Mark Scheme (Results)

January 2016

Pearson Edexcel International A Level in Core Mathematics C34 (WMA02/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 125.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- d… or dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper or ag- answer given
- $\sqsubset$ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however,

the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by ‘MR’ in the body of the script.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x = ...

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x = ...

2. Formula

Attempt to use correct formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c,$ $q \neq 0$, leading to x = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

Question Number	Scheme	Marks
1	$(3-2x)^{-4} = 3^{-4}\left(1-\frac{2}{3}x\right)^{-4}$ $\quad 3^{-4}$ or $\frac{1}{81}$	B1
	$=\frac{1}{81}\times\underline{\left(1+(-4)\left(-\frac{2}{3}x\right)+\frac{(-4)(-5)}{2}\left(-\frac{2}{3}x\right)^2+...\right)}$	<u>M1A1</u>
	$=\frac{1}{81}+\frac{8}{243}x+\frac{40}{729}x^2+...$	A1
		(4 marks)
	Alternative: $(3-2x)^{-4} = 3^{-4}+(-4)(3)^{-5}(-2x)+\frac{(-4)(-5)}{2}(3)^{-6}(-2x)^2+...$	B1 M1 A1
	$=\frac{1}{81}+\frac{8}{243}x+\frac{40}{729}x^2+...$	A1
		(4 marks)

B1 For taking out a factor of 3^{-4}

Evidence would be seeing either 3^{-4} or $\frac{1}{81}$ before the bracket.

M1 For the form of the binomial expansion with $n=-4$ and a term of (kx)

To score M1 it is sufficient to see just the second and third term with the correct coefficient multiplied by the correct power of *x*. Condone sign slips. Look for $...+(-4)(kx)+\frac{(-4)(-5)}{2!}(kx)^2...$

A1 Any (unsimplified) form of the binomial expansion. Ignore the factor before the bracket.

The bracketing must be correct but it is acceptable for them to recover from "missing" brackets for full marks.

Look for $1+(-4)\left(-\frac{2}{3}x\right)+\frac{(-4)(-5)}{2}\left(-\frac{2}{3}x\right)^2+$ or $1+\frac{8}{3}x+\frac{40}{9}x^2+$

A1 cao $=\frac{1}{81}+\frac{8}{243}x+\frac{40}{729}x^2+....$ Ignore any further terms.

Alternative

B1 For seeing either 3^{-4} or $\frac{1}{81}$ as the first term

M1 It is sufficient to see the second and third term (unsimplified or simplified) condoning missing brackets.

ie. Look for $...+(-4)(3)^{-5}(kx)+\frac{(-4)(-5)}{2}(3)^{-6}(kx)^2$

A1 Any (un simplified) form of the binomial expansion. $...+(-4)(3)^{-5}(-2x)+\frac{(-4)(-5)}{2}(3)^{-6}(-2x)^2$

A1 Must now be simplified cao

Question Number	Scheme	Marks
2 (a)	$\text{cosec}^2 x - \text{cosec}x - 12 = 0$ or $k = -12$	B1 [1]
(b)	$\text{cosec}^2 x - \text{cosec}\, x - 12 = 0$ so $(\text{cosec}x - 4)(\text{cosec}x + 3) = 0 \Rightarrow \text{cosec}x = ...$	M1
	$\sin x = \frac{1}{4}$ or $-\frac{1}{3}$	dM1
	$\Rightarrow x = 14.5°$ or $165.5°$ or $340.5°$ or $199.5°$	dM1, A1 A1 [5]
		(6 marks)

(a)

B1: Accept $\text{cosec}^2 x - \text{cosec}x - 12 = 0$ or $k = -12$. No working is required.
If they write $\text{cosec}^2 x - \text{cosec}x - 12 = 0$ followed by $k = 12$ allow isw

(b)

M1 Solves quadratic in cosec x by any method – factorising, formula (accept answers to 1 dp), completion of square. Correct answers (for cosec x of 4 and –3) imply this M mark.
Quadratic equations that have 'imaginary' roots please put into review.

dM1 Uses $\sin x = \frac{1}{\text{cosec}x}$ by taking the reciprocal of at least one of their previous answers
This is dependent upon having scored the first M1

dM1 For using arcsin to produce one answer inside the range 0 to 360 from their values.
Implied by any of $14.5°$ or $165.5°$ or $340.5°$ or $199.5°$ following (cosec x -4) (cosec x+3) =0

A1 Two correct answers inside the range 0 to 360

A1 All four answers in the range, $x = \text{awrt}\, 14.5°\ 165.5°\ 340.5°\ 199.5°$
Any extra solutions in the range withhold the last A mark.
Ignore any solutions outside the range $0 \leqslant x \leqslant 360°$

....

Radian solutions will be unlikely, but could be worth dM1 for one solution and dM1A1 A0 for all four solutions (maximum penalty is 1 mark) but accuracy marks are awarded for solutions to 3dp
FYI: Solutions awrt are 0.253, 2.889, 3.481, 5.943

....

The first two M marks may be achieved 'the other way around' if a candidate uses $\text{cosec}x = \frac{1}{\sin x}$ in line 1 and produces a quadratic in $\sin x$.

Award M1 for using $\text{cosec}x = \frac{1}{\sin x}$ (twice) and producing a quadratic in $\sin x$ and dM1 for solving as above.

Question Number	Scheme	Marks
3	Differentiates wrt x $\quad 3^x \ln 3 + 6\underline{\underline{\frac{dy}{dx}}} = \underline{\frac{3}{2}y^2 + 3xy\frac{dy}{dx}}$	B1 $\underline{\underline{\text{B1}}}$, $\underline{\text{M1, A1}}$
	Substitutes (2, 3) **AND** rearranges to get $\frac{dy}{dx}$ $\Rightarrow 9\ln 3 + 6\frac{dy}{dx} = \frac{27}{2} + 18\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{9\ln 3 - \frac{27}{2}}{12} = \frac{6\ln 3 - 9}{8}, = \frac{-9 + \ln 729}{8}$	M1 A1, A1
		(7)
		(7 marks)

B1 Differentiates $3^x \to 3^x \ln 3$ or $e^{x\ln 3} \to e^{x\ln 3}\ln 3$

B1 Differentiates $6y \to 6\frac{dy}{dx}$

M1 Uses the product rule to differentiate $\frac{3}{2}xy^2$. Evidence could be sight of $\underline{\frac{3}{2}y^2 + kxy\frac{dy}{dx}}$

If the rule is quoted it must be correct. It could be implied by u=.., u'=.., v =.., v'=.. followed by their vu'+uv'. For this M to be scored y^2 must differentiate to $ky\frac{dy}{dx}$, it cannot differentiate to $2y$.

