Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number	Candidate Number

Tuesday 18 June 2019

Morning (Time: 2 hours 30 minutes) | Paper Reference **WMA02/01**

Mathematics

International Advanced Level
Core Mathematics C34

You must have:
Mathematical Formulae and Statistical Tables (Blue), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 14 questions in this question paper. The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ►

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Answer ALL questions. Write your answers in the spaces provided.

1. $$f(x) = 2x^3 + x - 20$$

(a) Show that the equation $f(x) = 0$ can be rewritten as

$$x = \sqrt[3]{a - bx}$$

where a and b are positive constants to be determined. **(2)**

(b) Starting with $x_1 = 2.1$ use the iteration formula $x_{n+1} = \sqrt[3]{a - bx_n}$, with the numerical values of a and b, to calculate the values of x_2 and x_3 giving your answers to 3 decimal places. **(2)**

(c) Using a suitable interval, show that 2.077 is a root of the equation $f(x) = 0$ correct to 3 decimal places. **(2)**

(d) Hence state a root, to 3 decimal places, of the equation

$$2(x + 2)^3 + x - 18 = 0$$ **(1)**

(a) $f(x) = 2x^3 + x - 20 = 0$

$2x^3 = 20 - x$.

$x^3 = \frac{20 - x}{2}$

$x^3 = 10 - \frac{1}{2}x$

$x = \sqrt[3]{10 - \frac{1}{2}x}$ $a = 10$ $b = \frac{1}{2}$

(b) $x_{n+1} = \sqrt[3]{10 - \frac{1}{2}x_n}$ $x_1 = 2.1$

$n = 1$ $x_2 = \sqrt[3]{10 - \frac{1}{2}x_1} = \sqrt[3]{10 - \frac{1}{2}(2.1)}$

$x_2 = \underline{2.076}$

$x_3 = \sqrt[3]{10 - \frac{1}{2}x_2} = \sqrt[3]{10 - \frac{1}{2}(2.076)}$

$x_3 = \underline{2.077} = \underline{2.077}$

(c) $f(2.0765) = 2(2.0765)^3 + (2.0765) - 20$

$= -0.0163$ (−ve)

$f(2.0775) = 2(2.0775)^3 + (2.0775) - 20$

$= +0.0105$ (+ve)

∴ change in sign indicates root present. ∴ to 3dp root $= \underline{2.077}$

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Question 1 continued

$2(x+2)^3 + x - 18 = 0.$

$2(x+2)^3 + (x+2) - 20 = 0$

$x+2 = 2.077$

$\therefore x = 0.077$

(Total for Question 1 is ~~4~~ 7 marks)

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2. (a) Find $\int \frac{4x+3}{x}\,dx, \qquad x > 0$

(2)

(b) Given that $y = 25$ at $x = 1$, solve the differential equation

$$\frac{dy}{dx} = \frac{(4x+3)y^{\frac{1}{2}}}{x} \qquad x > 0, y > 0$$

giving your answer in the form $y = [g(x)]^2$.

(5)

a) $\int \frac{4x}{x}dx + 3\int \frac{1}{x}\,dx$

$= \int 4dx + 3\int \frac{1}{x}\,dx$

$= 4x + 3\ln x + c$

(b) $\frac{dy}{dx} = \left(\frac{4x+3}{x}\right) \times y^{1/2}$.

cross multiplying gives

$\frac{1}{y^{\frac{1}{2}}}dy = \left(\frac{4x+3}{x}\right)dx$

$\int y^{-1/2}\,dy = \int \left(\frac{4x+3}{x}\right)dx$

$\frac{y^{1/2}}{1/2} = \int \frac{4x}{x} + \frac{3}{x}\,dx$

$2y^{1/2} = 4x + 3\ln|x| + c$.

$y = 25$ when $x = 1$

$2\sqrt{25} = 4(1) + 3\ln(1) + c$

$2 \times 5 = 4 + c \qquad \underline{\underline{c = 6}}$

$2y^{1/2} = 4x + 3\ln x + 6$.

$\sqrt{y} = 2x + \frac{3}{2}\ln x + 3$

$y = (2x + 3/2\ln x + 3)^2$

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3. A curve C has parametric equations

$$x = \sqrt{3}\tan\theta, \quad y = \sec^2\theta, \quad 0 \leqslant \theta \leqslant \frac{\pi}{3}$$

The cartesian equation of C is

$$y = f(x), \quad 0 \leqslant x \leqslant k, \quad \text{where } k \text{ is a constant}$$

(a) State the value of k.

(1)

(b) Find f(x) in its simplest form.

(2)

(c) Hence, or otherwise, find the gradient of the curve at the point where $\theta = \frac{\pi}{6}$

(3)

(a) $0 \le \theta \le \pi/3$.

