Write your name here

Surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number	Candidate Number

Core Mathematics C34

Advanced

Tuesday 17 January 2017 – Morning
Time: 2 hours 30 minutes

Paper Reference
WMA02/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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Pearson

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1. Find an equation of the tangent to the curve

$$x^3 + 3x^2y + y^3 = 37$$

at the point (1, 3). Give your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(6)

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2.

$$f(x) = x^3 - 5x + 16$$

(a) Show that the equation $f(x) = 0$ can be rewritten as

$$x = (ax + b)^{\frac{1}{3}}$$

giving the values of the constants a and b.

(2)

The equation $f(x) = 0$ has exactly one real root α, where $\alpha = -3$ to one significant figure.

(b) Starting with $x_1 = -3$, use the iteration

$$x_{n+1} = (ax_n + b)^{\frac{1}{3}}$$

with the values of a and b found in part (a), to calculate the values of x_2, x_3 and x_4, giving all your answers to 3 decimal places.

(3)

(c) Using a suitable interval, show that $\alpha = -3.17$ correct to 2 decimal places.

(2)

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3. (a) Express $\dfrac{9+11x}{(1-x)(3+2x)}$ in partial fractions.

(3)

(b) Hence, or otherwise, find the series expansion of

$$\frac{9+11x}{(1-x)(3+2x)}, \quad |x| < 1$$

in ascending powers of x, up to and including the term in x^3.
Give each coefficient as a simplified fraction.

(6)

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Question 3 continued

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4. Given that

$$f(x) = \frac{4}{3x + 5}, \quad x > 0$$

$$g(x) = \frac{1}{x}, \quad x > 0$$

(a) state the range of f,

(2)

(b) find $f^{-1}(x)$,

(3)

(c) find $fg(x)$.

(1)

(d) Show that the equation $fg(x) = gf(x)$ has no real solutions.

(4)

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Question 4 continued

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5.

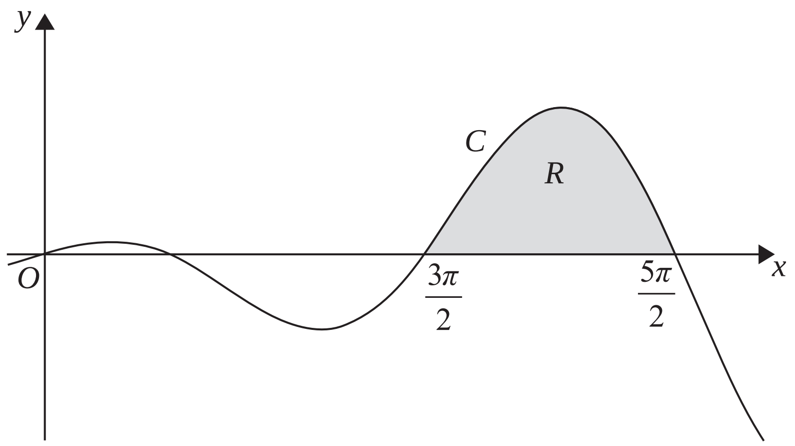

Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$y = x\cos x, \quad x \in \mathbb{R}$$

The finite region R, shown shaded in Figure 1, is bounded by the curve C and the x-axis for $\frac{3\pi}{2} \leqslant x \leqslant \frac{5\pi}{2}$

(a) Complete the table below with the exact value of y corresponding to $x = \frac{7\pi}{4}$ and with the exact value of y corresponding to $x = \frac{9\pi}{4}$

x	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π	$\frac{9\pi}{4}$	$\frac{5\pi}{2}$
y	0		2π		0

(1)

(b) Use the trapezium rule, with all five y values in the completed table, to find an approximate value for the area of R, giving your answer to 4 significant figures.

(3)

(c) Find

$$\int x\cos x\,dx$$

(3)

(d) Using your answer from part (c), find the exact area of the region R.

(2)

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Question 5 continued

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6. (i) Differentiate $y = 5x^2 \ln 3x, \quad x > 0$

(2)

(ii) Given that

$$y = \frac{x}{\sin x + \cos x}, \qquad -\frac{\pi}{4} < x < \frac{3\pi}{4}$$

show that

$$\frac{dy}{dx} = \frac{(1 + x)\sin x + (1 - x)\cos x}{1 + \sin 2x}, \qquad -\frac{\pi}{4} < x < \frac{3\pi}{4}$$

(4)

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Question 6 continued

Q6

(Total 6 marks)

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7.

