Pearson Edexcel

Mark Scheme (Results)

October 2018

Pearson Edexcel International Advanced Level in Core Mathematics C34 (WMA02/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 125

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- d... or dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper or ag- answer given
- $\sqsubset$ or d... The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \ldots$

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to $x = \ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.
Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
1(a)	$\cos\theta + 4\sin\theta = R\cos(\theta - \alpha)$	
	$R = \sqrt{1^2 + 4^2} = \sqrt{17}$	B1
	$\alpha = \arctan 4 = \text{awrt } 1.326$	M1A1
		(3)
(b)	$\sqrt{17}\cos(2\theta - 1.326) = 1.2 \Rightarrow \cos(2\theta - 1.326) = \dfrac{1.2}{\sqrt{17}}$	M1
	$\Rightarrow (2\theta - 1.326) = \pm 1.275.. \Rightarrow \theta = ...$	dM1
	$\theta = \text{awrt } 1.30$ or awrt 0.03	A1
	$2\theta - 1.326 = '1.275...'..$ **and** $'-1.275...'$	ddM1
	$\Rightarrow \theta = \text{awrt } 1.30$ **and** 0.03	A1
		(5)
		[8 marks]

(a)

B1 For $R = \sqrt{17}$. Condone $R = \pm\sqrt{17}$

M1 For $\alpha = \arctan(\pm 4)$ or $\alpha = \arctan\left(\pm\frac{1}{4}\right)$ leading to a solution of α

It is implied by $\alpha = \text{awrt } 76°$ or awrt 1.3 rads

Condone any solutions coming from $\cos\alpha = 1, \sin\alpha = 4$

If R has been used to find α award for only $\alpha = \arccos\left(\pm\frac{1}{'R'}\right)$ $\alpha = \arcsin\left(\pm\frac{4}{'R'}\right)$

A1 $\alpha = \text{awrt } 1.326$

(b)

M1 Using part (a) and proceeding as far as $\cos(2\theta \pm \text{their}1.326) = \dfrac{1.2}{\text{their } R}$.

Condone slips on the 1.2 and miscopying their 1.326

This may be implied by $(2\theta \pm \text{their}1.326) = \arccos\left(\dfrac{1.2}{\text{their } R}\right)$

Condone for this mark $\cos(\theta \pm \text{their}1.326) = \dfrac{1.2}{\text{their } R}$ or $\cos(2\theta \pm 2\times\text{their}1.326) = \dfrac{1.2}{\text{their } R}$

but $2\cos(\theta \pm \text{their}1.326) = \dfrac{1.2}{\text{their } R}$ is M0 and hence dM0…etc

dM1 Dependent upon the first M1. It is for a full method to find one value of θ within the range 0 to π from their principal value. Look for the correct order of operations, that is dealing with the "1.326" before the "2". Condone adding 1.326 instead of subtracting.

$$\cos(2\theta \pm \text{their}1.326) = ... \Rightarrow 2\theta \pm \text{their}1.326 = \beta \Rightarrow \theta = \frac{\beta \pm \text{their}1.326}{2}$$

A1 awrt $\theta = 1.30$ or $\theta =$ awrt 0.03 Only allow 1.3 if it is preceded by an answer that rounds to 1.30

ddM1 For a correct method to find a second value of θ (**for their** α) in the range 0 to π.
Eg $2\theta \pm 1.326 = '-\beta' \Rightarrow \theta =$ OR $2\theta \pm 1.326 = 2\pi + '\beta' \Rightarrow \theta =$ THEN MINUS π

A1 awrt $\theta = 1.30$ and $\theta =$ awrt 0.03. Only allow 1.3 if it is preceded by an answer that rounds to 1.30
Withhold this mark if there are extra solutions **in the range**.

Degree solution: Only lose the first time it occurs.

FYI. In degrees only lose the first A mark awrt (a) $\alpha = 75.964°$ and (b) $\theta_1 = 74.52°, \theta_2 = 1.44°$

Mixing degrees and radians only scores the first M in part (b)

Answers without working.

If $\sqrt{17}\cos(2\theta - 1.326) = 1.2$ is written down then all marks are available. (3 marks for one correct answer)

If there is no initial statement then score SC B1 B1 then 0,0, 0 for a maximum of 2, 1 for each solution.

Question Number	Scheme	Marks
2.	$x^3-4xy+2x+3y^2-3=0 \Rightarrow 3x^2 - 4x\frac{\mathrm{d}y}{\mathrm{d}x} - 4y + 2 + 6y\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	B1 M1 A1
	Substitute $(-3,2)$ $\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \left(-\frac{7}{8}\right)$	M1
	Uses gradient of normal $= -\frac{1}{\left.\mathrm{d}y/\mathrm{d}x\right\rvert_{x=-3}}$	dM1
	$y-2=\frac{8}{7}(x+3) \Rightarrow 8x-7y+38=0$	M1, A1
		[7 marks]

B1 Applies the product rule to $-4xy$ $\quad -4xy \to -4x\frac{\mathrm{d}y}{\mathrm{d}x} - 4y$

Accept exact alternatives such as $-4xy \to -4\left(x\frac{\mathrm{d}y}{\mathrm{d}x}+y\right)$ and allow if recovered from poor bracketing. You may see $-4xy \to -4x\,\mathrm{d}y - 4y\,\mathrm{d}x$

M1 Attempts the chain rule to $3y^2 \to Ay\frac{\mathrm{d}y}{\mathrm{d}x}$

You may see $3y^2 \to Ay\,\mathrm{d}y$

A1 For correct differentiation on $x^3+2x+3y^2-3 \Rightarrow 3x^2+2+6y\frac{\mathrm{d}y}{\mathrm{d}x}$

You may see $x^3+2x+3y^2-3 \Rightarrow 3x^2\mathrm{d}x+2\mathrm{d}x+6y\mathrm{d}y$

M1 Substitutes $(-3,2)$ into a differentiated form and attempts to find a numerical value of $\frac{\mathrm{d}y}{\mathrm{d}x}$

It is dependent upon the differentiated form having **exactly** two terms in $\frac{\mathrm{d}y}{\mathrm{d}x}$, **one** from $-4xy$ and **one** from $3y^2$ If the candidate attempts to rearrange and collect terms before substituting you may condone poor algebra.

dM1 Attempts to find a numerical value to the gradient of the normal. It is dependent upon the previous method mark and finding the negative reciprocal of the value of $\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right\rvert_{x=3}$

M1 Correct attempt at the form of the normal at $(-3,2)$ Eg. $y-2=$ their "$-\frac{1}{m}$"$(x+3)$

Condone one sign slip on the -2 or the +3. Condone, for this method, answers from poor differentiation (eg just having one $\frac{\mathrm{d}y}{\mathrm{d}x}$ term). This mark is for the method of finding the equation of a normal. If the form $y=mx+c$ is used it is for proceeding as far as $c = ...$

A1 $8x-7y+38=0$ Allow $k(8x-7y+38=0)$ where k is an integer

Note that the error $-4xy \to 4x\frac{\mathrm{d}y}{\mathrm{d}x}-4y$ can lead to $\frac{\mathrm{d}y}{\mathrm{d}x}=\infty$ which in turn gives a normal of $y=2$

This can potentially score B0 M1 A1 M1 dM1 M1 A0 for 5 out of 7

Question Number	Scheme	Marks
3.(a)	$\sec\theta° = \dfrac{1}{\cos\theta°} = \dfrac{1}{p}$ oe	B1
		(1)
(b)	$\sin(\theta - 90)° = \sin\theta°\cos 90° - \cos\theta°\sin 90° = -p$	M1A1
		(2)
(c)	$\sin 2\theta° = 2\sin\theta°\cos\theta°$	B1
	Uses $\sin\theta° = \sqrt{1-\cos^2\theta°} = \sqrt{1-p^2}, \Rightarrow \sin 2\theta° = 2p\sqrt{1-p^2}$	M1, A1
		(3)
		[6 marks]