A1 A completely correct differential of $\frac{3}{2}xy^2$. It need not be simplified.

M1 Substitutes $x = 2, y = 3$ into their expression containing a derivative to find a 'numerical' value for $\frac{dy}{dx}$

The candidate may well have attempted to change the subject. Do not penalise accuracy errors on this method mark

A1 Any correct numerical answer in the form $\frac{p\ln q - r}{s}$ where p, q, r and s are constants e.g. $\frac{9\ln 3 - \frac{27}{2}}{12}$

A1 Exact answer. Accept either $\frac{-9 + \ln 729}{8}$ or $\frac{\ln 729 - 9}{8}$

....

Note: There may be candidates who multiply by 2 first and start with $2\times 3^x + 12y = 3xy^2$

This is perfectly acceptable and the mark scheme can be applied in a similar way.

....

Question Number	Scheme	Marks
4(a)	$(V) = \pi\int_{-1}^{\frac{2}{3}} \frac{4}{(4+3x)^2}\,dx$	B1
	$(\pi)\int \frac{4}{(4+3x)^2}\,dx = (\pi)\left(-\frac{4}{3}(4+3x)^{-1}\right)$	M1A1
	$= (\pi)\left[-\frac{4}{3}(4+3x)^{-1}\right]_{-1}^{\frac{2}{3}} = (\pi)\left[-\frac{4}{3}(4+2)^{-1} - -\frac{4}{3}(4-3)^{-1}\right]$	M1
	$= \frac{10}{9}\pi$	A1 [5]
(b)	Length scale factor is 9 so volume scale factor is 9^3 So volume $= 9^3 \times \frac{10}{9}\pi = 810\pi$ or $2545\,(\text{cm}^3)$	M1 A1 [2]
		(7 marks)

(a)

B1 Need a correct statement including π and correct limits and dx. Allow $(V) = \pi\int_{-1}^{\frac{2}{3}} \left(\frac{2}{4+3x}\right)^2 dx$

Allow if candidate initially writes down $V = \pi\int \left(\frac{2}{(4+3x)}\right)^2 dx$ attempts to integrate and later uses the correct limits either way around. Also allow if the π is later multiplied by their $\int_{-1}^{\frac{2}{3}} \left(\frac{2}{4+3x}\right)^2 dx$

M1 Uses substitution or reverse chain rule to do integral achieving $\left(k(4+3x)^{-1}\right)$

A1 For $-\frac{4}{3}(4+3x)^{-1}$ They do not need π or the limits

M1 Substitutes the correct limits in a changed/integrated function and subtracts (either way around)

A1 This answer or equivalent fraction. Accept answer with recurring decimals ie $1.\dot{1}\pi$

(b)

M1 Attempts to multiply their answer in (a) by 729. May be implied by $\frac{10}{9}\pi \to 810$ (missing the π)

This may be implied by $(a) \times \left(\frac{15}{1+\frac{2}{3}}\right)^3$

A1 Any correct equivalent awrt 2540 or 2550 or 810π

Question Number	Scheme	Marks
5. (a)	$f(1) = -3, \quad f(2) = 2$	M1
	Sign change (and as $f(x)$ is continuous) therefore a root α lies in the interval $[1, 2]$	A1
		[2]
(b)	$f(x) = -x^3 + 4x^2 - 6 = 0 \Rightarrow x^2(4-x) = 6$	M1
	$\Rightarrow x^2 = \left(\frac{6}{4-x}\right)$ and so $x = \sqrt{\left(\frac{6}{4-x}\right)}$ *	A1*
		[2]
(c)	$x_2 = \sqrt{\left(\frac{6}{4-1.5}\right)}$	M1
	$x_2 = \text{awrt } 1.5492$,	A1
	$x_3 = \text{awrt } 1.5647$ **, and** $x_4 = \text{awrt } 1.5696 / 1.5697$	A1
		[3]
(d)	$f(1.5715) = -0.00254665...,\quad f(1.5725) = 0.0026157969$	
	Sign change (and as $f(x)$ is continuous) therefore a root α lies in the interval $[1.5715, 1.5725] \Rightarrow \alpha = 1.572$ (3 dp)	M1A1
		[2]
		(9 marks)

(a)

M1 Attempts to evaluate **both** $f(1)$ and $f(2)$ and achieves at least one of $f(1) = -3$ **or** $f(2) = 2$

If a smaller interval is chosen, eg 1.57 and 1.58, the candidate must refer back to the region 1 to 2

A1 Requires (i) both $f(1) = -3$ **and** $f(2) = 2$ correct,

(ii) sign change stated or equivalent Eg $f(1) \times f(2) < 0$ and

(iii) some form of conclusion which may be : or "so result shown" or qed or tick or equivalent

(b)

M1 Must either state f(x) = 0 or set $-x^3 + 4x^2 - 6 = 0$ before writing down at least the line equivalent to $\pm x^2(x-4) = \pm 6$

A1* Completely correct with all signs correct. There is no requirement to show $\frac{-6}{4-x} \to \frac{6}{x-4}$

Expect to see a minimum of the equivalent to $x^2 = \left(\frac{-6}{4-x}\right)$ and $x = \sqrt{\left(\frac{6}{x-4}\right)}$

Alternative working backwards

M1 **S**tarts with answer and squares, multiplies across and expands

$x = \sqrt{\left(\frac{6}{4-x}\right)} \Rightarrow x^2 = \frac{6}{4-x} \Rightarrow x^2(4-x) = 6 \Rightarrow 4x^2 - x^3 = 6$

A1 Completely correct $-x^3 + 4x^2 - 6 = 0$ **and** states "therefore f(x) = 0" or similar

(c) Ignore any reference to labelling. Mark as the first, second and third values given.