$0 \le x \le k$

to find k we use $\theta = \pi/3$.

$x = \sqrt{3}\tan(\pi/3) = 3$

$\underline{\underline{k = 3}}$

(b) $y = \sec^2\theta = 1 + \tan^2\theta$

$x = \sqrt{3}\tan\theta \rightarrow \tan\theta = \frac{x}{\sqrt{3}}$

$y = 1 + \left(\frac{x}{\sqrt{3}}\right)^2 \Rightarrow \underline{\underline{y = 1 + \frac{1}{3}x^2}}$

(c) $\frac{dy}{dx} = \frac{1}{3}(2x) = \frac{2}{3}x$

$\left.\frac{dy}{dx}\right|_{\theta = \pi/6} = \frac{2}{3}\left(\sqrt{3}\tan\left(\frac{\pi}{6}\right)\right)$

$\frac{dy}{dx} = \frac{2(1)}{3} = \underline{\underline{\frac{2}{3}}}$.

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4. The curve C has equation

$$3ye^{-2x} = 4x^2 + y^2 + 2$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

The point P on C has coordinates (0, 2).

(b) Find the equation of the normal to C at P giving your answer in the form $y = mx + c$, where m and c are constants to be found.

(3)

(a) $3ye^{-2x} = 4x^2 + y^2 + 2$

Implicitly differentiating gives;

$3\frac{dy}{dx} \cdot e^{-2x} + 3y \cdot -2 \cdot e^{-2x}$

$= 4(2x) + 2y \cdot \frac{dy}{dx}$

$\Rightarrow 3e^{-2x}\frac{dy}{dx} - 6ye^{-2x} =$

$8x + 2y\frac{dy}{dx}$

combining like terms.

$\frac{dy}{dx}(3e^{-2x} - 2y) = 8x + 6ye^{-2x}$

$\frac{dy}{dx} = \frac{8x + 6ye^{-2x}}{3e^{-2x} - 2y}$

(b) $m = \frac{dy}{dx}$ and $m_{\perp} = \frac{-1}{m}$

$m = \left.\frac{dy}{dx}\right|_{(0,2)} = \frac{8(0) + 6(2)e^{-2(0)}}{3e^{-2(0)} - 2(2)}$

$= \frac{12}{3-4} = -12$

$m_{\perp} = \frac{-1}{m} = \frac{-1}{-12} = \frac{1}{12}$

$y = mx + c$ (0, 2) y int.

$y = \frac{1}{12}x + 2$

$m = \frac{1}{12}$ $c = 2$

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5. A bath is filled with hot water. The temperature, θ°C, of the water in the bath, t minutes after the bath has been filled, is given by

$$\theta = 20 + Ae^{-kt}$$

where A and k are positive constants.

Given that the temperature of the water in the bath is initially 38 °C,

(a) find the value of A.

(2)

The temperature of the water in the bath 16 minutes after the bath has been filled is 24.5 °C.

(b) Show that $k = \frac{1}{8}\ln 2$

(4)

Using the values for k and A,

(c) find the temperature of the water 40 minutes after the bath has been filled, giving your answer to 3 significant figures.

(2)

(d) Explain why the temperature of the water in the bath cannot fall to 19 °C.

(1)

(a) At $t=0$ $\theta = 38$.

$\theta = 20 + Ae^{-kt}$.

$38 = 20 + Ae^{-k(0)}$

$38 = 20 + A$

$\underline{A = 18}$

(b) $t=16$ $\theta = 24.5$ $A = 18$

$\theta = 20 + Ae^{-kt}$.

$24.5 = 20 + 18 \cdot e^{-k \times 16}$

$4.5 = 18e^{-16k}$

$\frac{4.5}{18} = e^{-16k}$

$\frac{1}{4} = e^{-16k}$

$\ln\left(\frac{1}{4}\right) = -16k$

$\cancel{k} = \ln(2)^{-2} = -16k$.

$\frac{-2}{-16} \ln(2) = k$.

$k = \frac{1}{8}\ln 2$ as req.