(a)

B1 $\sec\theta° = \dfrac{1}{p}$ Accept $\sec\theta° = p^{-1}$

(b)

M1 Attempts $\sin(\theta-90)° = \sin\theta°\cos 90° \pm \cos\theta°\sin 90°$ with $\cos\theta° = p$ used

Alternatively uses $\sin(\theta-90)° = -\sin(90-\theta)° = -(\sin 90°\cos\theta° - \cos 90°\sin\theta°)$ with $\cos\theta° = p$ used

Similarly $\sin(\theta-90)° = -\sin(90-\theta)° = -\cos\theta°$ with $\cos\theta° = p$ used

Or $\sin(\theta-90)° = \sin\theta°\cos-90° \pm \cos\theta°\sin-90°$ with $\cos\theta° = p$ used

A1 $\sin(\theta-90)° = -p$.

Allow $-p$ for both marks as long as no incorrect work is used to generate this answer.

(c)

B1 States $\sin 2\theta° = 2\sin\theta°\cos\theta°$ or $\sin 2\theta° = \sin\theta°\cos\theta° + \sin\theta°\cos\theta°$

M1 Attempts $\sin^2\theta + \cos^2\theta = 1$ in part (c) with $\cos\theta° = p$ to get $\sin\theta°$ in terms of *p*.

Only accept $\sin\theta° = 1 - p$ if a version involving squares has been seen first.

Allow $\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin\theta = \sqrt{1-p}$ as a slip. (We have seen the Pythagorean identity)

You may see an attempt using a right angled triangle. The same scheme may be applied.

A1 $\sin 2\theta° = 2p\sqrt{1-p^2}$ but NOT $\sin 2\theta° = \pm 2p\sqrt{1-p^2}$

or equivalent such as $\sin 2\theta° = 2\sqrt{p^2 - p^4}$, $2p\sqrt{(1+p)(1-p)}$ or $\sqrt{4p^2 - 4p^4}$

Final answer (do not isw here).

$\sin 2\theta° = 2p\sqrt{1-p^2} = 2p(1-p)$ is B1 M1 A0

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B1 Alternatively attempts to use $\sin^2 2\theta + \cos^2 2\theta = 1$ and $\cos 2\theta = 2\cos^2\theta - 1$

M1 $\sin 2\theta = \sqrt{1-\cos^2 2\theta} = \sqrt{1-(1-2p^2)^2}$

A1 Usually $\sqrt{4p^2 - 4p^4}$ via this method

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Question Number	Scheme	Marks
4 (a)	Sets $8x - xe^{3x} = 0$	
	$\Rightarrow e^{3x} = 8 \Rightarrow 3x = \ln 8 \Rightarrow x = \dfrac{1}{3}\ln 8 = \ln 2$	M1A1
		(2)
(b)	$\dfrac{dy}{dx} = 8 - \left(\underline{3xe^{3x} + e^{3x}}\right)$	<u>M1</u> A1
	Sets $\dfrac{dy}{dx} = 0 \Rightarrow (1+3x)e^{3x} = 8$	M1
	$\Rightarrow e^{3x} = \dfrac{8}{(1+3x)} \Rightarrow x = \dfrac{1}{3}\ln\left(\dfrac{8}{1+3x}\right)$	dM1, A1*
		(5)
(c)	$x_1 = \dfrac{1}{3}\ln\left(\dfrac{8}{1+3\times 0.4}\right) =$ awrt 0.430	M1A1
	$x_2 =$ awrt 0.417, $x_3 =$ awrt 0.423	A1
		(3)
		[10 marks]

(a)

M1 Attempts to solve $e^{3x}=8$ using a correct order of operations. (Eg. some may take $\sqrt[3]{\ }$ first)
Allow for $3x=\ln 8$ Condone a slip on the 8. It may be implied by answers awrt 0.693

A1 $x=\ln 2$ Note that $x=\frac{1}{3}\ln 8$ is M1 A0

(b)

M1 Attempts to differentiate the $-xe^{3x}$ term to $\pm Axe^{3x}\pm Be^{3x}$ using the product rule.
If the rule is quoted it must be correct.

A1 Correct derivative $\frac{dy}{dx}=8-\left(3xe^{3x}+e^{3x}\right)$ with correct bracketing or $\frac{dy}{dx}=8-3xe^{3x}-e^{3x}$

M1 States or sets $\frac{dy}{dx}=0$ (which may be implied) and takes out a common factor of e^{3x} reaching a form $(\ldots\pm\ldots)e^{3x}=\ldots$ or $e^{3x}=\frac{\ldots}{(\ldots\pm\ldots)}$

dM1 Dependent upon the **both** previous M's, it is scored for using correct ln work, moving from

$$(\ldots\pm\ldots)e^{3x}=\ldots\Rightarrow \boldsymbol{x}= \quad\text{or}\quad e^{3x}=\frac{\ldots}{(\ldots\pm\ldots)}\Rightarrow \boldsymbol{x}=$$

A1* Reaches $x=\frac{1}{3}\ln\left(\frac{8}{1+3x}\right)$, $x=\frac{1}{3}\ln\left(\frac{8}{3x+1}\right)$ or $x=\frac{1}{3}\ln\frac{8}{3x+1}$ oe with correct work and no errors or omissions (See scheme for necessary steps that need to be seen)

$\frac{dy}{dx}=8-\left(\underline{3xe^{3x}+e^{3x}}\right)\Rightarrow 3xe^{3x}+e^{3x}=8$ is an example where there is missing step. (No $\frac{dy}{dx}=0$)

$\frac{dy}{dx}=0\Rightarrow 3xe^{3x}+e^{3x}=8\Rightarrow e^{3x}=\frac{8}{3x+1}$ is also an example where there is missing step. (No attempt to show the factorised line $(1+3x)e^{3x}=8$)

$\frac{dy}{dx}=0\Rightarrow 3xe^{3x}+e^{3x}=8\Rightarrow 3x+1.e^{3x}=8\Rightarrow e^{3x}=\frac{8}{3x+1}$ is also an example where there is a missing bracket (for factorisation)

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If the first M1 isn't scored for an attempt at the product rule a special case M0A0M1dM0 A0 may be awarded for setting their $\frac{dy}{dx}=0$ and proceeding to a form $e^{3x}=\ldots$

For example, if $\frac{dy}{dx}=8-3xe^{3x}$ it would be for proceeding to $e^{3x}=\frac{8}{3x}$

………

(c)

M1 Calculates x_1 from the given iterative formula. May be implied by $\frac{1}{3}\ln\left(\frac{8}{1+3\times 0.4}\right)$ or awrt 0.43

A1 awrt 0.430 Allow answer written as 0.43

A1 awrt $x_2=0.417$, $x_3=0.423$. NB. The subscripts are not important.