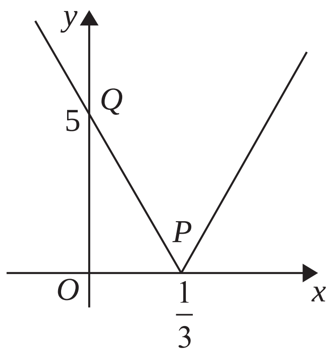

Figure 2

Figure 2 shows a sketch of the graph of $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The point $P\left(\frac{1}{3}, 0\right)$ is the vertex of the graph.

The point $Q\,(0, 5)$ is the intercept with the y-axis.

Given that $\mathrm{f}(x) = |ax + b|$, where a and b are constants,

(a) (i) find all possible values for a and b,

(ii) hence find an equation for the graph.

(4)

(b) Sketch the graph with equation

$$y = \mathrm{f}\left(\frac{1}{2}x\right) + 3$$

showing the coordinates of its vertex and its intercept with the y-axis.

(3)

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Question 7 continued

Q7

(Total 7 marks)

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8. (a) Using the trigonometric identity for $\tan(A + B)$, prove that

$$\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}, \qquad x \neq (2n+1)30°, \qquad n \in \mathbb{Z}$$

(4)

(b) Hence solve, for $-30° < x < 30°$,

$$\tan 3x = 11 \tan x$$

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

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Question 8 continued

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9.

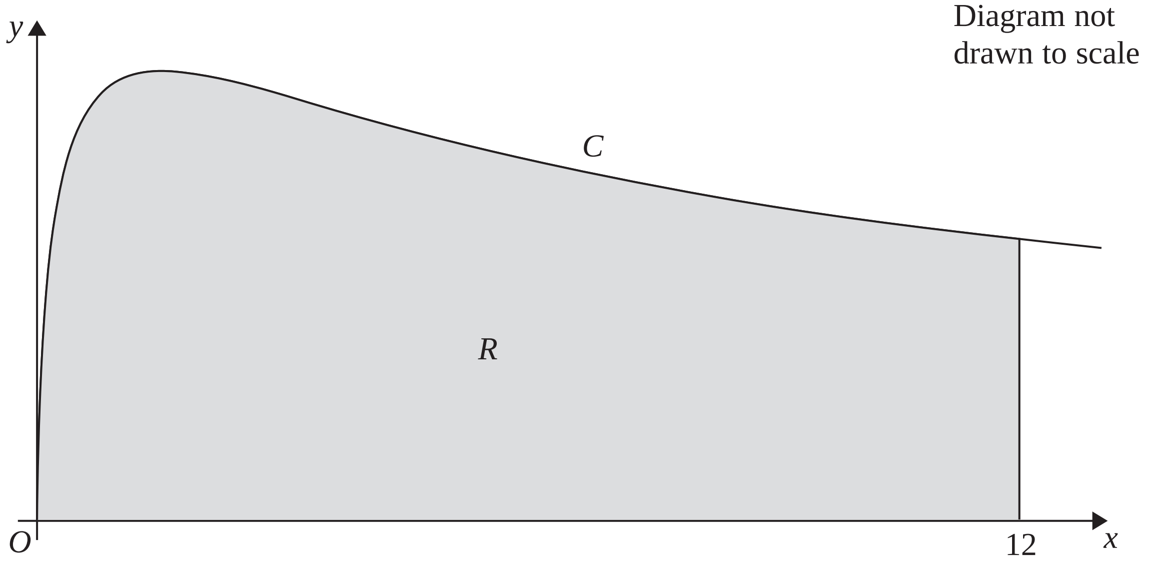

Figure 3

(a) By using the substitution $u = 2x + 3$, show that

$$\int_0^{12} \frac{x}{(2x+3)^2}\,dx = \frac{1}{2}\ln 3 - \frac{2}{9}$$

(7)

The curve C has equation

$$y = \frac{9\sqrt{x}}{(2x+3)}, \quad x > 0$$

The finite region R, shown shaded in Figure 3, is bounded by the curve C, the x-axis and the line with equation $x = 12$. The region R is rotated through 2π radians about the x-axis to form a solid of revolution.

(b) Use the result of part (a) to find the exact value of the volume of the solid generated.

(2)

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Question 9 continued

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10. A population of insects is being studied. The number of insects, N, in the population, is modelled by the equation

$$N = \frac{300}{3 + 17e^{-0.2t}} \qquad t \in \mathbb{R},\ t \geqslant 0$$

where t is the time, in weeks, from the start of the study.

