Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number | Candidate Number

Wednesday 7 November 2018

Morning (Time: 2 hours 30 minutes) | Paper Reference **WMA02/01**

Core Mathematics C34

Advanced

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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Pearson

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1. (a) Write $\cos\theta + 4\sin\theta$ in the form $R\cos(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

(b) Hence solve, for $0 \leqslant \theta < \pi$, the equation

$$\cos 2\theta + 4\sin 2\theta = 1.2$$

giving your answers to 2 decimal places.

(5)

(a) $R\cos(\theta - \alpha)$.

$R = \sqrt{1^2+4^2} = \sqrt{17}$.

$\theta = \tan^{-1}\left(\frac{4}{1}\right) = 1.33 \text{ rad}$. $\underline{1.326 \text{ (3dp)}}$

$\therefore \Rightarrow \cancel{\sqrt{17}\cos(\theta - 1.326\alpha)}$.

$\sqrt{17}\cos(\theta - 1.326)$.

(b) $\underbrace{\cos 2\theta + 4\sin 2\theta} = 1.2$

$R\cos(2\theta - \alpha) = 1.2$

$\sqrt{17}\cos(2\theta - 1.326) = 1.2$

$\cos(2\theta - 1.326) = \frac{1.2}{\sqrt{17}}$

$2\theta - 1.326 = \pm 1.275$

$\theta_1 = (1.275 + 1.326)/2$

$\theta_2 = (-1.275 + 1.326)/2$.

$\theta_1 = \underline{\underline{1.30}}$

$\theta_2 = \underline{\underline{0.03}}$

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2. A curve C has equation

$$x^3 - 4xy + 2x + 3y^2 - 3 = 0$$

Find an equation of the normal to C at the point $(-3, 2)$, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(7)

Implicit differentiation

$3x^2 - 4\left(y + x \cdot \frac{dy}{dx}\right) + 2 + 6y\frac{dy}{dx} = 0$

$3x^2 - 4y - 4x\frac{dy}{dx} + 2 + 6y\frac{dy}{dx} = 0$

$-4x\frac{dy}{dx} + 6y\frac{dy}{dx} = 4y - 3x^2 - 2$

$\frac{dy}{dx}(-4x + 6y) = 4y - 3x^2 - 2$

$\frac{dy}{dx} = \frac{4y - 3x^2 - 2}{-4x + 6y}$

when $x = -3$ $y = 2$

$\frac{dy}{dx} = \frac{4(2)^2 - 3(-3)^2 - 2}{-4(-3) + 6(2)}$

$\frac{dy}{dx} = \frac{-7}{8}$.

$\therefore$ grad of normal $= \frac{8}{7}$.

$y - 2 = \frac{8}{7}(x + 3)$

$y = \frac{8}{7}x + \frac{24}{7} + 2$

$y = \frac{8}{7}x + \frac{38}{7}$.

$7y = 8x + 38$.

$7y - 8x - 38 = 0$

3. Given

$$\cos\theta° = p, \text{ where } p \text{ is a constant and } \theta° \text{ is acute}$$

use standard trigonometric identities to find, in terms of p,

(a) $\sec\theta°$ **(1)**

(b) $\sin(\theta - 90)°$ **(2)**

(c) $\sin 2\theta°$ **(3)**

Write each answer in its simplest form.

(a) $\frac{1}{\cos\theta} = \sec\theta$

$= \frac{1}{p}$

(b) $\sin(\theta - 90) =$

$\sin\theta\cos 90 - \cos\theta\sin 90$

$\cos 90 = 0$

$\therefore = -\cos\theta\sin 90$

$\sin 90 = 1$

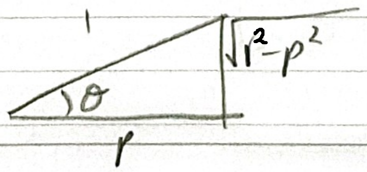
$\therefore = -\cos\theta$

$= -p$

(c) $\sin 2\theta = 2\sin\theta\cos\theta$

$\cos\theta = \frac{a}{h}$

$= \frac{p}{1}$



$\sin\theta = \frac{o}{h} = \frac{\sqrt{1-p^2}}{1}$

$\sin\theta = \sqrt{1-p^2}$

$\cos\theta = p$

$\therefore 2p\sqrt{1-p^2} = 2\sin\theta\cos\theta$

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4.