Question Number	Scheme	Marks
5. (a)	$4x^2+5x+3 = A(1-x)^2 + B(x+2)(1-x) + C(x+2)$	B1
	Sub $x=1$ $\quad C=4$	M1
	$x=-2$ $\quad \Rightarrow A=1$ $\quad$ any two constants correct	A1
	Coefficients of x^2	
	$4 = A - B \Rightarrow B=-3$ $\quad$ all three constants correct	A1
		(4)
(b)(i)	$\int \left(\frac{1}{(x+2)} - \frac{3}{(1-x)} + \frac{4}{(1-x)^2} \right) dx = \underline{\ln(x+2) + 3\ln(1-x)} + 4(1-x)^{-1}$ $\quad (+c)$ oe	M1 M1 A1ft
(ii)	$\int_0^{\frac{1}{2}} \frac{4x^2+5x+1}{(x+2)(1-x)^2} dx = \left[\ln(x+2) + 3\ln(1-x) + 4(1-x)^{-1} \right]_0^{\frac{1}{2}}$	
	$= \left(\ln\frac{5}{2} + 3\ln\frac{1}{2} + 8 \right) - (\ln 2 + 3\ln 1 + 4)$	M1
	$= \ln \left(\frac{\frac{5}{2} \times \left(\frac{1}{2}\right)^3}{2} \right) + \ldots$	M1
	$= 4 + \ln\left(\frac{5}{32}\right)$	A1 **(6)**
		[10 marks]

(a)

B1 Writes $4x^2+5x+3=A(1-x)^2+B(x+2)(1-x)+C(x+2)$

This may be implied by the sight of two equivalent fractions or via work leading to the constants

M1 Substitutes $x=1$ or $x=-2$ or equivalent and attempts to find the value of one constant.

It can be scored after scoring B0.

Eg condone the use of $4x^2+5x+3=A(1-x)^2(1-x)+B(x+2)(1-x)+C(x+2)$ or similar

Alternatively attempts to equate coefficients of x^2, x and constant terms to produce and solve simultaneous equations to find the value of one constant.

A1 Any two constants correct

A1 All three constants correct

(b)(i)

M1 For $\int \frac{A}{x+2} \to ..\ln(x+2)$ and $\int \frac{B}{1-x} \to ..\ln(1-x)$

M1 For $\int \frac{C}{(1-x)^2} \to ..(1-x)^{-1}$

A1ft All three of their integrals correct, following through on incorrect constants (but not zero's)

There must be some attempt to write in the simplest form. (Cannot leave $--\ 4(1-x)^{-1}$ for instance)

(b)(ii)

M1 Substitutes both $x=\frac{1}{2}$ and $x=0$ into their answer for (b)(i) which involves lns and subtracts (either way around).

M1 Uses correct ln work to combine **their** ln terms

A1 cao $=4+\ln\left(\frac{5}{32}\right)$ Note that the decimal equivalent $4+\ln 0.15625$ is correct

Question Number	Scheme	Marks
6. (a)	$\left\{\sqrt{\left(\frac{1+2x}{1-x}\right)}\right\} = (1+2x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}$	B1
	$=\left(1+\left(\frac{1}{2}\right)(2x)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(2x)^2+...\right)\times\left(1+\left(-\frac{1}{2}\right)(-x)+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-x)^2+...\right)$	M1 A1
	$=\left(1+x-\frac{1}{2}x^2+...\right)\times\left(1+\frac{1}{2}x+\frac{3}{8}x^2+...\right)$	A1
	$=1+\frac{1}{2}x+\frac{3}{8}x^2+x+\frac{1}{2}x^2-\frac{1}{2}x^2+...$	M1
	$=1+\frac{3}{2}x+\frac{3}{8}x^2$.	A1 *
		(6)
(b)	$\sqrt{\left(\frac{1+\left(\frac{2}{10}\right)}{1-\left(\frac{1}{10}\right)}\right)}=\frac{2}{3}\sqrt{3}$	B1
		(1)
(c)	Sub $x=\frac{1}{10}\Rightarrow "k"\sqrt{3}=1+\frac{3}{2}\left(\frac{1}{10}\right)+\frac{3}{8}\left(\frac{1}{10}\right)^2$	M1
	so, $\sqrt{3}\approx\frac{2769}{1600}$	A1
		(2) **[9 marks]**
Alt 6. (a)	$\left\{\sqrt{\left(\frac{1+2x}{1-x}\right)}\right\}=1+\frac{3}{2}x+\frac{3}{8}x^2\Rightarrow(1+2x)^{\frac{1}{2}}=\left(1+\frac{3}{2}x+\frac{3}{8}x^2\right)(1-x)^{\frac{1}{2}}$	B1
	$\Rightarrow\left(1+\left(\frac{1}{2}\right)(2x)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(2x)^2+...\right)=\left(1+\frac{3}{2}x+\frac{3}{8}x^2\right)\left(1+\left(\frac{1}{2}\right)(-x)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(-x)^2\right.$ -	M1 A1 A1
	$\Rightarrow\left(1+x-\frac{1}{2}x^2+...\right)=\left(1+\frac{3}{2}x+\frac{3}{8}x^2\right)\left(1-\frac{1}{2}x-\frac{1}{8}x^2+...\right)$	
	$\Rightarrow\left(1+x-\frac{1}{2}x^2+...\right)=1+x-\frac{1}{2}x^2$ Hence true	M1A1*
		(6)

(a)

B1 For writing the given expression in index form $(1+2x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}$ It may be implied by working but it must be a form that can lead to the answer.

Do not allow $\frac{(1+2x)^{\frac{1}{2}}}{(1-x)^{\frac{1}{2}}}$ for this mark unless the expanded $(1-x)^{\frac{1}{2}}$ is subsequently set with index -1

M1 Score for the form of the binomial expansion with index $\frac{1}{2}$ or $-\frac{1}{2}$

Eg $=\left[1+\left(\frac{1}{2}\right)(**x)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(**x)^2+...\right]$ or $=\left[1+\left(-\frac{1}{2}\right)(**x)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}(**x)^2+...\right]$

A1 Correct unsimplified form for one expression

A1 The correct simplified form for both expressions seen.
This mark may be implied if a correct final expression is found following correct working.
There doesn't need to be an implication that these expressions are to be multiplied.

M1 For multiplying terms in the first expansion by terms in the second expansion.
Expect to see an attempt to find the six terms required to produce the given solution. Allow terms in x^3 and greater to be seen which don't need to be correct.
Follow through on their expansions but condone "changes" in an attempt to reach the given solution. (See Practice Items)

A1* Correct solution only $= 1 + \frac{3}{2}x + \frac{3}{8}x^2$

(b)

B1 $\frac{2}{3}\sqrt{3}$ or statement $k = \frac{2}{3}$ seen in (b)

(c)

M1 Sub $x = \frac{1}{10}$ into both sides of (a) $\Rightarrow "k"\sqrt{3} = 1 + \frac{3}{2}\left(\frac{1}{10}\right) + \frac{3}{8}\left(\frac{1}{10}\right)^2$ or $\sqrt{\frac{1+2/10}{1-1/10}} = 1 + \frac{3}{2}\left(\frac{1}{10}\right) + \frac{3}{8}\left(\frac{1}{10}\right)^2$

Do not allow $k = 1$

A1 $\sqrt{3} \approx \frac{2769}{1600}$ or exact equivalent. Condone $\sqrt{3} \approx \frac{1600}{923}$ which follows (b) $= \frac{2}{\sqrt{3}}$

....