Using the model,

(a) find the number of insects at the start of the study,

(1)

(b) find the number of insects when $t = 10$,

(2)

(c) find the time from the start of the study when there are 82 insects.
(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(4)

(d) Find, by differentiating, the rate, measured in insects per week, at which the number of insects is increasing when $t = 5$. Give your answer to the nearest whole number.

(3)

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Question 10 continued

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11. (a) Express $35 \sin x - 12 \cos x$ in the form $R \sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R, and give the value of α, in radians, to 4 significant figures.

(3)

(b) Hence solve, for $0 \leqslant x < 2\pi$,

$$70 \sin x - 24 \cos x = 37$$

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(4)

$$y = \frac{7000}{31 + (35 \sin x - 12 \cos x)^2}, \quad x > 0$$

(c) Use your answer to part (a) to calculate

(i) the minimum value of y,

(ii) the smallest value of x, $x > 0$, at which this minimum value occurs.

(4)

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Question 11 continued

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12. In freezing temperatures, ice forms on the surface of the water in a barrel. At time t hours after the start of freezing, the thickness of the ice formed is x mm. You may assume that the thickness of the ice is uniform across the surface of the water.

At 4pm there is no ice on the surface, and freezing begins.

At 6pm, after two hours of freezing, the ice is 1.5 mm thick.

In a simple model, the rate of increase of x, in mm per hour, is assumed to be constant for a period of 20 hours.

Using this simple model,

(a) express t in terms of x,

(2)

(b) find the value of t when $x = 3$

(1)

In a second model, the rate of increase of x, in mm per hour, is given by the differential equation

$$\frac{dx}{dt} = \frac{\lambda}{(2x+1)} \text{ where } \lambda \text{ is a constant and } 0 \leqslant t \leqslant 20$$

Using this second model,

(c) solve the differential equation and express t in terms of x and λ,

(3)

(d) find the exact value for λ,

(1)

(e) find at what time the ice is predicted to be 3 mm thick.

(2)

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Question 12 continued

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13.

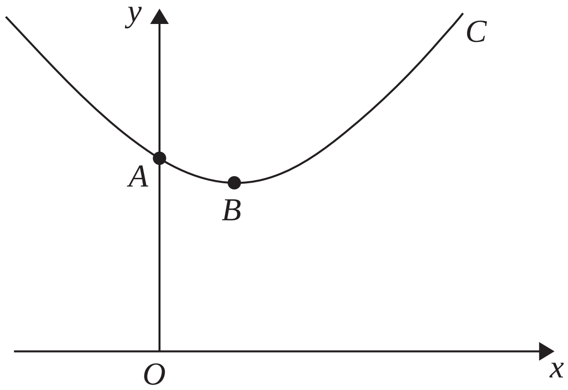

Figure 4

The curve C shown in Figure 4 has parametric equations

$$x = 1 + \sqrt{3}\tan\theta, \qquad y = 5\sec\theta, \qquad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

The curve C crosses the y-axis at A and has a minimum turning point at B, as shown in Figure 4.

(a) Find the exact coordinates of A.

(3)

(b) Show that $\dfrac{dy}{dx} = \lambda \sin\theta$, giving the exact value of the constant λ.

(4)

(c) Find the coordinates of B.

(2)

(d) Show that the cartesian equation for the curve C can be written in the form

$$y = k\sqrt{(x^2 - 2x + 4)}$$

where k is a simplified surd to be found.

(3)

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Question 13 continued

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14. *ABCD* is a parallelogram with *AB* parallel to *DC* and *AD* parallel to *BC*.
The position vectors of *A*, *B*, *C*, and *D* relative to a fixed origin *O* are **a**, **b**, **c** and **d** respectively.

Given that

$$\mathbf{a} = \mathbf{i} + \mathbf{j} - 2\mathbf{k}, \quad \mathbf{b} = 3\mathbf{i} - \mathbf{j} + 6\mathbf{k}, \quad \mathbf{c} = -\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$$

(a) find the position vector **d**,

(3)

(b) find the angle between the sides *AB* and *BC* of the parallelogram,

(4)

(c) find the area of the parallelogram *ABCD*.

(2)

The point *E* lies on the line through the points *C* and *D*, so that *D* is the midpoint of *CE*.

(d) Use your answer to part (c) to find the area of the trapezium *ABCE*.

(2)

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Question 14 continued

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