M1 An attempt to substitute $x_0 = 1.5$ into the iterative formula. Eg. Sight of $\sqrt{\left(\frac{6}{4-1.5}\right)}$ or $x_2 = \text{awrt}1.55$

A1 $x_2 = \text{awrt } 1.5492$

A1 **Both** $x_3 = \text{awrt } 1.5647$ **and** $x_4 = \text{awrt } 1.5696$ ***or*** 1.5697

(d)

M1 Choose suitable interval for *x*, e.g. $[1.5715,\ 1.5725]$ and at least one attempt to evaluate f(*x*) not the iterative formula. A minority of candidate may choose a tighter range which should include1.57199 (alpha to 5dp). This would be acceptable for both marks, provided the conditions for the A mark are met. Continued iteration is M0

A1 Needs

(i) both evaluations correct to 1 sf, (either rounded or truncated)

Eg $f(1.5715) = -0.003$ rounded $f(1.5715) = -0.002$ truncated

(ii) sign change stated or equivalent Eg $f(a) \times f(b) < 0$ and

(iii)some form of conclusion which may be : or “so result shown” or qed or tick or equivalent

x	f(x)
1	-3
1.1	-2.491
1.2	-1.968
1.3	-1.437
1.4	-0.904
1.5	-0.375
1.6	0.144
1.7	0.647
1.8	1.128
1.9	1.581
2	2

x	f(x)
1.5715	-0.002546651
1.5716	-0.002030342
1.5717	-0.001514047
1.5718	-0.000997766
1.5719	-0.0004815
1.572	3.4752E-05
1.5721	0.00055099
1.5722	0.001067213
1.5723	0.001583422
1.5724	0.002099617
1.5725	0.002615797

Question Number	Scheme	Marks
6(a)	$320\ (°C)$	B1 [1]
(b)	$T=180 \Rightarrow 300e^{-0.04t}=160, \Rightarrow e^{-0.04t}=\frac{160}{300}(\text{awrt } 0.53)$	M1, A1
	$t=\frac{1}{-0.04}\ln\left(\frac{160}{300}\right)$ *or* $\frac{1}{0.04}\ln\left(\frac{300}{160}\right)$	dM1
	15.7 (minutes) cao	A1cso [4]
(c)	$\frac{dT}{dt}=(-0.04)\times 300e^{-0.04t} = (-0.04)\times(T-20)$	M1 A1
	$=\frac{20-T}{25}$ *	A1* [3]
		(8 marks)
Alt **(b)**	Puts T = 180 so 180 = $300e^{-0.04t}+20$ and $300e^{-0.04t}=160$	M1
	$\ln 300 - 0.04t = \ln 160 \Rightarrow t = ..,\ \frac{\ln 300 - \ln 160}{0.04}$	dM1, A1
	15.7 (minutes) cao	A1cso [4]

(a)

B1 320 cao - do not need ° C

(b)

M1 Substitutes T= 180 and proceeds to a form $Ae^{-0.04t}=B$ or $Ce^{0.04t}=D$

Condone slips on the power for this mark. For example condone $Ae^{-0.4t}=B$

A1 For $e^{-0.04t}=\frac{160}{300}$ or $e^{0.04t}=\frac{300}{160}$ or exact equivalent such as $e^{-0.04t}=\frac{8}{15}$

Accept decimals here $e^{-0.04t}=0.53..$ or $e^{0.04t}=1.875$

dM1 Dependent upon having scored the first M1, it is for moving from $e^{kt}=c, c>0 \Rightarrow t=\frac{\ln c}{k}$

A1 15.7 correct answer and correct solution only. Do not accept awrt

(c)

M1 Differentiates to give $\frac{dT}{dt}=ke^{-0.04t}$. Condone $\frac{dT}{dt}=ke^{-0.4t}$ following $T=300e^{-0.4t}+20$

This can be achieved from $T=300e^{-0.04t}+20 \Rightarrow t=\frac{1}{-0.04}\ln\left(\frac{T-20}{300}\right) \Rightarrow \frac{dt}{dT}=\frac{k}{(T-20)}$ for M1

A1 Correct derivative and correctly eliminates t to achieve $\frac{dT}{dt}=(-0.04)\times(T-20)$ oe

If candidate changes the subject it is for $\frac{dt}{dT}=\frac{-25}{(T-20)}$ oe

Alternatively obtains the correct derivative, substitutes T in $\frac{dT}{dt}=\frac{20-T}{25} \to \frac{dT}{dt}=-12e^{-0.04t}$ and compares. To score the A1* under this method there must be a statement.

A1* Obtains printed answer correctly – no errors

Question Number	Scheme	Marks
7.(a)	$\frac{dy}{dx}=\frac{(x^2+1)\times\frac{6x}{x^2+1}-6x\times\ln(x^2+1)}{(x^2+1)^2}$ Or $\frac{6x\left(1-\ln(x^2+1)\right)}{(x^2+1)^2}$	M1A1 [2]
(b)	$\frac{dy}{dx}=0 \Rightarrow \cancel{(x^2+1)}3\frac{2x}{\cancel{(x^2+1)}}-3\ln(x^2+1)(2x)=0$	M1
	$\ln(x^2+1)=1$ *so* $x=\sqrt{e-1}$	M1A1
	$y=\frac{3}{e}$	ddM1A1 [5]
(c)	$\frac{3}{2}\ln 2$ or 1.0397	B1 [1]
(d)	$\frac{1}{2}\times 1\times\{\underline{\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots}\}$	B1 oe
	$\frac{1}{2}\times 1\times\left\{\underline{0+\frac{3}{10}\ln 10+2\left(\frac{3}{2}\ln 2+\frac{3}{5}\ln 5\right)}\right\}$	M1
	$\left\{=\tfrac{1}{2}(0.6907755279..+4.010767..)\right\}$	
	= 2.351 (awrt 4 sf)	A1 [3]
		(11 marks)

(a)

M1 Applies the Quotient rule, a form of which appears in the formula book, to $\frac{3\ln(x^2+1)}{(x^2+1)}$

If the formula is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted nor implied by their working, meaning that terms are written out $u=3\ln\left(x^2+1\right), v=x^2+1, u'=..,v'=...$ followed by their $\frac{vu'-uv'}{v^2}$, then only accept answers of the form

$\frac{dy}{dx}=\frac{(x^2+1)A\frac{x}{x^2+1}-Bx\ln(x^2+1)}{(x^2+1)^2}$ *or* $\frac{Ax-Bx\ln(x^2+1)}{(x^2+1)^2}$. Condone invisible brackets for the M.

Alternatively applies the product rule with $u=3\ln(x^2+1), v=\left(x^2+1\right)^{-1}$

If the formula is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted nor implied by their working, meaning that terms are written out $u=3\ln(x^2+1), v=\left(x^2+1\right)^{-1}, u'=..,v'=...$ followed by their $vu'+uv'$, then only accept answers of the form

$\left(x^2+1\right)^{-1}\times A\frac{x}{x^2+1}+\left(x^2+1\right)^{-2}\times Bx\ln(x^2+1)$.

Condone invisible brackets for the M.

A1 Any fully correct (unsimplified) form of $f'(x)$. Remember to isw.