You may see a variety of solutions to part (a). Please consider carefully when marking.
Example: Mark in this order

B1: $\left\{\sqrt{\left(\frac{1+2x}{1-x}\right)}\right\} = \sqrt{\left(\frac{1-x+3x}{1-x}\right)} = \sqrt{\left(1+\frac{3x}{1-x}\right)} = \left(1+\frac{3x}{1-x}\right)^{\frac{1}{2}}$

M1: For one attempt at the binomial expansion $\left(1+\frac{*x}{1-x}\right)^{\frac{1}{2}} = 1 + \frac{1}{2}\times\frac{*x}{1-x} + \frac{\frac{1}{2}\times-\frac{1}{2}}{2}\left(\frac{*x}{1-x}\right)^2$

condoning slips on the bracketing

A1: Completely correct intermediate form. $\left(1+\frac{3x}{1-x}\right)^{\frac{1}{2}} = 1 + \frac{1}{2}\times\frac{3x}{1-x} + \frac{\frac{1}{2}\times-\frac{1}{2}}{2}\left(\frac{3x}{1-x}\right)^2$

M1: For a second use of the binomial expansion. It is dependent upon a correct first use.

$$\left(1+\frac{*x}{1-x}\right)^{\frac{1}{2}} = 1 + \frac{*x}{2}\times(1-x)^{-1} - \frac{*x^2}{8}(1-x)^{-2} = 1 + \frac{*x}{2}\times(1+x+(x^2)) - \frac{*x^2}{8}\times(1+(2x+3x^2))$$

Expect to see a correct use of the binomial expansion in both TERMS.

A1: $\left(1+\frac{*x}{1-x}\right)^{\frac{1}{2}} = 1 + \frac{3x}{2}\times(1-x)^{-1} - \frac{9x^2}{8}(1-x)^{-2} = 1 + \frac{3x}{2}\times(1+x+...) - \frac{9x^2}{8}\times(1+...)$

A1: $= 1 + \frac{3}{2}x + \frac{3}{8}x^2$

Question Number	Scheme	Marks
7 (a)	$y=\ln(1-\cos 2x)\Rightarrow \frac{dy}{dx}=\frac{2\sin 2x}{1-\cos 2x}$	M1A1
	$\Rightarrow \frac{dy}{dx}=\frac{4\sin x\cos x}{1-\left(1-2\sin^2 x\right)},=\frac{4\sin x\cos x}{2\sin^2 x}=2\cot x$	M1, A1
		(4)
(b)	$2\cot x=2\sqrt{3}\Rightarrow \tan x=\frac{1}{\sqrt{3}}$	
	$x=\arctan\left(\frac{1}{\sqrt{3}}\right)\Rightarrow x=\frac{\pi}{6}$	M1A1
	$y=\ln\left(1-\cos\left(\frac{2\pi}{6}\right)\right)=\ln\frac{1}{2}$ or $-\ln 2$	M1A1
		(4) **[8 marks]**
7 (a)alt I	$y=\ln(1-\cos 2x)\Rightarrow y=\ln(2\sin^2 x)\Rightarrow \frac{dy}{dx}=\frac{4\sin x\cos x}{2\sin^2 x}$	M1A1
	$\Rightarrow \frac{dy}{dx}=2\frac{\cos x}{\sin x}=2\cot x$	M1, A1
		(4)
7 (a)alt II	$y=\ln(2\sin^2 x)\Rightarrow y=\ln 2+2\ln\sin x\Rightarrow \frac{dy}{dx}=0+\frac{2\cos x}{\sin x}$	M1A1
	$\Rightarrow \frac{dy}{dx}=2\frac{\cos x}{\sin x}=2\cot x$	M1, A1
		(4)
7 (a)alt III	$y=\ln(1-\cos 2x)\Rightarrow e^y=1-\cos 2x\Rightarrow e^y\frac{dy}{dx}=2\sin 2x$	M1A1
	Then as main scheme	M1, A1 **(4)**

(a)

M1 For differentiating $y=\ln(1-\cos 2x)$ to $\frac{dy}{dx}=\frac{\pm A\sin 2x}{1-\cos 2x}$

A1 $\frac{dy}{dx}=\frac{2\sin 2x}{1-\cos 2x}$ oe

M1 Uses the double angle identities $\sin 2x=2\sin x\cos x$ and $\cos 2x=1-2\sin^2 x$

The double angle for $\cos 2x$ may be implied by sight of "$1-1-2\sin^2 x$" on the denominator as we can condone the missing bracket.

A1 Simplifies to show that $\Rightarrow \frac{dy}{dx}=2\cot x$ showing at least one correct intermediate line between $\frac{dy}{dx}=\frac{4\sin x\cos x}{1-\left(1-2\sin^2 x\right)}$ and $2\cot x$ usually $\frac{4\sin x\cos x}{2\sin^2 x}$ or $\frac{2\cos x}{\sin x}$

In the alternative versions the double angle identity (or identities) are seen before the differentiation.

For example.
(i) This one shows incorrect differentiation and is scored M0 A0 M1 A0

$$y = \ln(1-\cos 2x) \Rightarrow y = \ln(2\sin^2 x) \Rightarrow \frac{dy}{dx} = \frac{1}{2\sin^2 x}$$

(ii) This one can be awarded the method mark for differentiation and is scored M1 A0 M1 A0

$$y = \ln(1-\cos 2x) \Rightarrow y = \ln(2\sin^2 x) \Rightarrow \frac{dy}{dx} = \frac{2\sin x\cos x}{2\sin^2 x} = \cot x$$

The two accuracy marks are linked and cannot really be awarded separately M1 A1 M1 A0 would be difficult to score via this method.

(b)

M1 Uses $\cot x = \frac{1}{\tan x}$ and proceeds to find x

A1 $x = \frac{\pi}{6}$. Ignore additional (incorrect) values such as $x = \frac{5\pi}{6}$

Do not accept $30°$ for this mark. Do not allow this for candidates who guess $k = 2$

M1 Substitutes their value of x in $y = \ln(1-\cos 2x)$

A1 For $y = \ln\frac{1}{2}$ or $-\ln 2$ following $x = \frac{\pi}{6}$ or $30°$

Do not allow this for candidates who guess $k = 2$

Withhold this final mark if there is another value (x, y) given within the range

Question Number	Scheme	Marks
8(i)	$\int x\sin x\,\mathrm{d}x = -x\cos x + \int \cos x\,\mathrm{d}x$	M1
	$= -x\cos x + \sin x\,(+c)$	dM1A1
		(3)
(ii)(a)	$\mathrm{d}x = \sec\theta\tan\theta\,\mathrm{d}\theta$	B1
	$\int_1^2 \sqrt{1-\frac{1}{x^2}}\,\mathrm{d}x = \int_0^{\frac{\pi}{3}} \sqrt{1-\cos^2\theta}\,\sec\theta\tan\theta\,\mathrm{d}\theta$	
	$= \int_0^{\frac{\pi}{3}} \sqrt{\sin^2\theta} \times \frac{1}{\cos\theta}\tan\theta\,\mathrm{d}\theta = \int_0^{\frac{\pi}{3}} \tan^2\theta\,\mathrm{d}\theta$	M1 A1*
		(3)
(b)	$\int \sqrt{1-\frac{1}{x^2}}\,\mathrm{d}x = \int \tan^2\theta\,\mathrm{d}\theta = \int \left(\sec^2\theta - 1\right)\mathrm{d}\theta = \tan\theta - \theta$	M1A1
	$\int_1^2 \sqrt{1-\frac{1}{x^2}}\,\mathrm{d}x = \left[\tan\theta - \theta\right]_{\theta=0}^{\theta=\frac{\pi}{3}} = \tan\left(\frac{\pi}{3}\right) - \frac{\pi}{3} = \sqrt{3} - \frac{\pi}{3}$	dM1A1
		(4) **[10 marks]**

(i)

M1 Attempts integration by parts the correct way $\int x\sin x\,\mathrm{d}x = \pm x\cos x \pm \int \cos x\,\mathrm{d}x$

dM1 Integrates again to $\int x\sin x\,\mathrm{d}x = \pm x\cos x \pm \sin x \quad (+c)$

A1 $= -x\cos x + \sin x\,(+c)$ with no need for + c

$= -\cos x.x + \sin x\,(+c)$ can be accepted

Note that international centres sometimes teach a "tabular method" "D-I method"