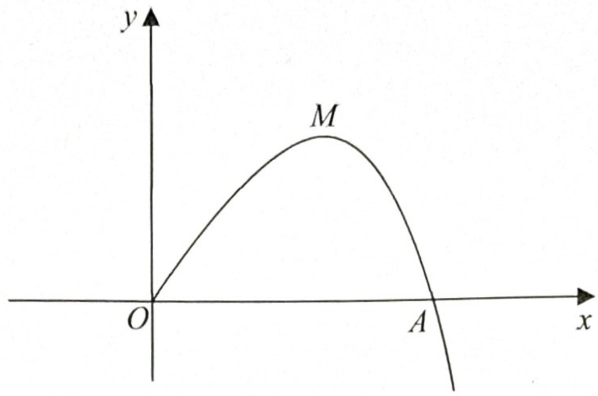

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = 8x - xe^{3x},\ x \geqslant 0$

The curve meets the x-axis at the origin and cuts the x-axis at the point A.

(a) Find the exact x coordinate of A, giving your answer in its simplest form.
(2)

The curve has a maximum turning point at the point M.

(b) Show, by using calculus, that the x coordinate of M is a solution of

$$x = \frac{1}{3}\ln\left(\frac{8}{1+3x}\right)$$
(5)

(c) Use the iterative formula

$$x_{n+1} = \frac{1}{3}\ln\left(\frac{8}{1+3x_n}\right)$$

with $x_0 = 0.4$ to calculate the values of x_1, x_2 and x_3, giving your answers to 3 decimal places.
(3)

(a) X int Y = 0 .

$0 = 8x - xe^{3x}$

$x(8 - e^{3x}) = 0$

$x = 0$ (NOT the x co-ordinate we want).

$8 - e^{3x} = 0$

$8 = e^{3x}$

$\ln 8 = 3x$

$x = \frac{1}{3}\ln 8$.

$(\frac{1}{3}\ln 8, 0)$.

$\frac{1}{3}\ln 8 = \ln 2$ $\therefore (\ln 2, 0)$.

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Question 4 continued

(b) X_M at $\frac{dy}{dx} = 0$.

$y = 8x - xe^{3x}$.

$\frac{dy}{dx} = 8 - [e^{3x} + x \cdot 3e^{3x}]$

$= 8 - e^{3x} - x \cdot 3e^{3x}$

$= 8 - e^{3x}(1+3x) = 0$

$\cancel{e^{3x}(1+3x)}$

$e^{3x} = \frac{8}{1+3x}$

$3x = \ln\left(\frac{8}{1+3x}\right)$

$X_M = \frac{1}{3}\ln\left(\frac{8}{1+3x}\right)$ as req.

(c) $x_0 = 0.4$ replace into iterative formula.

$x_1 = \frac{1}{3}\ln\left(\frac{8}{1+3(0.4)}\right)$

$x_1 = 0.430$

$x_2 = \frac{1}{3}\ln\left(\frac{8}{1+3(0.43)}\right)$

$x_2 = 0.417$

$x_3 = \frac{1}{3}\ln\left(\frac{8}{1+3(0.417)}\right)$

$x_3 = 0.423$.

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5.

$$f(x) = \frac{4x^2 + 5x + 3}{(x+2)(1-x)^2} \equiv \frac{A}{(x+2)} + \frac{B}{(1-x)} + \frac{C}{(1-x)^2}$$

(a) Find the values of the constants A, B and C.

(4)

(b) (i) Hence find $\int f(x)\,dx$.