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Question 5 continued

(c) $t=40$ $\quad \theta = 20+18e^{-\frac{1}{8}\ln 2 \cdot t}$

$\theta = 20+18e^{-\frac{40}{8}\ln 2}$

$\theta = \underline{\underline{20.6}}\,°C\ (3sf)$

(d) $\theta = 20 + (+ve)$

$\theta > 20 \quad \theta \neq 19$

$\Rightarrow$ as logs can't be negative

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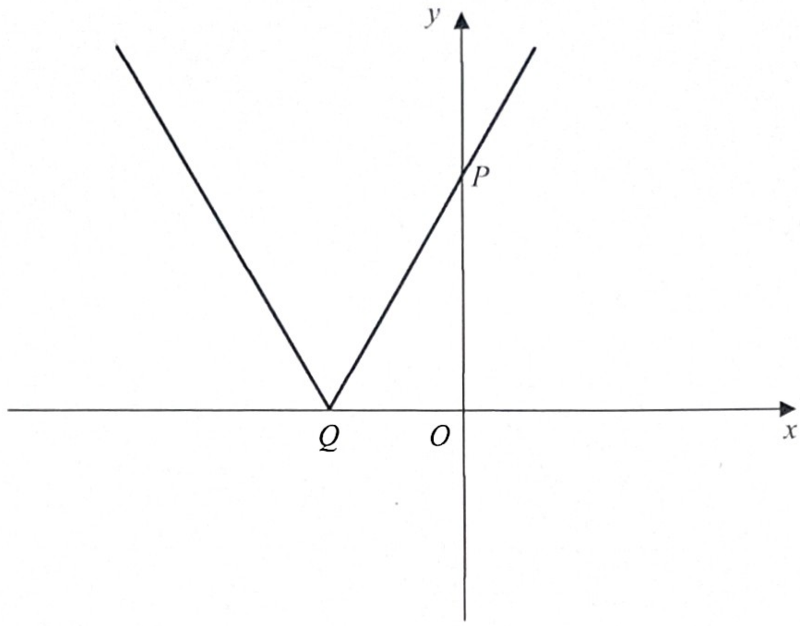

Figure 1

Figure 1 shows a sketch of the graph with equation $y = |4x + 10a|$, where a is a positive constant.

The graph cuts the y-axis at the point P and meets the x-axis at the point Q as shown.

(a) (i) State the coordinates of P.

(ii) State the coordinates of Q.

(2)

(b) A copy of Figure 1 is shown on page 15. On this copy, sketch the graph with equation

$$y = |x| - a$$

Show on the sketch the coordinates of each point where your graph cuts or meets the coordinate axes.

(2)

(c) Hence, or otherwise, solve the equation

$$|4x + 10a| = |x| - a$$

giving your answers in terms of a.

(3)

(a) At Q → y=0

$0 = 4x + 10a$.

$x = -\frac{5}{2}a$ ∴ $Q = (-\frac{5}{2}a, 0)$

at P → x=0 $y = 4(0) + 10a$

$y = 10a$.

P → $(0, 10a)$

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Question 6 continued

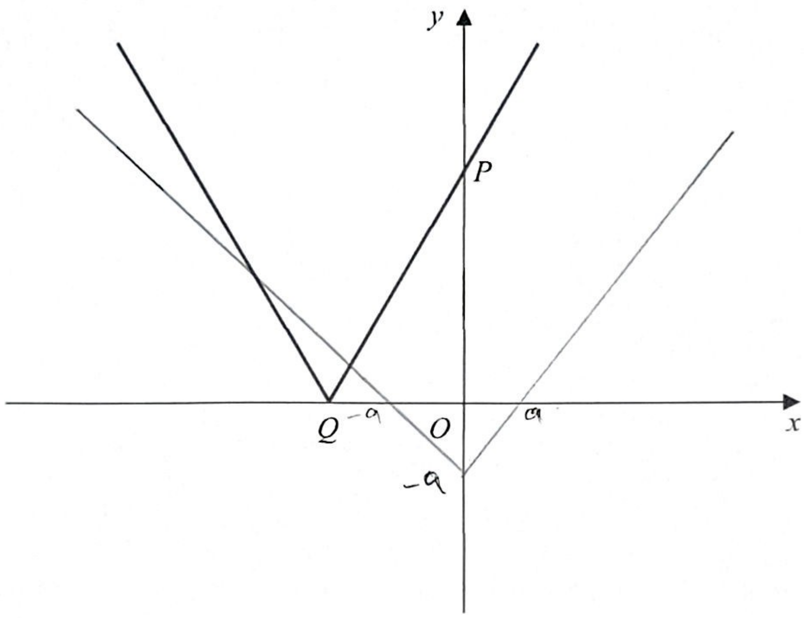

Figure 1

(c) $|4x+10a| = |x|-a$

$\pm(4x+10a) = -x-a$.

For +

$4x+10a = -x-a$

$5x = -11a$

$x = \frac{-11a}{5}$.

For −ve

$-4x-10a = -x-a$

$-3x = 9a$.

$x = -3a$

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7. (a) Express $5\cos\theta - 3\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α, in radians, to 4 decimal places.

(3)

The height of sea water, H metres, on a harbour wall is modelled by the equation

$$H = 6 + 2.5\cos\left(\frac{4\pi t}{25}\right) - 1.5\sin\left(\frac{4\pi t}{25}\right), \qquad 0 \leqslant t < 12$$

where t is the number of hours after midday.