	Diff	Int
+	x	$\sin x$
-	1	$-\cos x$
+	0	$-\sin x$

If there is no evidence of an intermediate line, a tabular method, or integration by parts score M1 dM1 A1 for a fully correct answer
AND M0 dM0 A0 for an incorrect answer

(ii)(a)

B1 $\mathrm{d}x = \sec\theta\tan\theta\,\mathrm{d}\theta$ or equivalent. Eg accept $\frac{\mathrm{d}x}{\mathrm{d}\theta} = \sec\theta\tan\theta$

M1 Substitutes $x = \sec\theta$ into $\sqrt{1-\frac{1}{x^2}}$ and simplifies to $\sin\theta$, $\frac{1}{\operatorname{cosec}\theta}$, $\frac{\tan\theta}{\sec\theta}$ or $\sqrt{\sin^2\theta}$ $\sqrt{\frac{1}{\operatorname{cosec}^2\theta}}$ $\sqrt{\frac{\tan^2\theta}{\sec^2\theta}}$ This may be implied if the $\sqrt{1-\frac{1}{x^2}}$ has been adapted

A1* Completes proof.

This is a show that question and you should expect the $\sec\theta$ to be either replaced by $\frac{1}{\cos\theta}$ or allow to be cancelled as seen below

$$\int \frac{\tan\theta}{\sec\theta} \times \sec\theta\tan\theta\,\mathrm{d}\theta = \int \tan^2\theta\,\mathrm{d}\theta$$

Expect to see the correct limits and correct notation **within their solution.** (Condone incorrect notation in jottings/working at the side of their solution)
The limits can just appear without the need to see calculations.
A notational error is $\tan^2\theta$ being written $\tan\theta^2$ without a bracket
It is possible to do this question from rhs to lhs but marks in a similar way.

(ii)(b)

M1 $\int \tan^2\theta\,\mathrm{d}\theta = \int (\pm\sec^2\theta \pm 1)\,\mathrm{d}\theta = \pm\tan\theta \pm \theta$

A1 $\int \tan^2\theta\,\mathrm{d}\theta = \tan\theta - \theta$

dM1 Uses the limit(s) $\frac{\pi}{3}$ (and 0) in a function of the form $\pm\tan\theta \pm \theta$

A1 cso $\sqrt{3} - \frac{\pi}{3}$

Question Number	Scheme	Marks
9 (a)	$P = 450$	B1 **(1)**
(b)	$315 = \dfrac{900\mathrm{e}^{\frac{1}{4}t}}{3\mathrm{e}^{\frac{1}{4}t} - 1} \Rightarrow 45\mathrm{e}^{\frac{1}{4}t} = 315$	M1 A1
	$\Rightarrow \mathrm{e}^{\frac{1}{4}t} = 7 \Rightarrow t = 4\ln 7$	M1A1 **(4)**
(c)(i)	$\dfrac{\mathrm{d}P}{\mathrm{d}t} = \dfrac{\left(3\mathrm{e}^{\frac{1}{4}t} - 1\right) \times 225\mathrm{e}^{\frac{1}{4}t} - 900\mathrm{e}^{\frac{1}{4}t} \times \frac{3}{4}\mathrm{e}^{\frac{1}{4}t}}{\left(3\mathrm{e}^{\frac{1}{4}t} - 1\right)^2} = \left(\dfrac{-225\mathrm{e}^{\frac{1}{4}t}}{\left(3\mathrm{e}^{\frac{1}{4}t} - 1\right)^2}\right)$	M1A1
(ii)	$\left.\dfrac{\mathrm{d}P}{\mathrm{d}t}\right\vert_{t=8} = \dfrac{\left(3\mathrm{e}^{2} - 1\right) \times 225\mathrm{e}^{2} - 900\mathrm{e}^{2} \times \frac{3}{4}\mathrm{e}^{2}}{\left(3\mathrm{e}^{2} - 1\right)^2} = \dfrac{-225\mathrm{e}^2}{\left(3\mathrm{e}^{2} - 1\right)^2} = -3.71$	M1A1 **(4)** **[9 marks]**

(a)

B1 $(P =)450$

(b)

M1 Substitutes $P = 315$, cross multiplies to reach a form $A\mathrm{e}^{\frac{1}{4}t} = B$ oe

A1 $45\mathrm{e}^{\frac{1}{4}t} = 315$ or equivalent such as $\mathrm{e}^{\frac{1}{4}t} = 7$. Note $\mathrm{e}^{-\frac{1}{4}t} = \frac{1}{7}$ is correct

M1 $\mathrm{e}^{\frac{1}{4}t} = D\ (D > 0) \Rightarrow t = ...$ using ln's. Allow equivalent working from $\mathrm{e}^{-\frac{1}{4}t} = E,\ E > 0$

Equivalent work may be seen. $\ln 45 + \frac{1}{4}t = \ln 315 \Rightarrow t = ...$

A1 $t = 4\ln 7$ or equivalent such as $t = 2\ln 49$, $\ln 2401$ but not $t = 4\ln\left(\frac{315}{45}\right)$ (Scheme requires $a\ln k$)

(c)(i)

M1 Attempts to apply the quotient rule on $P = \dfrac{900e^{\frac{1}{4}t}}{3e^{\frac{1}{4}t} - 1}$ with $u = 900e^{\frac{1}{4}t}, v = 3e^{\frac{1}{4}t} - 1$ to reach an expression of the required form (see below). Condone slips on the coefficients.

Score for $\dfrac{dP}{dt} = \dfrac{\left(3e^{\frac{1}{4}t} - 1\right) \times Ae^{\frac{1}{4}t} - 900e^{\frac{1}{4}t} \times Be^{\frac{1}{4}t}}{\left(3e^{\frac{1}{4}t} - 1\right)^2}$, $\quad A, B > 0$

Award this mark if the candidate incorrectly multiples out before writing down their $\dfrac{dP}{dt}$

$$u = 900e^{\frac{1}{4}t}, u' = 225e^{\frac{1}{4}t}, v = 3e^{\frac{1}{4}t} - 1, v' = \frac{3}{4}e^{\frac{1}{4}t} \Rightarrow \frac{dP}{dt} = \frac{\left(3e^{\frac{1}{4}t} - 1\right) \times 225e^{\frac{1}{4}t} - 675e^{\frac{1}{4}t^2}}{\left(3e^{\frac{1}{4}t} - 1\right)^2}$$

Alternatively applies the product rule on $900e^{\frac{1}{4}t} \times \left(3e^{\frac{1}{4}t} - 1\right)^{-1}$ or chain rule on $900\left(3 - e^{-\frac{1}{4}t}\right)^{-1}$

For the product rule look for $Pe^{\frac{1}{4}t} \times \left(3e^{\frac{1}{4}t} - 1\right)^{-1} \pm Qe^{\frac{1}{4}t} \times e^{\frac{1}{4}t}\left(3e^{\frac{1}{4}t} - 1\right)^{-2}$ oe

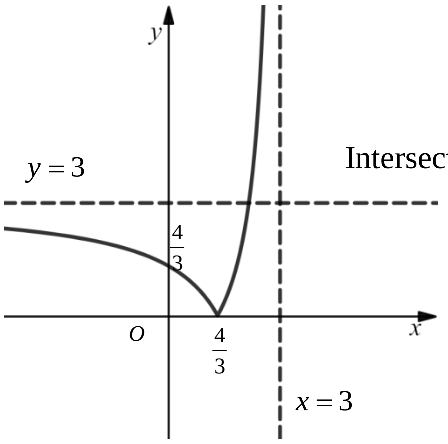
A1 $\dfrac{dP}{dt} = \dfrac{\left(3e^{\frac{1}{4}t} - 1\right) \times 225e^{\frac{1}{4}t} - 900e^{\frac{1}{4}t} \times \frac{3}{4}e^{\frac{1}{4}t}}{\left(3e^{\frac{1}{4}t} - 1\right)^2}$ which may be left unsimplified.