(ii) Find the exact value of $\int_0^{\frac{1}{2}} f(x)\,dx$, writing your answer in the form $p + \ln q$, where p and q are constants.

(6)

(a) $\frac{4x^2+5x+3}{(x+2)(1-x)^2} = \frac{A(1-x)^2+B(x+2)(1-x)+C(x+2)}{(x+2)(1-x)^2}$

$4x^2+5x+3 = A(1-x)^2+B(x+2)(1-x)+C(x+2)$

let $x=1$

$4(1)^2+5(1)+3 = A(0)+B(0)+C(1+2)$

$12 = 3C$

$C = 4$

let $x=-2$

$4(-2)^2+5(-2)+3 = A(1-(-2))^2+B(0)+C(0)$

$16-10+3 = 9A$

$9 = 9A$

$A = 1$

coefficient of $x^2 = 4$ $\quad Ax^2 - Bx^2 = 4x^2$

$\therefore 4 = A-B$

$B = -3$

$\therefore f(x) = \frac{1}{(x+2)} - \frac{3}{(1-x)} + \frac{4}{(1-x)^2}$

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Question 5 continued

b i) $\int f(x)\,dx$.

$= \int \frac{1}{x+2} - \frac{3}{1-x} + \frac{4}{(1-x)^2}\,dx$

$= \ln(x+2) - 3 \cdot -1 \ln(1-x)$

$+ 4 \cdot -1 \cdot -1 (1-x)^{-1} + C$

$\ln(x+2) + 3\ln(1-x) + \frac{4}{(1-x)} + C$.

ii) $\int_0^{1/2} f(x)\,dx$

$= \left[\ln(x+2) + 3\ln(1-x) + \frac{4}{(1-x)}\right]_0^{1/2}$

$= \left[\ln\left(\frac{1}{2}+2\right) + 3\ln\left(1-\frac{1}{2}\right) + \frac{4}{1-\frac{1}{2}}\right]$

$- \left[\ln(2) + 3\ln(1) + \frac{4}{1}\right]$

$\left[\ln\frac{5}{2} + 3\ln\frac{1}{2} + 8\right] -$

$\left[\ln 2 + 4\right]$

$= \ln\left(\frac{\frac{5}{2}\cdot\left(\frac{1}{2}\right)^3}{2}\right) + 8 - 4$

$= 4 + \ln\left(\frac{5}{32}\right)$.

Leave blank

6. (a) Use binomial expansions to show that, for $|x| < \frac{1}{2}$

$$\sqrt{\frac{1+2x}{1-x}} \approx 1 + \frac{3}{2}x + \frac{3}{8}x^2$$

(6)

(b) Find the exact value of $\sqrt{\frac{1+2x}{1-x}}$ when $x = \frac{1}{10}$

Give your answer in the form $k\sqrt{3}$, where k is a constant to be determined.

(1)

(c) Substitute $x = \frac{1}{10}$ into the expansion given in part (a) and hence find an approximate value for $\sqrt{3}$

Give your answer in the form $\frac{a}{b}$ where a and b are integers.

(2)

$(1+x)^n = 1+nx+\frac{n(n-1)}{2!}x^2$

$\sqrt{\frac{1+2x}{1-x}} = (1+2x)^{1/2} \cdot (1-x)^{-1/2}$

$(1+2x)^{1/2}$ $n=\frac{1}{2}$ $x=2x$

$(1-x)^{-1/2}$ $n=\frac{-1}{2}$ $x=-x$

$(1+2x)^{1/2} = 1+\left(\frac{1}{2}\right)(2x) + \frac{\frac{1}{2}\cdot\left(\frac{1}{2}-1\right)}{2}(2x)^3$

$= 1+x+\left(\frac{-1}{2}\right)x^2$

$(1-x)^{-1/2} = 1+\left(\frac{-1}{2}\right)(-x) + \frac{\left(-\frac{1}{2}\right)\left(\frac{-1}{2}-1\right)}{2}(-x)^2$

$1+\frac{1}{2}x+\frac{3}{8}x^2$

$\left(1+x-\frac{x^2}{2}\right)\cdot\left(1+\frac{x}{2}+\frac{3}{8}x^2\right)$

$= 1+\frac{x}{2}+\frac{3}{8}x^2+x+\cancel{\frac{x^2}{2}}-\cancel{\frac{x^2}{2}}$

$= 1+\frac{3}{2}x+\frac{3}{8}x^2$ as req.

