of A and B.

(2)

(b) Find the values of t at which the curve passes through the origin.

(2)

(c) Find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t, and hence find the gradient of the curve when $t = \dfrac{\pi}{6}$

(4)

(d) Show that the cartesian equation for the curve C can be written in the form

$$y^2 = ax^2(b - x^2)$$

where a and b are integers to be determined.

(4)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

P 5 1 4 0 3 A 0 3 2 4 8

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Leave blank

Question 11 continued

Leave blank

Question 11 continued

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Leave blank

Question 11 continued

Q11

(Total 12 marks)

P 5 1 4 0 3 A 0 3 5 4 8

Leave blank

12. (a) Express $2\sin x - 4\cos x$ in the form $R\sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$

Give the exact value of R and give the value of α, in radians, to 3 significant figures.
(3)

In a town in Norway, a student records the number of hours of daylight every day for a year. He models the number of hours of daylight, H, by the continuous function given by the formula

$$H = 12 + 4\sin\left(\frac{2\pi t}{365}\right) - 8\cos\left(\frac{2\pi t}{365}\right), \qquad 0 \leqslant t \leqslant 365$$

where t is the number of days since he began recording.

(b) Using your answer to part (a), or otherwise, find the maximum and minimum number of hours of daylight given by this formula. Give your answers to 3 significant figures.
(3)

(c) Use the formula to find the values of t when $H = 17$, giving your answers to the nearest integer.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)
(6)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Leave blank

Question 12 continued

Leave blank

Question 12 continued

Leave blank

Question 12 continued

(Total 12 marks)

Q12

Leave blank

13.

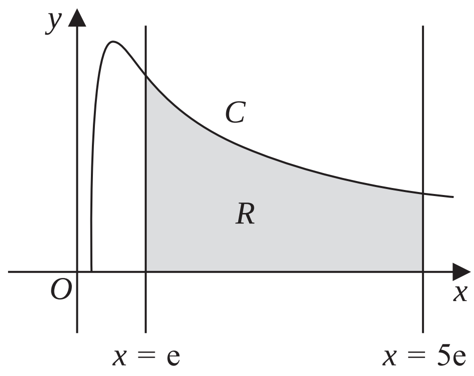

Figure 4

Figure 4 shows a sketch of part of the curve C with equation

$$y = \frac{1}{2x}\ln 2x, \qquad x > \frac{1}{2}$$

The finite region R, shown shaded in Figure 4, is bounded by the curve C, the x-axis and the lines with equations $x = \mathrm{e}$ and $x = 5\mathrm{e}$.

The table below shows corresponding values of x and y for $y = \frac{1}{2x}\ln 2x$. The values for y are given to 4 significant figures.

x	e	2e	3e	4e	5e
y	0.3114	0.2195	0.1712	0.1416	0.1215

(a) Use the trapezium rule with all the y values in the table to find an approximate value for the area of R, giving your answer to 3 significant figures.

(3)

(b) Using the substitution $u = \ln 2x$, or otherwise, find $\int \frac{1}{2x}\ln 2x \, \mathrm{d}x$

(3)

(c) Use your answer to part (b) to find the true area of R, giving your answer to 3 significant figures.

(2)

(d) Using calculus, find an equation for the tangent to the curve at the point where $x = \frac{\mathrm{e}^2}{2}$, giving your answer in the form $y = mx + c$ where m and c are exact multiples of powers of e.

(5)

Leave blank

Question 13 continued

Leave blank

Question 13 continued

Leave blank

Question 13 continued

(Total 13 marks)

Q13

Leave blank

14. The volume of a spherical balloon of radius r cm is V cm^3, where $V = \frac{4}{3}\pi r^3$

(a) Find $\frac{\mathrm{d}V}{\mathrm{d}r}$

(1)

The volume of the balloon increases with time t seconds according to the formula

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{9000\pi}{(t+81)^{\frac{5}{4}}} \qquad t \geqslant 0$$

(b) Using the chain rule, or otherwise, show that

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{k}{r^n(t+81)^{\frac{5}{4}}} \qquad t \geqslant 0$$

where k and n are constants to be found.

(2)

Initially, the radius of the balloon is 3 cm.

(c) Using the values of k and n found in part (b), solve the differential equation

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{k}{r^n(t+81)^{\frac{5}{4}}} \qquad t \geqslant 0$$

to obtain a formula for r in terms of t.

(6)

(d) Hence find the radius of the balloon when $t = 175$, giving your answer to 3 significant figures.

(1)

(e) Find the rate of increase of the radius of the balloon when $t = 175$. Give your answer to 3 significant figures.

(2)

Leave blank

Question 14 continued

Leave blank

Question 14 continued

Leave blank

Question 14 continued

P 5 1 4 0 3 A 0 4 7 4 8

Turn over

Leave blank

Question 14 continued

Q14

(Total 12 marks)

TOTAL FOR PAPER: 125 MARKS

END