$\dfrac{dP}{dt} = 225e^{\frac{1}{4}t} \times \left(3e^{\frac{1}{4}t} - 1\right)^{-1} - 675e^{\frac{1}{4}t} \times e^{\frac{1}{4}t}\left(3e^{\frac{1}{4}t} - 1\right)^{-2}$ which may be left unsimplified.

(c)(ii)

M1 Substitutes $t = 8$ into their $\dfrac{dP}{dt}$ and calculates a value for $\dfrac{dP}{dt}$

A1 -3.71 only. Note that this is **not** awrt
If the candidate subsequently writes 3.71 this is A0

Question Number	Scheme	Marks
10 (a)	$y<3$	B1 **(1)**
(b)(i)	$\left(\frac{4}{3},0\right)$	B1
(ii)	$\left(0,\frac{4}{3}\right)$	B1 **(2)**
(c)	$\text{gg}(x)=\dfrac{3\times\frac{3x-4}{x-3}-4}{\frac{3x-4}{x-3}-3}=\dfrac{3(3x-4)-4(x-3)}{3x-4-3(x-3)}=\dfrac{5x}{5}=x$	M1dM1A1 **(3)**
(d)	 Shape Intersects y - axis at $\left(0,\frac{4}{3}\right)$ meets x- axis at $\left(\frac{4}{3},0\right)$ Asymptotes at $x=3$ and $y=3$	B1 B1ft B1 **(3)**
(e)	$\dfrac{3x-4}{x-3}=-8\Rightarrow 3x-4=-8x+24\Rightarrow x=\dfrac{28}{11}$	M1dM1A1 **(3)** **[12 marks]**

(a)

B1 Accept $y<3$, $\text{g}(x)<3$, $\text{g}<3$, $-\infty<y<3$ $(-\infty,3)$

(b)(i)

B1 $\left(\frac{4}{3},0\right)$ Allow candidate to state $x=\frac{4}{3}, y=0$ or state $x=\frac{4}{3}, \text{g}(x)=0$

(b)(ii)

B1 $\left(0,\frac{4}{3}\right)$ Allow candidate to state $x=0, y=\frac{4}{3}$ or state $x=0, \text{g}(x)=\frac{4}{3}$

SC: For candidates who in(i) write just x or $A=\frac{4}{3}$ and in (ii) write just y or $B=\frac{4}{3}$ score B1 B0 SC

This SC should also be used for candidates who embed the values. So for example in (i) show $0=\frac{3x-4}{x-3}\Rightarrow x=\frac{4}{3}$ and in (ii) show $y=\frac{3\times0-4}{0-3}\Rightarrow y=\frac{4}{3}$ as there is no explicit statement in (i) $x=0$ and (ii) $y=0$

(c)

M1 Attempts to substitute g into g. $gg(x)=\dfrac{3\times\frac{3x-4}{x-3}-4}{\frac{3x-4}{x-3}-3}$ is sufficient

An alternative is using $g(x)=3+\dfrac{5}{x-3}\Rightarrow gg(x)=3+\dfrac{5}{3+\frac{5}{x-3}-3}$

dM1 Multiplies by all terms on the numerator and all terms on the denominator by $(x-3)$ to form a fraction of the form $\dfrac{ax+b}{cx+d}$ Condone poor bracketing

Al $gg(x)=x$

(d)

B1 Correct shape with cusp (not a minimum) at *A*. The curve must appear to have the same asymptote as the original curve (at $x=3$ and $y=3$) It should have the correct curvature on the rhs and not appear to bend back on itself

B1ft Intersects *y* - axis at $\left(0,\dfrac{4}{3}\right)$ and meets *x*- axis at $\left(\dfrac{4}{3},0\right)$.

Follow through on coordinates from (b). Allow this to be marked *A*, *B* as long as the coordinates of *A* and *B* were correct.

B1 Gives the equation of both asymptotes as $x=3$ and $y=3$

(e)

M1 For writing down a correct equation leading to a solution of $|g(x)|=8$

Allow $\dfrac{3x-4}{x-3}=-8$ oe so allow $-\dfrac{3x-4}{x-3}=8$, $\dfrac{-3x+4}{x-3}=8$, $\dfrac{3x-4}{3-x}=8$ and $\left(\dfrac{3x-4}{x-3}\right)^2=64$

dM1 Solves an allowable equation (see above) by cross multiplying, collecting terms to reach $x=..$

Do not allow a candidate to score this mark from $-\dfrac{3x-4}{x-3}=8\Rightarrow\dfrac{-3x+4}{-x+3}=8$ This scores M1 M0

A1 $x=\dfrac{28}{11}$ **oee only**

If both values are found $\dfrac{3x-4}{x-3}=\pm8\Rightarrow x=4,\dfrac{28}{11}$ then this mark is scored only when "4" is deleted or $\dfrac{28}{11}$ is chosen as **the** answer

....

Solution from squaring in (c)

M1: $\left(\dfrac{3x-4}{x-3}\right)^2=64$

dM1: $\Rightarrow Ax^2+Bx+c=0$ and solves by usual methods

FYI the correct quadratic is $55x^2-360x+560=0\Rightarrow(11x-28)(x-4)=0$

A1: Selects $x=\dfrac{28}{11}$

Question Number	Scheme	Marks
11 (a)	l_2: $\mathbf{r}=\begin{pmatrix}0\\0\\0\end{pmatrix}+\mu\begin{pmatrix}-1\\4\\3\end{pmatrix}$	M1A1
		(2)
(b)(i)	$A=(2,3,-1)$	B1
(ii)	$B=(-1,15,8)$	B1
		(2)
(c)	Uses a correct pair of gradients $a=\begin{pmatrix}2\\3\\-1\end{pmatrix}$ and $b=k\begin{pmatrix}-1\\4\\3\end{pmatrix}$ or vice versa	M1
	$a.b=\lvert a\rvert\lvert b\rvert\cos\theta \Rightarrow -2+12-3=\sqrt{14}\sqrt{26}\cos\theta \Rightarrow \theta=$ awrt $68.5°$	dM1, A1
		(3)
(d)	$AB=\sqrt{3^2+12^2+9^2}$ OR $AB=3\times\text{'}\sqrt{26}\text{'}$	M1
	Area = $\text{'}OA\times AB\times\sin(c)\text{'}=\sqrt{14}\times3\sqrt{26}\times\sin68.5°=$ awrt $53\left(\text{units}^2\right)$	M1A1
		(3)
		[10 marks]

(a)

M1 Scored for the **rhs** of the equation with gradient $k\times\begin{pmatrix}-1\\4\\3\end{pmatrix}$ **and** containing the point $\begin{pmatrix}0\\0\\0\end{pmatrix}$.

A1 Correct equation with lhs $\mathbf{r}=\begin{pmatrix}0\\0\\0\end{pmatrix}+\mu\begin{pmatrix}-1\\4\\3\end{pmatrix}$ oe such as $\begin{pmatrix}x\\y\\z\end{pmatrix}=\mu\begin{pmatrix}-1\\4\\3\end{pmatrix}$.

Allow for gradient any multiple of $\begin{pmatrix}-1\\4\\3\end{pmatrix}$ Allow with any scalar parameter inc λ.