(b) $\sqrt{\frac{1+2x}{1-x}}$ when $x=0.1$

$\sqrt{\frac{1+2(0.1)}{1-0.1}} = \frac{2}{3}\sqrt{3}$

$\therefore k=\frac{2}{3}$

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Question 6 continued

(c) $\frac{2}{3}\sqrt{3} = 1 + \frac{3}{2}\left(\frac{1}{10}\right) + \frac{3}{8}\left(\frac{1}{10}\right)^2$

$\frac{2}{3}\sqrt{3} = \frac{923}{800}$

$\sqrt{3} = \frac{923}{800} \times \frac{3}{2}$

$= \frac{2769}{1600}$

Turn over

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7. A curve has equation

$$y = \ln(1 - \cos 2x), \quad x \in \mathbb{R}, 0 < x < \pi$$

Show that

(a) $\frac{dy}{dx} = k\cot x$, where k is a constant to be found. **(4)**

Hence find the exact coordinates of the point on the curve where

(b) $\frac{dy}{dx} = 2\sqrt{3}$ **(4)**

$y = \ln(1-\cos 2x)$

$\frac{dy}{dx} = \frac{1}{1-\cos 2x} \times 2 \cdot \sin 2x$

$= \frac{2\sin 2x}{1-\cos 2x}$

$\sin 2x = 2\sin x \cos x$.

$\cos 2x = 1 - 2\sin^2 x$

$= \frac{2 \times 2\sin x \cos x}{1-(1-2\sin^2 x)}$

$= \frac{\cancel{2} \times 2\cancel{\sin x}\cos x}{\cancel{2}\sin x \sin x}$

$= \frac{2\cos x}{\sin x}$

$= 2\cot x$ as req

$\underline{\underline{k = 2}}$

(b) $\cancel{2}\cot x = \cancel{2}\sqrt{3}$

$\frac{1}{\tan x} = \sqrt{3}$.

$\tan x = \frac{1}{\sqrt{3}}$

$x = \frac{1}{6}\pi, \frac{7}{6}\pi$ (not in range)

$\therefore x = \frac{1}{6}\pi$.

$y = \ln\left(1 - \cos\frac{2\pi}{6}\right)$

$y = \ln(1 - 1/2)$

$y = \ln(1/2)$

point $= \left(\frac{1}{6}\pi, \ln\left(\frac{1}{2}\right)\right)$

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8. (i) Find $\int x\sin x \, dx$ **(3)**

(ii) (a) Use the substitution $x = \sec\theta$ to show that

$$\int_1^2 \sqrt{1 - \frac{1}{x^2}} \, dx = \int_0^{\frac{\pi}{3}} \tan^2\theta \, d\theta$$

(3)

(b) Hence find the exact value of

$$\int_1^2 \sqrt{1 - \frac{1}{x^2}} \, dx$$

(4)

i) $\int x \sin x \, dx$, u, $\frac{dv}{dx}$

$u = x$ $v = -\cos x$

$\frac{du}{dx} = 1$ $\frac{dv}{dx} = \sin x$

$uv - \int v \frac{du}{dx}$

$x \cdot \cancel{\sin x}\ -\cos x - \int -\cos x \, dx$

$-x\cos x$ $\cancel{x\sin x} + \int \cos x \, dx$

$-x\cos x$ $\cancel{x\sin x} + \sin x + c$

ii) $x = \sec\theta$

$\frac{dx}{d\theta} = \sec\theta\tan\theta$

$\int_1^2 \sqrt{1 - \frac{1}{x^2}} \, dx$

when $x = 1$

$1 = \sec\theta$

$\theta = 0$

when $x = 2$

$2 = \sec\theta$

$\cos\theta = 1/2$

$\theta = \pi/3$ new limits: $\frac{\pi}{3}$ and 0

$\int_0^{\pi/3} \sqrt{\left(1 - \frac{1}{\sec^2\theta}\right)} \sec\theta\tan\theta \, d\theta$