Using quotient rule look for variations of $\frac{dy}{dx} = \frac{(x^2+1)\times\frac{6x}{x^2+1} - 6x\times\ln(x^2+1)}{(x^2+1)^2}$

Using the product rule look for $\frac{dy}{dx} = (x^2+1)^{-1}\times\frac{6x}{x^2+1} - (x^2+1)^{-2}\times 2x\times 3\ln(x^2+1)$

(b)

M1 Setting their numerator (with more than one term) of their $f'(x) = 0$ and proceeds to a form that does not include fractional terms.
If the product rule has been applied in (a) they also need an equation without fractions to score this.
Allow all marks in part (b) if **denominato**r was incorrect in (a), for example v rather than v^2 in their quotient rule.

M1 Proceeds using correct work to $\ln(x^2+1) = A \Rightarrow x = ..$

A1 $x = \sqrt{e-1}$ achieved from a $\pm$ correct numerator. Ie condone it arising from $\pm(vu' - uv')$

dM1 Dependent upon both M's having been scored. It is for substituting in their value of x (which may be decimal) and finding a value of y from the correct function

A1 Correct solution only $y = \frac{3}{e}$ and no other solution for $x > 0$. Ignore solutions $x \leqslant 0$

(c)

B1 $\frac{3}{2}\ln 2$ or 1.0397 or exact equivalent such as $\frac{1}{2}\ln 8$

(d)

B1 for h = 1. This is implied by $\frac{1}{2}\times 1$ or $\frac{1}{2}$ outside the (main) bracket

M1 For inside the brackets: $0 + \frac{3}{10}\ln 10 + 2\left(\frac{3}{2}\ln 2 + \frac{3}{5}\ln 5\right)$ You can follow through on their $\frac{3}{2}\ln 2$
The decimal equivalent is $0 + 0.691 + 2(1.040 + 0.966)$
Allow if you have an invisible bracket. That is you see $\frac{1}{2}\times 0 + \frac{3}{10}\ln 10 + 2\left(\frac{3}{2}\ln 2 + \frac{3}{5}\ln 5\right)$

A1 awrt 2.351

Question Number	Scheme	Marks
8(a)	**Either** $\mathrm{f}(\theta)=9\cos^2\theta+\sin^2\theta=9\cos^2\theta+1-\cos^2\theta$	M1
	$=8\cos^2\theta+1=8\dfrac{(\cos 2\theta+1)}{2}+1$	M1
	$=5+4\cos 2\theta$	A1 [3]
	Or $\mathrm{f}(\theta)=9\dfrac{(\cos 2\theta+1)}{2}+1\dfrac{(1-\cos 2\theta)}{2}$	M1 M1
	$=5+4\cos 2\theta$	A1 [3]
(b)	**Either :Way1** splits as $\int_0^{\frac{\pi}{2}} a\theta^2\,\mathrm{d}\theta+\int_0^{\frac{\pi}{2}} b\theta^2\cos 2\theta\,\mathrm{d}\theta$	
	$\int_0^{\frac{\pi}{2}} b\theta^2\cos 2\theta\,\mathrm{d}\theta=\ ...\theta^2\sin 2\theta\pm\int...\theta\sin 2\theta\mathrm{d}\theta$	M1
	$=\ =..\theta^2\sin 2\theta\pm..\theta\cos 2\theta\pm\int..\cos 2\theta\mathrm{d}\theta$	dM1
	Integral = $\underline{\underline{\left[2\theta^2\sin 2\theta+2\theta\cos 2\theta-\sin 2\theta\right]}}+\frac{5}{3}\theta^3$	$\underline{\underline{\text{A1}}}$ **B1ft**
	Use limits to give $\left[\dfrac{5\left(\frac{\pi}{2}\right)^3}{3}-\pi\right]-[0]\quad=\quad\left[\dfrac{5\pi^3}{24}-\pi\right]$	ddM1 A1 [6]
		(9 marks)
1st 4 marks	**Or: Way 2** $\int_0^{\frac{\pi}{2}}\theta^2\,\mathrm{f}(\theta)\mathrm{d}\theta=\int_0^{\frac{\pi}{2}}\theta^2(a+b\cos 2\theta)\mathrm{d}\theta\ =$	
	$=\theta^2(..\theta\pm..\sin 2\theta)-\int..\theta(..\theta\pm..\sin 2\theta)\mathrm{d}\theta$	M1
	$=\theta^2(...\theta\pm..\sin 2\theta)-..\theta\left(..\theta^2\pm...\cos 2\theta\right)\pm\int.\left(..\theta^2\pm...\cos 2\theta\right)\mathrm{d}\theta$	dM1
	$=\theta^2(5\theta+2\sin 2\theta)-2\theta\left(\dfrac{5\theta^2}{2}-\cos 2\theta\right)+\left(\dfrac{5\theta^3}{3}-\sin 2\theta\right)$	A1 **B1ft**
1st 4 marks	**Or: Way 3 Way 2 that goes back to Way One**	
	$\int_0^{\frac{\pi}{2}}\theta^2(a+b\cos 2\theta)\mathrm{d}\theta=\theta^2(..\theta\pm..\sin 2\theta)-\int..\theta(..\theta\pm..\sin 2\theta)\mathrm{d}\theta$	M1
	$=\theta^2(..\theta\pm..\sin 2\theta)-\left(\int...\theta^2\mathrm{d}\theta\right)\pm\int..\theta\sin 2\theta\mathrm{d}\theta$	
	$=\theta^2(..\theta\pm..\sin 2\theta)-\left(\int...\theta^2\mathrm{d}\theta\right)\pm..\theta\cos 2\theta\pm\int..\cos 2\theta\mathrm{d}\theta$	dM1
	$=\theta^2(5\theta+2\sin 2\theta)-\dfrac{10}{3}\theta^3+2\theta\cos 2\theta-\sin 2\theta$	A1 **B1ft**

(a)

M1 Uses $\sin^2\theta = 1-\cos^2\theta$ or $\cos^2\theta = 1-\sin^2\theta$ to reach an expression in either $\sin^2\theta$ or $\cos^2\theta$

M1 Attempts to use the double angle formula $\cos 2\theta = \pm1 \pm 2\sin^2\theta$ or $\cos 2\theta = \pm2\cos^2\theta \pm 1$ to convert their expression in $\sin^2\theta$ or $\cos^2\theta$ to form an expression $a + b\cos 2\theta$

A1 cao $= 5 + 4\cos 2\theta$

....