Condone another constant appearing so $\mathbf{r}=\begin{pmatrix}1k\\-4k\\-3k\end{pmatrix}+\mu\begin{pmatrix}-1\\4\\3\end{pmatrix}$ is acceptable

It must be an equation so $\mathbf{r}=$ or $\begin{pmatrix}x\\y\\z\end{pmatrix}=$ must be on the lhs

(b)(i)

B1 $A=(2,3,-1)$ Accept in vector notation

(b)(ii)

B1 $B=(-1,15,8)$ Accept in vector notation

(c)

M1 Attempts to use a correct pair of gradient vectors. Eg uses their $\boldsymbol{a} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$ and their $\boldsymbol{b} = k\begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix}$ or vice versa, Condone one sign slip only

dM1 Correct method for finding acute angle from $a.b = |a||b|\cos\theta$

Expect to see an attempt proceeding to $\cos\theta = ...$ condoning one slip on $a.b$ and an attempt at squaring and adding for $|a| \& |b|$ If there is no method shown for $|a|$ *or* $|b|$ then expect at least one to be correct. It is dependent upon the previous M

A1 $\theta =$ awrt $68.5°$ Note that 1.2 is the radian answer and scores A0.

Allow awrt $68.5°$ coming from $180° - 111.5°$

(d)

M1 Uses a correct method of finding distance *AB* Accept $\sqrt{(x-x_1)^2+(y-y_1)^2+(z-z_1)^2}$ or $3\times\begin{vmatrix} -1 \\ 4 \\ 3 \end{vmatrix}$

M1 Uses a correct method of finding the area of *OABD*

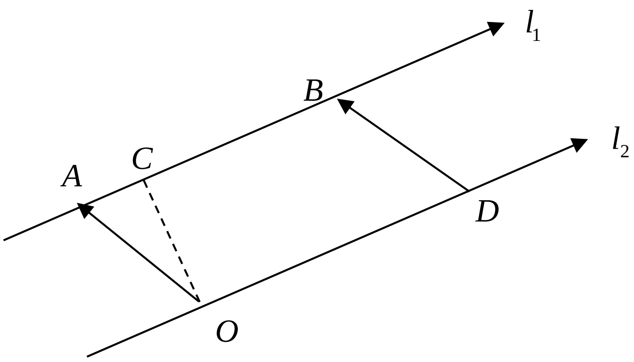
Accept '$OA\times AB\times \sin(c)$' or two triangles

Note that '$OA\times AB\times\sin(180-c)$' $= \sqrt{14}\times 3\sqrt{26}\times\sin 111.5° =$ awrt $53\left(\text{units}^2\right)$ is also correct

A1 awrt 53 Accept the exact answer $9\sqrt{35}$ and then isw

....

There are various other ways of finding are *OABD*



Eg by finding the perpendicular distance between *AB* and *OD*

M1 Uses a correct method of finding distance *AB*

M1 It must be a full method. The values are given for a check. (It is a method mark!)

Sets up a point *C* on l_1 with coordinates $(2-\lambda, 3+4\lambda, -1+3\lambda)$

Uses the fact that *OC* and *AB* are perpendicular

$$\begin{pmatrix} 2-\lambda \\ 3+4\lambda \\ -1+3\lambda \end{pmatrix}.\begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix} = 0 \Rightarrow \lambda = -\frac{7}{26}$$

Finds point $C = \left(\frac{59}{26}, \frac{25}{13}, -\frac{47}{26}\right)$ and hence distance $OC = \sqrt{\frac{315}{26}}$

And then multiplies *AB* by *OC*

A1 AWRT 53

Note: It is possible to find the area by finding the perpendicular distance between *AO* and *BD*

Please scan through the whole response and mark M1 dM1 A1where the first M is a full method to find the perpendicular distance.

Similarly, it is possible to attempt $AO\times OB\sin AOB$ Use the same scoring as above. M1 dM1 A1

Question Number	Scheme	Marks
Qu 12 (a)	$t=1 \Rightarrow x=2,\ y=8$	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y/\mathrm{d}t}{\mathrm{d}x/\mathrm{d}t} = \frac{9-3t^2}{14t}$	M1A1
	Equation of tangent is $y-8=\frac{6}{14}(x-2)$	M1
	$3x-7y+50=0$	A1
		(5)
(b)(i)	At A, $x=-5$;	B1
(ii)	$y=0 \Rightarrow t(9-t^2)=t(3-t)(3+t)=0$	
	$t=0, 3, -3$	M1
	At $t=3$, $x=7(3)^2-5=58$ or $(\text{At } t=-3,\ x=7(-3)^2-5=58)$	
	At B, $x=58$	A1
		(3)
(c)	$\int y\,\mathrm{d}x = \int y\frac{\mathrm{d}x}{\mathrm{d}t}\mathrm{d}t = \int t(9-t^2)14t\,\mathrm{d}t$	M1
	$=\int(126t^2-14t^4)\mathrm{d}t$	A1
	$=\frac{126t^3}{3}-\frac{14t^5}{5}\ (+C) \quad (=42t^3-2.8t^5\ (+C))$	M1
	Either $2\times\left[\frac{126t^3}{3}-\frac{14t^5}{5}\right]_0^3 = 2\times(42\times3^3-2.8\times3^5)=907.2$	ddM1 A1
	Or $\left[\frac{126t^3}{3}-\frac{14t^5}{5}\right]_{-3}^3 = (42\times3^3-2.8\times3^5)-(42\times(-3)^3-2.8\times(-3)^5)=907.2$	
		(5) **[13 marks]**

(a)

B1 At $t=1 \Rightarrow x=2,\ y=8$. Score if $(2,8)$ is used in the tangent equation

M1 Attempts $\frac{dy}{dx}=\frac{dy/dt}{dx/dt}$. Condone for this mark slips in attempts at multiplying out $t(9-t^2)$ before differentiating or slips in the product rule (eg bracketing errors) but do not allow $\frac{dy}{dt}=1\times-2t$

A1 $\frac{dy}{dx}=\frac{9-3t^2}{14t}$ or exact equivalent.

M1 A valid attempt at a tangent to C at $t = 1$ Allow $y-"8"="\frac{6}{14}"(x-"2")$

A1 Allow $k(3x-7y+50=0)$ where k is an integer

(b)

B1 $A=(-5,0)$ Allow $x=-5$ and $A=-5$

M1 For attempting to solve $t(9-t^2)=0$ to produce a non zero value for t and substitute the value into $x=7t^2-5$

A1 $B=(58,0)$ Allow $x=58$ and $B=58$

(c)

M1 Attempts area = $\int y\frac{dx}{dt}dt$. Do not be concerned with limits

A1 Area $=\int(126t^2-14t^4)dt$. Do not be concerned with limits but it must be multiplied out

Alternatively (rare) accept $\int 14t^2(9-t^2)\,dt$ by parts with u' being one of $14t^2$ or $(9-t^2)$ and v being the other

M1 Integrates to a form At^3+Bt^5 $(+c)$. Condone slips on the coefficients only

ddM1 A full method to find the area of R using their limits. Accept either $2\times[....]_0^3$ or $[....]_{-3}^3$

Dependent upon both M's

A1 907.2 (units^2) or equivalent 4536/5

Attempts made from a Cartesian equation:

(a) Allow slips, but the M1 should be awarded for an attempt at the product and chain rules on functions of the type $y=\sqrt{\frac{x+5}{7}}\left(9-\frac{x+5}{7}\right)$ or $y=\frac{(x+5)^{\frac{1}{2}}}{\sqrt{7}}\times\frac{(58-x)}{7}$

or the chain rules on a function of this type $\frac{9}{\sqrt{7}}(x+5)^{\frac{1}{2}}-\frac{1}{7\sqrt{7}}(x+5)^{\frac{3}{2}}$