$= \int_0^{\pi/3} \sqrt{1 - \cos^2\theta} \cdot \sec\theta\tan\theta \, d\theta$

$= \int_0^{\pi/3} \sin\theta \cdot \sec\theta\tan\theta \, d\theta$

$\sin\theta \cdot \frac{1}{\cos\theta} = \tan\theta$

$= \int_0^{\pi/3} \tan^2\theta \, d\theta$ as req.

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Question 8 continued

b) $\int_0^{\pi/3} \tan^2\theta \; d\theta$.

$\int_0^{\pi/3} (\sec^2\theta - 1)\; d\theta$.

$= \left[\tan\theta - \theta\right]_0^{\pi/3}$

$= \left[\tan\frac{\pi}{3} - \frac{\pi}{3}\right] - 0$

$= \sqrt{3} - \frac{\pi}{3}$

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9. A rare species of mammal is being studied. The population P, t years after the study started, is modelled by the formula

$$P = \frac{900e^{\frac{1}{4}t}}{3e^{\frac{1}{4}t} - 1}, \quad t \in \mathbb{R}, \quad t \geqslant 0$$

Using the model,

(a) calculate the number of mammals at the start of the study, **(1)**

(b) calculate the exact value of t when $P = 315$

Give your answer in the form $a \ln k$, where a and k are integers to be determined. **(4)**

(c) (i) Find $\frac{dP}{dt}$

(ii) Hence find the value of $\frac{dP}{dt}$ when $t = 8$, giving your answer to 2 decimal places. **(4)**

(a) P at $t=0$.

$P = \frac{900e^0}{3e^0 - 1} = \frac{900}{2} = \underline{\underline{450}}$

(b) $315 = \frac{900e^{\frac{1}{4}t}}{3e^{1/4t} - 1}$

$315(3e^{1/4t} - 1) = 900e^{1/4t}$

$315 \cdot 3e^{1/4t} - 315 = 900e^{1/4t}$.

$945e^{1/4t} - 900e^{1/4t} = 315$

$45e^{1/4t} = 315$

$e^{1/4t} = 7$.

$\frac{1}{4}t = \ln 7$.

$t = \underline{\underline{4\ln 7}}$

(c) $P = \frac{900e^{1/4t}}{3e^{1/4t} - 1} = 900e^{1/4t} \cdot (3e^{1/4t} - 1)^{-1}$

$\frac{dP}{dt} = 900e^{1/4t} \cdot \frac{1}{4} \times (3e^{1/4t} - 1)^{-1}$

$+ 900e^{1/4t} \times -1(3e^{1/4t} - 1)^{-2} \times 3e^{1/4t} \times \frac{1}{4}$

$= \frac{225e^{1/4}}{(3e^{1/4} - 1)} + \frac{225 \times 3 \times -1 \times e^{1/4t} \times e^{1/4t}}{(3e^{1/4t} - 1)^2}$

$\frac{dP}{dt} = \frac{\cancel{225e^{1/4} \times 3e^{1/4t}} - 225e^{1/4t} - 3 \times 225e^{1/4t}}{(3e^{1/4t} - 1)^2}$. $e^{1/4t}$.

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Question 9 continued

$\frac{dP}{dt} = \frac{-225e^{1/4t}}{(3e^{1/4t}-1)^2}$

(d) $\frac{dP}{dt}$ when $t=8$.

$\frac{-225e^{1/4 \times 8}}{(3e^{1/4 \times 8}-1)^2}$

$= \frac{-225e^2}{(3e^2-1)^2}$

$= -3.71$

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10.