Alternative

M1 One attempted application of double angle formula on either $\sin^2\theta$ or $\cos^2\theta$. See above for rules

M1 A second attempted applications of double angle formula to form an expression $a + b\cos 2\theta$

A1 cao $= 5 + 4\cos 2\theta$

(b) Note: On e pen this is marked up M1 M1 A1 M1 M1 A1. We are scoring it M1 M1 A1 B1 M1 A1

M1 An attempt at using integration by parts the correct way around.
IF THE CANDIDATE DOES NOT STATE OR IMPLY AN INCORRECT FORMULA ACCEPT

In Way One look for $\int b\theta^2 \cos 2\theta \, d\theta \rightarrow \pm..\theta^2 \sin 2\theta \pm \int ..\theta \sin 2\theta d\theta$

In Way Two look for $\int \theta^2 (a + b\cos 2\theta) d\theta \rightarrow \left[\theta^2(..\theta \pm ..\sin 2\theta)\right] - \int ..\theta(..\theta \pm ..\sin 2\theta) d\theta$

dM1 Dependent upon M1 having been scored, it is for an attempted use of integration by parts the correct way around for a second time.

In Way One look for

$$\rightarrow \pm..\theta^2 \sin 2\theta \pm ..\theta \cos 2\theta \pm \int ..\cos 2\theta d\theta$$

In Way Two look for

$$\rightarrow \left[\theta^2(..\theta \pm ..\sin 2\theta)\right] - ..\theta(..\theta^2 \pm ..\cos 2\theta) \pm \int (..\theta^2 \pm ..\cos 2\theta) d\theta$$

Way 3 : You may see a candidate multiplying out their second integral and reverting to a type one integral.

$$\rightarrow \left[\theta^2(..\theta \pm ..\sin 2\theta)\right] \pm ..\theta^3 \pm ...\theta \cos 2\theta \pm \int ..\cos 2\theta \, d\theta$$

A1 cao $\left[2\theta^2 \sin 2\theta + 2\theta \cos 2\theta - \sin 2\theta\right]$ Accept in any unsimplified form

B1ft $\int a\theta^2 \, d\theta \rightarrow a\dfrac{\theta^3}{3}$ It is scored for the term independent of the trigonometrical terms.

ddM1 Dependent upon both previous M's. For using **both** limits although you may not see the 0.
A decimal answer of 3.318 following correct working implies this mark

A1 cso. Note that a correct answer does not necessarily imply a correct solution

Question Number	Scheme	Marks
9(a)	$\dfrac{3x^2-4}{x^2(3x-2)} \equiv \dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{3x-2}$	
	$\dfrac{2}{x^2}, \dfrac{-6}{3x-2}$ $(B = 2,\ C = -6)$	B1, B1,
	$3x^2-4 \equiv Ax(3x-2)+B(3x-2)+Cx^2 \Rightarrow A = ..$	M1
	$\dfrac{3}{x}$ $(A = 3)$ is one of the fractions	A1
		[4]
(b)	$\displaystyle\int \frac{1}{y}\,dy = \int \frac{3x^2-4}{x^2(3x-2)}\,dx$	B1
	$\ln y = \displaystyle\int \frac{A}{x}+\frac{B}{x^2}+\frac{C}{3x-2}\,dx$	M1
	$= A\ln x - \dfrac{B}{x} + \dfrac{C}{3}\ln(3x-2)$ $(+k)$	M1A1ft
	$y = e^{A\ln x - \frac{B}{x} + \frac{C}{3}\ln(3x-2) + D}$ or $y = De^{A\ln x - \frac{B}{x} + \frac{C}{3}\ln(3x-2)}$	M1
	$y = Kx^3(3x-2)^{-2}e^{-\frac{2}{x}}$ or $\dfrac{Kx^3e^{-\frac{2}{x}}}{(3x-2)^2}$ or $\dfrac{e^k x^3 e^{-\frac{2}{x}}}{(3x-2)^2}$ oe	A1cso
		[6] **(10 marks)**

(a)

B1 For either $+\frac{2}{x^2}$ or $\frac{-6}{3x-2}$ being one of the "partial" fractions

B1 For two of the partial fractions being $+\frac{2}{x^2}$ **and** $\frac{-6}{3x-2}$

M1 Need three terms in pfs and correct method either compares coefficients or substitutes a value to obtain A
Look for $3x^2-4 \equiv Ax(3x-2)+B(3x-2)+Cx^2 \Rightarrow A = ..$

A1 $\frac{3}{x}$

(b)

B1: Separates variables correctly. No need for integral signs

M1 Integrates left hand side to give ln*y* and uses their partial fractions from part (a) (may only have two pf 's)

M1 Obtains two ln terms and one reciprocal term on rhs (need not have constant of integration for this mark) (must have 3 pf 's here). Condone a missing bracket on the ln(3*x*-2)

A1ft Correct (unsimplified) answer for rhs for their *A*, *B* and *C* (do not need constant of integration at this stage)

M1 For undoing the logs correctly to get y = …. now need constant of integration.

Accept $y = e^{A\ln x - \frac{B}{x} + \frac{C}{3}\ln(3x-2)+d}$ OR $y = De^{A\ln x - \frac{B}{x} + \frac{C}{3}\ln(3x-2)}$ BUT NOT $y = e^{A\ln x - \frac{B}{x} + \frac{C}{3}\ln(3x-2)} + D$

A1 cso One of the forms of the answer given in the scheme o.e.

....

Special case: For students who use two partial fractions

Very common incorrect solutions using two partial fractions are

$\frac{3x^2-4}{x^2(3x-2)} \equiv \frac{A}{x^2} + \frac{B}{3x-2} = \boldsymbol{\frac{2}{x^2} + \frac{3}{3x-2}}$ using substitution and comparing terms in x^2

or $\frac{3x^2-4}{x^2(3x-2)} \equiv \frac{A}{x^2} + \frac{B}{3x-2} = \boldsymbol{\frac{2}{x^2} + \frac{-6}{3x-2}}$ using substitution

Both of these will scoring B1B1M0A0 in SC in (a)

In part (b) this could score B1, M1 M0 A0 M1 A0 for a total of 5 out of 10.

For the final M1 they must have the correct form $y = e^{-\frac{\dots}{x} + \dots\ln(3x-2)+D}$ or $y = De^{-\frac{\dots}{x} + \dots\ln(3x-2)}$ or equivalent

....