A1 Scored for $\frac{dy}{dx}=\frac{9}{2\sqrt{7}}(x+5)^{-\frac{1}{2}}-\frac{3}{14\sqrt{7}}(x+5)^{\frac{1}{2}}$ or $\frac{dy}{dx}=\frac{(x+5)^{-\frac{1}{2}}}{2\sqrt{7}}\times\frac{(58-x)}{7}-\frac{(x+5)^{\frac{1}{2}}}{7\sqrt{7}}$ oe

(c)

Alt I (c) **Cartesian**	$\int y\,dx=\int\sqrt{\frac{x+5}{7}}\left(9-\frac{x+5}{7}\right)dx,=\int\frac{9}{\sqrt{7}}(x+5)^{\frac{1}{2}}-\frac{1}{7\sqrt{7}}(x+5)^{\frac{3}{2}}\,dx$	M1, A1
	$\frac{6}{\sqrt{7}}(x+5)^{\frac{3}{2}}-\frac{2}{35\sqrt{7}}(x+5)^{\frac{5}{2}}$	M1
	$2\times\left[\frac{6}{\sqrt{7}}(x+5)^{\frac{3}{2}}-\frac{2}{35\sqrt{7}}(x+5)^{\frac{5}{2}}\right]_{-5}^{58}=907.2$	ddM1 A1
Alt II (c) **Cartesian**	Via parts $=\int\frac{(x+5)^{\frac{1}{2}}}{\sqrt{7}}\times\frac{(58-x)}{7}dx$ with $u=\frac{(58-x)}{7}$ $\frac{dv}{dx}=\frac{(x+5)^{\frac{1}{2}}}{\sqrt{7}}$	M1, A1
	Integrates by parts to the form $=\frac{2(58-x)(x+5)^{\frac{3}{2}}}{21\sqrt{7}}+\frac{4(x+5)^{\frac{5}{2}}}{105\sqrt{7}}$	M1
	$2\times\left[=\frac{2(58-x)(x+5)^{\frac{3}{2}}}{21\sqrt{7}}+\frac{4(x+5)^{\frac{5}{2}}}{105\sqrt{7}}\right]_{-5}^{58}=907.2$	ddM1 A1

M1 A full attempt to get y in terms of x AND forms $\int y\,dx$

A1 A correct expression for the area and in a form that can be integrated. If by parts is used the correct selection must be made for u and v'

M1 Raises powers correctly to a form $P(x+5)^{\frac{3}{2}}+Q(x+5)^{\frac{5}{2}}$

Or by parts $C(58-x)(x+5)^{\frac{3}{2}}+D(x+5)^{\frac{5}{2}}$

ddM1 A full method to find the area of R using their limits. Dependent upon both M's

Question Number	Scheme	Marks
13 (a)	$\dfrac{dV}{dr}=4\pi r^2$	B1
		(1)
(b)	Uses $\dfrac{dV}{dt}=\dfrac{dV}{dr}\times\dfrac{dr}{dt}\Rightarrow\dfrac{20}{V\left(0.05t+1\right)^3}=4\pi r^2\times\dfrac{dr}{dt}$	M1
	$\Rightarrow\dfrac{dr}{dt}=\dfrac{15}{4\pi^2r^5\left(0.05t+1\right)^3}$	dM1, A1
		(3)
(c)	$\dfrac{dV}{dt}=\dfrac{20}{V\left(0.05t+1\right)^3}\Rightarrow\int V\,dV=\int\dfrac{20dt}{\left(0.05t+1\right)^3}$	B1
	$\dfrac{V^2}{2}=\dfrac{20(0.05t+1)^{-2}}{-0.1}+c$	M1M1A1
	Sub $V=1$ when $t=0\Rightarrow 0.5=-200+c\Rightarrow c=200.5$	M1
	$\Rightarrow V^2=401-\dfrac{400}{\left(0.05t+1\right)^2}$	A1
		(6)
(d)	Sub $t=20$ into $\left(V^2\right)=401-\dfrac{400}{\left(1+1\right)^2}\left(=301\right)$	M1
	Sub $V=\sqrt{301}$ into $V=\dfrac{4}{3}\pi r^3\Rightarrow r=\text{awrt }1.61\text{(m)}$	dM1,A1
		(3)
		[13 marks]

(a)

B1 $\dfrac{dV}{dr}=4\pi r^2$ Do not accept $\dfrac{dV}{dr}=\dfrac{4}{3}\pi\times 3r^2$

(b)

M1 Uses $\dfrac{dV}{dt}=\dfrac{dV}{dr}\times\dfrac{dr}{dt}$ or equivalent correctly. (It is not enough to just state this)

Follow through on their $\dfrac{dV}{dr}$

dM1 Makes $\dfrac{dr}{dt}$ the subject and attempts to replace V with $\frac{4}{3}\pi r^3$

A1 $\dfrac{dr}{dt}=\dfrac{15}{4\pi^2r^5\left(0.05t+1\right)^3}$ Allow $\dfrac{dr}{dt}=\dfrac{A}{B\pi^2r^5\left(0.05t+1\right)^3}$ or $\dfrac{dr}{dt}=\dfrac{A}{B\pi^2r^5\left(\frac{1}{20}t+1\right)^3}$

where A and B are integers and $\dfrac{A}{B}$ cancels to $\dfrac{15}{4}$

(c)

B1 Separates variables to achieve $\int V\,\mathrm{d}V = \int \frac{20\mathrm{d}t}{(0.05t+1)^3}$ or equivalent (with or without the integral sign)

M1 Integrates lhs to a form aV^2 with or without a constant

M1 Integrates rhs to a form $b(0.05t+1)^{-2}$ with or without a constant

A1 $\frac{V^2}{2} = \frac{20(0.05t+1)^{-2}}{-0.1} + c$ or equivalent including a constant

M1 Substitutes $V = 1$ and $t = 0$ into an integrated expression of the form $pV^2 = q(0.05t+1)^n + c$, where n is an integer, to find the constant c

A1 $V^2 = 401 - \frac{400}{(0.05t+1)^2}$ or equivalent $V^2 = 401 - \frac{160000}{(t+20)^2}$

…………………………………………………………………………………………………..

Note that there is a solution to part (c) via definite integration that marks similarly

$$\int_1^V V\,\mathrm{d}V = \int_0^t \frac{20\mathrm{d}t}{(0.05t+1)^3} \Rightarrow \left[\frac{V^2}{2}\right]_1^V = \left[\frac{20(0.05t+1)^{-2}}{-0.1}\right]_0^t$$ is B1 M1 M1 A1 as before

with the next M1 scored when $\frac{V^2}{2} - \frac{1}{2} = \frac{20(0.05t+1)^{-2}}{-0.1} + 200$ oe

……………………………………………………………………………………………………

(d)

M1 Substitutes $t = 20$ into their equation for V^2 which includes a numerical constant and finds a value for V or V^2 (This cannot be scored for impossible values ie when $V^2 < 0$)

dM1 Substitutes their V into $V = \frac{4}{3}\pi r^3 \Rightarrow r = ...$ Alt substitutes their V^2 into $V^2 = \frac{16}{9}\pi^2 r^6 \Rightarrow r = ...$

A1 cso $r = \text{awrt } 1.61(\text{m})$ in

Watch for candidates who solve (d) using the differential equation $\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{15}{4\pi^2 r^5 (0.05t+1)^3}$ and find r in terms of t. This is an acceptable method (although unlikely) but do consider this kind of solution carefully.

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