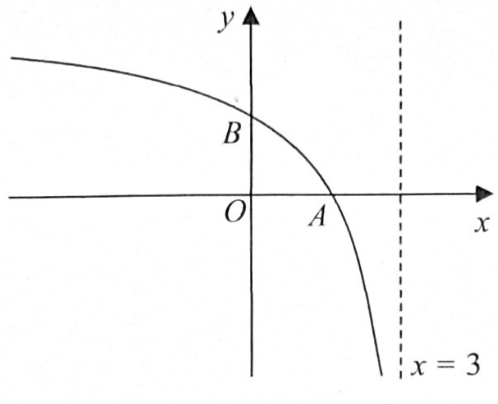

Figure 2

Figure 2 shows a sketch of part of the graph with equation $y = g(x)$, where

$$g(x) = \frac{3x-4}{x-3}, \quad x \in \mathbb{R}, \quad x < 3$$

The graph cuts the x-axis at the point A and the y-axis at the point B, as shown in Figure 2.

(a) State the range of g. **(1)**

(b) State the coordinates of

(i) point A

(ii) point B **(2)**

(c) Find $gg(x)$ in its simplest form. **(3)**

(d) Sketch the graph with equation $y = |g(x)|$

On your sketch, show the coordinates of each point at which the graph meets or cuts the axes and state the equation of each asymptote. **(3)**

(e) Find the exact solution of the equation $|g(x)| = 8$ **(3)**

a) $g(x) < 3$

(b) x int $y = 0$.

$3x - 4 = 0$

$x = 4/3$. $(4/3, 0)$

ii) y int $x = 0$.

$\frac{3(0)-4}{0-3} = \frac{4}{3}$.

$(0, 4/3)$

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Question 10 continued

(c) $gg(x)$

$= \dfrac{3\left(\frac{3x-4}{x-3}\right)-4}{\frac{3x-4}{x-3}-3}$

$= \dfrac{3(3x-4)-4(x-3)}{3x-4 \quad -3(x-3)}$

$= \dfrac{9x-\cancel{12}-4x+\cancel{12}}{\cancel{3x}-4-\cancel{3x}+9}$

$= \dfrac{5x}{5} = \underline{\underline{x}}$

(d) $|g(x)| = 8.$

$\dfrac{3x-4}{x-3} = 8.$ | $\dfrac{3x-4}{x-3} = -8$

① .

$3x-4 = 8x-24$

$5x = 20$

$\underline{x = 4}$

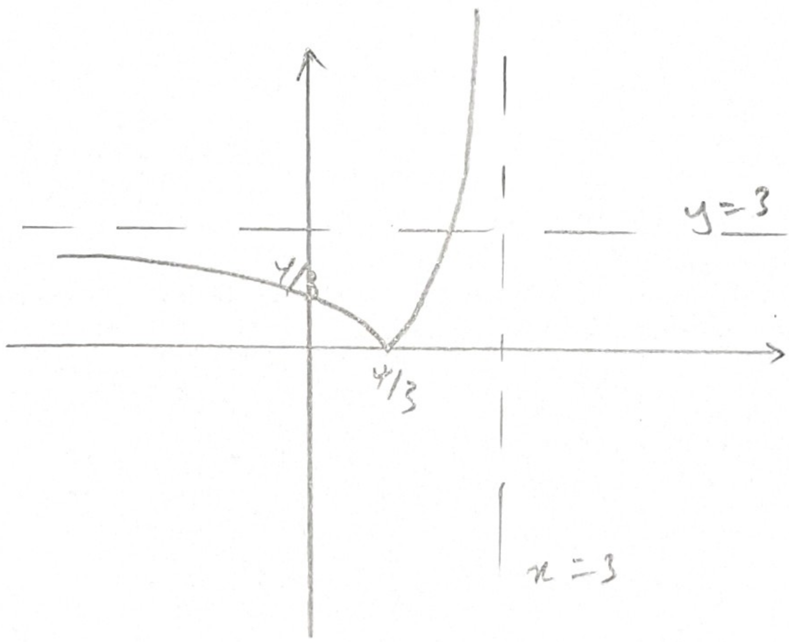
② $3x-4 = -8x+24.$

$11x = 28$

$x = \dfrac{28}{11}$

since $x < 3$

$x = \dfrac{28}{11}$ only



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11. Relative to a fixed origin O, the line l_1 is given by the equation

$$l_1: \quad \mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix}$$

where λ is a scalar parameter.