Question Number	Scheme		Marks
10. (a)	$R = \sqrt{34}$		B1
	$\tan\alpha = \frac{5}{3}$		M1
	$\Rightarrow \alpha = 1.03$	awrt 1.03	A1
			[3]
(b)	$3\sin 2x + 5\cos 2x = 4 \Rightarrow \sqrt{34}\sin(2x + 1.03) = 4$		
	$\sin(2x + "1.03") = \frac{4}{"\sqrt{34}"} \quad (= 0.68599...)$		M1
	One solution in range Eg. $2x + "1.03" = 2\pi + \arcsin\left(\frac{4}{"\sqrt{34}"}\right) \Rightarrow x = ...$		M1
	Either x = awrt 3.0 **or** awrt 0.68		A1
	Second solution in range Eg $2x + "1.03" = \pi - \arcsin\left(\frac{4}{"\sqrt{34}"}\right) \Rightarrow x = ...$		M1
	Both x = awrt 2sf 3.0 **and** 0.68		A1
			[5]
(c)	Greatest value is $4\left(\sqrt{34}\right)^2 + 3 = 139$		M1 A1
	Least value is $4(0) + 3 = 3$	.	M1 A1
			[4]
			(12 marks)

(a)

B1 $R=\sqrt{34}$ Condone $\pm\sqrt{34}$

M1 For $\tan\alpha=\pm\frac{5}{3}$ or $\tan\alpha=\pm\frac{3}{5}$ This may be implied by awrt 1.0 rads or awrt 59 degrees

If R is used to find α only accept $\cos\alpha=\pm\frac{3}{their\ R}$ or $\sin\alpha=\pm\frac{5}{their\ R}$

A1 accept $\alpha=$ awrt 1.03; also accept $\sqrt{34}\sin(2x+1.03)$.

If the question is done in degrees only the first accuracy mark is withheld. The answer in degrees (59.04) is A0

(b) On epen this is marked up M1M1M1A1A1. We are scoring it M1M1A1M1A1

M1 For reaching $\sin(2x\pm\text{their }\alpha)=\frac{4}{\text{their }R}$ (Uses part (a) to solve equation)

It may be implied by $(2x\pm\text{their }\alpha)=\arcsin\left(\frac{4}{\text{their }R}\right)$ = 0.75 rads

M1 For an attempt at one solution in the range. It is acceptable to find the negative solution, -0.14 and add π

Look for $2x\pm\text{their }\alpha=2\pi+\arcsin\left(\frac{4}{\text{their }R}\right)\Rightarrow x=..$ (correct order of operations)

Alternatively $2x\pm\text{their }\alpha=\pi-\arcsin\left(\frac{4}{\text{their }R}\right)\Rightarrow x=..$

A1 Awrt 3.0 **o**r awrt 0.68. Condone 3 for 3.0. In degrees accept awrt 38.8 or 172.1

M1 For an attempt at a second solution in the range. This can be scored from their "$\arcsin\left(\frac{4}{\text{their }R}\right)$"

Look for $2x\pm\text{their }\alpha=\pi-\text{their }\arcsin\left(\frac{4}{\text{their }R}\right)\Rightarrow x=..$ (correct order of operations)

Or $2x\pm\text{their }\alpha=2\pi+\text{their }\arcsin\left(\frac{4}{\text{their }R}\right)\Rightarrow x=..$

A1 Awrt 3.0 **AND** awrt 0.68 in radians or awrt 38.8 and awrt 172.1 in degrees. Condone 3 for 3.0

(c) (i)

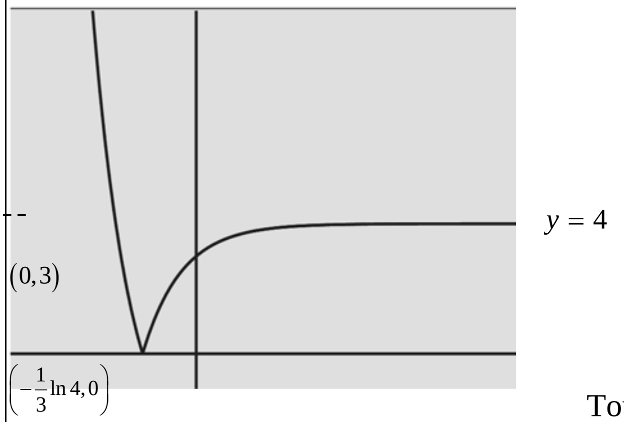
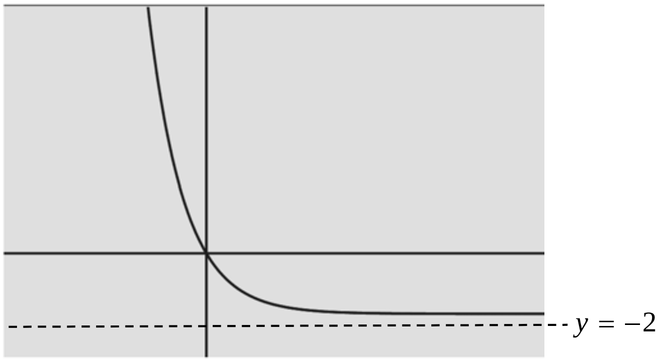
M1 Attempts to find $4(R)^2+3$

A1 139 cao

(c)(ii)

M1 Uses 0 for minimum value. Accept $4(0)^2+3$

A1 3

Question Number	Scheme	Marks
11(a)		
	Shape	B1
	Asymptote $y = 4$	B1
	y intercept $(0,3)$	B1
	Touches x axis at $\left(-\frac{1}{3}\ln 4, 0\right)$	B1
		[4]
(b)		
	Shape	B1
	Asymptote $y = -2$	B1
	Passes though origin	B1
		[3]
(c)	$f(x) > -4$	B1
		[1]
(d)	$y = e^{-3x} - 4 \quad \Rightarrow e^{-3x} = y + 4$	M1
	$\Rightarrow -3x = \ln(y+4)$ and $x =$	dM1
	$f^{-1}(x) = -\frac{1}{3}\ln(x+4)$ or $\ln\frac{1}{(x+4)^{\frac{1}{3}}}$, $\quad (x > -4)$ cao	A1
		[3]
(e)	$fg(x) = e^{-3\ln\left(\frac{1}{x+2}\right)} - 4$	M1
	$= (x+2)^3 - 4$, $\quad = x^3 + 6x^2 + 12x + 4$	dM1, A1
		[3]
		(14 marks)

(a)

B1 For a correct shape. The curve must lie completely in first and second quadrants with a cusp on – ve x axis and approaching an asymptote as x becomes large and positive. The curvature must be correct on both sections. The lh section does not need to extend above the asymptote. On the rh section do not allow if the curve drops below the y intercept.
See practice items for clarification

B1 The asymptote of the curve is given as $y = 4$. Do not award if a second asymptote is given or the curve does not appear to have an asymptote at $y = 4$

B1 y intercept is (0, 3). Allow if given in the body of their answer say as A= .
3 sufficient if given on the y-axis. Condone (3,0) being marked on the correct axis.
Do not award if there are two intercepts

B1 x intercept is $(-1/3 \ln 4, 0)$. Allow if given in the body of their answer say as B= .
$-1/3 \ln 4$ is sufficient if given on the x axis. Do not award if there are two intercepts

(b)

B1 shape – similar to original graph (do not try to judge the stretch)

B1 Equation of the asymptote given as $y = -2$ Do not award if a second asymptote is given or the curve does not appear to have an asymptote here.