The line l_2 passes through the origin and is parallel to l_1

(a) Find a vector equation for l_2

(2)

The point A and the point B both lie on l_1 with parameters $\lambda = 0$ and $\lambda = 3$ respectively.

Write down

(b) (i) the coordinates of A,

(ii) the coordinates of B.

(2)

(c) Find the size of the acute angle between OA and l_1

Give your answer in degrees to one decimal place.

(3)

The point D lies on l_2 such that $OABD$ is a parallelogram.

(d) Find the area of $OABD$, giving your answer to the nearest whole number.

(3)

$l_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix}$

bi) when $\lambda = 0$.

$l_1 r = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + 0 \begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix}$

$A = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$

$A = (2, 3, -1)$

when $\lambda = 3$.

$\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + 3 \begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 15 \\ 8 \end{pmatrix} \rightarrow 13.$

$B = (-1, 15, 8)$

(c)

$a \cdot b = |a||b| \cos\theta$.

$\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix} = \sqrt{4+9+1} \times \sqrt{1+16+9} \times \cos\theta$.

$\cos\theta = \dfrac{-2+12-3}{\sqrt{14} \times \sqrt{26}}$

$\theta = \underline{\underline{68.5^\circ}}$

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Question 11 continued

(d) Area: OA × AB × sin θ

$\overrightarrow{AB} = \vec{B} - \vec{A}$

$= (-1, 15, 8) - (2, 3, -1)$

$AB = (-3, 12, 9)$

$\text{Area} = \sqrt{2^2 + 3^2 + (-1)^2} \times \sqrt{(-3)^2 + 12^2 + 9^2} \times \sin 68.5$

$= \sqrt{14} \times \sqrt{9 + 144 + 81} \times \sin 68.5$

$= 53.3$

Area = 53

12.

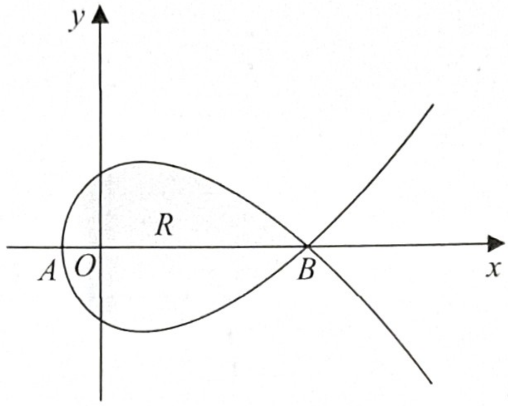

Figure 3

Figure 3 shows a sketch of part of the curve C with parametric equations

$$x = 7t^2 - 5, \qquad y = t(9 - t^2), \qquad t \in \mathbb{R}$$

(a) Find an equation of the tangent to C at the point where $t = 1$

Write your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(5)

The curve C cuts the x-axis at the points A and B, as shown in Figure 3

(b) (i) Find the x coordinate of the point A.

(ii) Find the x coordinate of the point B.

(3)

The region R, shown shaded in Figure 3, is enclosed by the loop of the curve C.

(c) Use integration to find the area of R.

(5)

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$.

$x = 7t^2 - 5$.

$\frac{dx}{dt} = 14t$.

$y = t(9 - t^2)$

$\frac{dy}{dt} = 9 - 3t^2$

$\frac{dy}{dx} = 9 - 3t^2 \times \frac{1}{14t}$

$= \frac{9 - 3t^2}{14t}$ at $t = 1$

$\frac{9-3}{14} = \frac{6}{14} = \frac{3}{7}$

$y = \frac{3}{7}x + c$. $(2, 8)$

$8 = \frac{6}{7} + c$ $c = 50/7$.