B1 Curve passes through O only.

(c)

B1 cao $f(x) > -4$. Accept alternatives such as $(-4, \infty)$
Note that $f(x) \geq -4$ is B0. Accept range or y for $f(x)$

(d)

M1 For an attempt to make x (or a switched y) the subject of the formula. For this to be scored they must make e^{-3x} or e^{3x} as subject. Allow numerical slips.

dM1 This is dependent upon the first M being scored. It is for undoing the exp correctly by using ln's. Condone imaginary brackets for this mark. Accept x being given as a function of y and involving ln's. If the rhs is written $\ln(4x)$ this implies taking the ln of each term which is dM0

A1 This is cao. Accept any correct equivalent. The domain is not required for this mark but the bracket is. Accept y =..

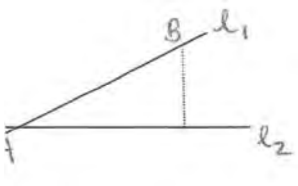
(e)

M1 Correct order of operations. Allow for $e^{\pm 3\ln\frac{1}{(x+2)}} - 4$

dM1 Dependent Method Mark. Simplifies this expression using firstly the power law and then the fact that $e^{\ln ..} = ..$ to reach $fg(x)$ as a function of x.

You may condone sign errors here so tolerate $e^{-3\ln\left(\frac{1}{x+2}\right)} - 4 \rightarrow \frac{1}{(x+2)^3} - 4$

A1 Correct expansion to give this answer.

Question Number	Scheme	Marks
12 (a)	$\begin{pmatrix}12\\-4\\5\end{pmatrix}+\lambda\begin{pmatrix}5\\-4\\2\end{pmatrix}=\begin{pmatrix}2\\2\\0\end{pmatrix}+\mu\begin{pmatrix}0\\6\\3\end{pmatrix}\Rightarrow$ $\begin{matrix}12+5\lambda=\ \ 2\\-4-4\lambda=2+6\mu\\5+2\lambda=3\mu\end{matrix}$ **any two** of these	M1
	Full method to find either λ **or** μ	M1
	(1) $\Rightarrow\lambda=-2$	
	Sub $\lambda=-2$ into (2) to give $\mu=\frac{1}{3}$ (need both)	A1
	Check values in 3rd equation $5+2(-2)=\ 3(\frac{1}{3})$ and make statement eg. True	B1
	Position vector of intersection is $\begin{pmatrix}12\\-4\\5\end{pmatrix}-2\begin{pmatrix}5\\-4\\2\end{pmatrix}$ **OR** $\begin{pmatrix}2\\2\\0\end{pmatrix}+\frac{1}{3}\begin{pmatrix}0\\6\\3\end{pmatrix}=\begin{pmatrix}2\\4\\1\end{pmatrix}$	M1,A1
		[6]
(b)	$\cos\theta=\dfrac{\begin{pmatrix}5\\-4\\2\end{pmatrix}.\begin{pmatrix}0\\6\\3\end{pmatrix}}{\sqrt{5^2+(-4)^2+2^2}\sqrt{6^2+3^2}}=\dfrac{-18}{45}=-0.4$	M1 A1
	So acute angle is 66.4 degrees	A1
		[3]
(c)	When $\lambda=-1$ this gives $\begin{pmatrix}7\\0\\3\end{pmatrix}$ so B lies on l_1	B1
		[1]
		
(d)	**Way 1:** $AB=\sqrt{45}$	M1 A1
	$h=\sqrt{45}\times\sin 66.4$	M1
	h = 6.15	A1
		[4]
	Way 2: $\overrightarrow{XB}:\ =\pm\begin{pmatrix}5\\-2-6\mu\\3-3\mu\end{pmatrix}$	M1A1
	Find $\overrightarrow{XB}.\begin{pmatrix}0\\6\\3\end{pmatrix}=0\Rightarrow\mu=\left(-\frac{1}{15}\right)$ followed by calculation of $\lvert\overrightarrow{XB}\rvert$ by Pythag	M1
	h = 6.15	A1
		[4]
		(14 marks)

(a)

M1 For writing down **any two equations** that give the coordinates of the point of intersection. Accept two of $12+5\lambda = 2$, $-4-4\lambda = 2+6\mu$, $5+2\lambda = 3\mu$ condoning slips.

M1 A full method to find **either** λ **or** μ.

A1 **Both** values correct $\mu = \frac{1}{3}$ and $\lambda = -2$

B1 The correct values must be substituted into **both** sides of the third equation. There must be some minimal statement (a tick will suffice) that the values are the same. This can also be scored via the substitution of $\mu = \frac{1}{3}$ $\lambda = -2$ into **both** of the equations of the lines but there must be the same minimal statement.

M1 Substitutes their value of λ into l_1 to find the coordinates or position vector of the point of intersection. It is dependent upon having scored second method mark. Alternatively substitutes their value of μ into l_2 to find the coordinates or position vector of the point of intersection.

A1 Correct answer only. Accept as a vector or a coordinate. Accept (2, 4, 1) (A correct answer here implies previous M mark). Note that the correct answer can be achieved by solving just the first equation.

(b)

M1 A clear attempt to use the correct formula for $a.b = |a||b|\cos\theta$ (where a and b are the gradient vectors)
Expect to see $5\times0+-4\times6+2\times3 = \sqrt{5^2+(-4)^2+2^2}\times\sqrt{6^2+3^2}\cos\theta$ allowing for slips.