$y = \frac{3}{7}x + \frac{50}{7}$.

$7y - 3x - 50 = 0$

Leave blank

Question 12 continued

bi) x int $y=0$.

$0 = t(9-t^2)$

$t=0$ $\underline{t=\pm 3}$

$x = 7t^2 - 5$.

at $t=0$.

$7(0)-5 = \underline{-5}$

at $t=3$

$7(3)^2 - 5$

$= 58$.

$(-5,0)$ $(58,0)$

A $x=0$ B $x=58$

(c) $2\int_{x=-5}^{x=58} y\,dx$ = area

$2\int_0^3 y \cdot \frac{dx}{dt} \cdot dt$

→ when $x=58$, $t=0$

when $x=-5$, $t=3$

∴ new limits : 3 & 0

$= 2\int_0^3 t(9-t^2) \cdot 14t \cdot dt$

$= 2\int_0^3 (9\times14\times t^2 - 14t^4)\, dt$

$2\left[\frac{9\times14}{3}t^3 - \frac{14}{5}t^5\right]_0^3$

$2\left[\frac{9\times14(3)^3}{3} - \frac{14}{5}(3)^5\right] - 0$

$\underline{\underline{907.2}}$

Leave blank

13. The volume of a spherical balloon of radius r m is V m^3, where $V = \frac{4}{3}\pi r^3$

(a) Find $\frac{dV}{dr}$ **(1)**

Given that the volume of the balloon increases with time t seconds according to the formula

$$\frac{dV}{dt} = \frac{20}{V(0.05t+1)^3}, \qquad t \geqslant 0$$

(b) find an expression in terms of r and t for $\frac{dr}{dt}$ **(3)**

Given that $V = 1$ when $t = 0$

(c) solve the differential equation

$$\frac{dV}{dt} = \frac{20}{V(0.05t+1)^3}$$

giving your answer in the form $V^2 = f(t)$. **(6)**

(d) Hence find the radius of the balloon at time $t = 20$, giving your answer to 3 significant figures. **(3)**

a) $V = \frac{4}{3}\pi r^3$.

$\frac{dv}{dr} = 4\pi r^2$

b) $\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dv}{dt}$.

$= \frac{1}{4\pi r^2} \times \frac{20}{v(0.05t+1)^3}$

$= \frac{5}{v\pi r^2(0.05t+1)^3}$

$V = \frac{4}{3}\pi r^3$.

$= \frac{5}{\frac{4}{3}\pi r^3 . \pi r^2(0.05t+1)^3}$

$\frac{15}{\pi^2 r^5 (0.05t+1)^3}$

(c) $\frac{dv}{dt} = \frac{20}{v(0.05t+1)^3}$

Leave blank

Question 13 continued

$$\int v\,dv = \int 20(0.05t+1)^{-3}\,dt$$

$$\frac{v^2}{2} = 20 \times \frac{(0.05t+1)^{-2}}{-2} \times \frac{1}{0.05} + c$$

$$v^2 = \frac{-20}{0.05(0.05t+1)^2} + c$$

$$v^2 = \frac{-400}{(0.05t+1)^2} + c$$

$v = 1 \quad t = 0$

$1 = -400 + c \quad c = 401$

$$v^2 = 401 - \frac{400}{\cancel{(0.05t+1)}\,(0.05t+1)^2}$$

(d) $$v^2 = 401 - \frac{400}{(0.05t+1)^2}$$

$$v^2 = 401 - \frac{400}{(0.05\times 20+1)^2}$$

$= 301$

$v = \sqrt{301}$

$$V = \frac{4}{3}\pi r^3.$$

$$r^3 = \sqrt[3]{\frac{3\times\sqrt{301}}{4\pi}}.$$

$r = 1.61\text{ m}$

Turn over