A1 For $\cos\theta = \pm 0.4$. This may be implied by 66.4 or 113.6. Also accept $\cos\theta = \frac{0-24+6}{\sqrt{45}\sqrt{45}}$ oe

A1 cao for awrt 66.4

(c)

B1 States or uses $\lambda = -1$ and checks all 3 coordinates

(d) **Way 1:**

M1 Finds distance AB using a correct method
Using Pythagoras look for $\sqrt{(7-\text{"}2\text{"})^2+(0-\text{"}4\text{"})^2+(3-\text{"}1\text{"})^2}$ or 'one' gradient $\sqrt{5^2+(-4)^2+2^2}$

A1 Correct answer $(AB) = \sqrt{45}$ or $(AB) = 3\sqrt{5}$ or (AB) = awrt 6.71

M1 Reaches $h = \sqrt{45}\times\sin 66.4$ with their values for AB and $\sin\theta$ to find h

A1 awrt 6.15 (allow also if it follows 113.6). The exact answer of $\frac{3}{5}\sqrt{105}$ is fine

Way 2: Setting up a point *X* on l_2

M1 Finds distance XB^2 or vector $\overrightarrow{XB}$ using a correct method. For this to be scored $X = (2, 2+6\mu, 3\mu)$ $B = (7, 0, 3)$ and there must be an attempt at differences

A1 Correct answer $h^2 = |\overrightarrow{XB}|^2 = 45\mu^2+6\mu+38$ or $\overrightarrow{XB} = \pm(-5, 2+6\mu, 3\mu-3)$

M1 Find minimum value of h by completion of square or differentiation giving h =
Alternatively by the vector method uses $\overrightarrow{XB}.\begin{pmatrix}0\\6\\3\end{pmatrix} = 0 \Rightarrow \mu = ..$ followed by substitution of this $\mu\left(=-\frac{1}{15}\right)$
into l_2 to find length of $BX = \left|\pm\begin{pmatrix}-5\\1.6\\-3.2\end{pmatrix}\right|$ using pythagoras' theorem.

A1 awrt 6.15

Question Number	Scheme	Marks
13 (a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y/\mathrm{d}t}{\mathrm{d}x/\mathrm{d}t} = \frac{2\cos t}{-12\sin 2t}$	M1
	$= \frac{2\cos t}{-24\sin t\cos t}$	dM1
	$= \frac{2\cancel{\cos t}}{-24\sin t\cancel{\cos t}} = -\frac{1}{12}\ \text{cosec}\ t$	M1 A1 [4]
(b)	When $t = \frac{\pi}{3}$, $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{12\times \sqrt{3}/2} = \left(-\frac{\sqrt{3}}{18}\right)$	M1 A1
	So Normal has gradient $-\frac{1}{m} = 6\sqrt{3}$	M1
	When $t = \frac{\pi}{3}$, $x = -3$ and $y = \sqrt{3}$	B1
	Equation of normal is $y - \sqrt{3} = 6\sqrt{3}(x+3)$ so $y = 6\sqrt{3}x + 19\sqrt{3}$	M1 A1 [6]
(c)	$x = 6(1-2\sin^2 t) \Rightarrow x = \mathrm{f}(y)$	M1
	So $x = 6 - 3y^2$ or $\mathrm{f}(y) = 6 - 3y^2$	dM1 A1 [3]
(d)	$-2 < y < 2$ or $k = 2$	B1 [1] **(14 marks)**
Alt (a)	Via cartesian must start with $x = A \pm By^2$ or $y = \sqrt{C \pm Dx}$	
	$\frac{\mathrm{d}x}{\mathrm{d}y} = ky$ or $\frac{\mathrm{d}y}{\mathrm{d}x} = b\left(2 - \frac{x}{3}\right)^{-\frac{1}{2}}$ then as before	M1
	followed by correct (double angle) substitution	dM1
Alt (b)	Must start with $x = A \pm By^2$ or $y = \sqrt{C \pm Dx}$	
	$\frac{\mathrm{d}x}{\mathrm{d}y} = -6y$ or $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{6}\left(2 - \frac{x}{3}\right)^{-\frac{1}{2}}$	
	For substituting their $y = \sqrt{3}$ into a $\frac{\mathrm{d}x}{\mathrm{d}y}$ of the form Py	1st M1
	Or alternatively substituting their $x = -3$ into a $\frac{\mathrm{d}y}{\mathrm{d}x}$ of the form $P(Q \pm Rx)^{-\frac{1}{2}}$	
	For using the 'correct numerical' grad of the normal either $-\frac{\mathrm{d}x}{\mathrm{d}y}$ or $-\frac{1}{\mathrm{d}y/\mathrm{d}x}$	2nd M1

(a)

M1 Differentiates both x and y wrt t and establishes $\frac{{dy}/{dt}}{{dx}/{dt}} = \frac{\pm A\cos t}{\pm B\sin 2t}$

They may use any double angle formula for cos first. Condone sign slips in this formula

Eg $\cos 2t = \pm 2\cos^2 t \pm 1$ to get $\frac{{dy}/{dt}}{{dx}/{dt}} = \frac{\pm A\cos t}{\pm B\sin t\cos t}$

dM1 Correct double angle formula used $\sin 2t = 2\sin t\cos t$

In the alternative method the correct double angle formula must have been used

M1 Cancels cos$t$ and replaces 1/sin$t$ by cosec$t$ correctly achieving a form $\frac{dy}{dx} = \lambda \text{cosec}t$

A1 cao $\frac{dy}{dx} = -\frac{1}{12}\text{cosec}t$

(b)

M1 Substitute $t = \frac{\pi}{3}$ into their $\frac{dy}{dx} = \lambda\text{cosec}t$

A1 $\frac{dy}{dx} = -\frac{1}{12\times {\sqrt{3}}/{2}}$ or exact equivalent. It may be implied by normal gradient of $6\sqrt{3}$

Accept decimals here. $\frac{dy}{dx} = -0.096$ or implied by normal gradient of 10.4

M1 Use of negative reciprocal in finding the gradient of the normal.

B1 for x = -3, $y = \sqrt{3}$

M1 Correct method for line equation using their **normal** gradient and their (-3, $\sqrt{3}$) allowing a sign slip on one of their coordinates.

Look for $y - y_1 = -\left.\frac{dx}{dy}\right|_{t=\frac{\pi}{3}}(x - x_1)$ or $x - x_1 = -\left.\frac{dy}{dx}\right|_{t=\frac{\pi}{3}}(y - y_1)$

If the candidate uses $y = mx + c$ they must proceed as far as c =.. for this mark

A1 cao $y = 6\sqrt{3}x + 19\sqrt{3}$

(c)

M1 Attempts to use the double angle formula $\cos 2t = \pm 1 \pm 2\sin^2 t$ leading to an equation linking x and y

If $\cos 2t = \cos^2 t - \sin^2 t$ is initially used there must be an attempt to replace the $\cos^2 t$ by $1 - \sin^2 t$

dM1 Uses correct $\cos 2t = 1 - 2\sin^2 t$ and attempts to replace sin$t$ by $\frac{y}{2}$ and cos2t by $\frac{x}{6}$

Condone poor bracketing in cases such as $\cos 2t = 1 - 2\sin^2 t \Rightarrow \frac{x}{6} = 1 - 2\frac{y}{2}^2$

A1 Correct equation. Accept $x = 6 - 3y^2$ or $f(y) = 6 - 3y^2$

(d)

B1 States $k = 2$ or writes the range of y as $-2 < y < 2$

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