(b) Calculate the times at which the model predicts that the height of sea water on the harbour wall will be 4.6 metres. Give your answers to the nearest minute.

(6)

a) $R\cos(\theta+\alpha) =$

~~Rcost~~

$R(\cos\theta\cos\alpha - \sin\theta\sin\alpha)$

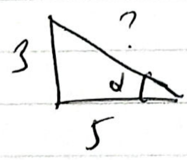
$5\cos\theta - 3\sin\theta$

$R\cos\alpha = 5$ --- ①

$R\sin\alpha = 3$ --- ②

$\frac{②}{①} = \tan\alpha = \frac{3}{5}$

$\alpha = \underline{0.5405}$



$\sqrt{3^2+5^2} = \sqrt{34} \rightarrow R$

b) $H = 6 + \frac{1}{2}(5\cos\theta - 3\sin\theta)$

$H = 6 + \frac{1}{2}R\cos(\theta+\alpha)$

$H = 4.6$ and $\theta = \frac{4\pi t}{25}$

$\alpha = 0.5404$ rad

$4.6 = 6 + \frac{1}{2}\sqrt{34}\cos(\theta + 0.5404)$

$\cos(\theta + 0.5404) = -0.4802$

$\theta + 0.5404 = \cos^{-1}(-0.4802)$

$\theta + 0.5404 = 2.0717,\ 2\pi - 2.0717$

$\theta + 0.5404 = 2.0717,\ 4.2115$

$\theta = 1.5313,\ 3.671$

but $\theta = \frac{4\pi t}{25}$

$t = \frac{3.671 \times 25}{4\pi} = 7.3\text{hrs} \rightarrow \underline{438\text{min}}$

$t = \frac{1.5313 \times 25}{4\pi} = 3.05\text{hr}$

$= \underline{183\text{ min}}$

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8.

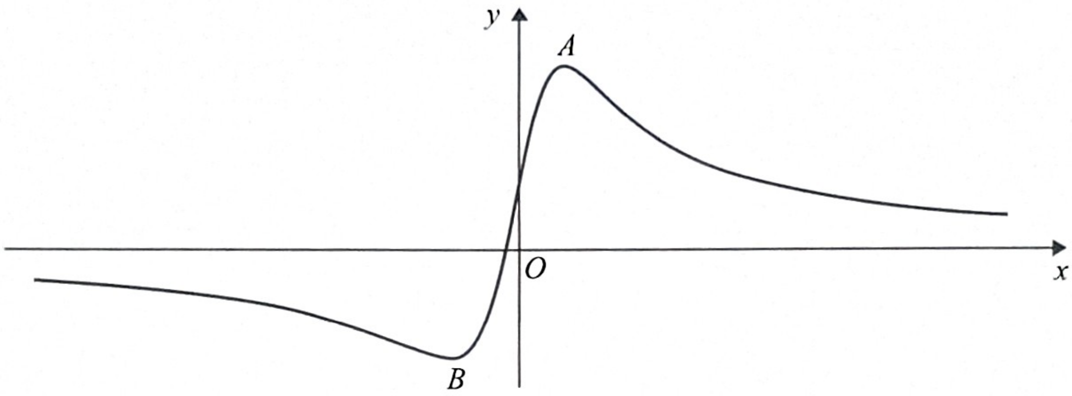

Figure 2

Figure 2 shows a sketch of part of the curve $y = f(x)$, where

$$f(x) = \frac{6x + 2}{3x^2 + 5}, \qquad x \in \mathbb{R}$$

(a) Find $f'(x)$, writing your answer as a single fraction in its simplest form.

(3)

The curve has two turning points, a maximum at point A and a minimum at point B, as shown in Figure 2.

(b) Using part (a), find the coordinates of point A and the coordinates of point B.

(4)

(c) State the coordinates of the maximum turning point of the function with equation

$$y = f(2x) + 4 \qquad x \in \mathbb{R}$$

(2)

(d) Find the range of the function

$$g(x) = \frac{6x + 2}{3x^2 + 5}, \qquad x \leqslant 0$$

(2)

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{(g(x))^2}$$

$$f(x) = \frac{6x+2}{(3x^2+5)}$$

$$f'(x) = \frac{(3x^2+5)\cdot 6 - (6x+2).2\cdot 3x}{(3x^2+5)^2}$$

$$= \frac{18x^2+30-36x^2-12x}{(3x^2+5)^2}$$

$$= \frac{-18x^2+30-12x}{(3x^2+5)^2}$$

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Question 8 continued

$$= \frac{-6(3x^2+2x-5)}{(3x^2+5)^2} \rightarrow f'(x)$$

(b) $f'(x)=0$

$$\frac{-6(3x^2+2x-5)}{(3x^2+5)^2} = 0$$

$-6(3x^2+2x-5)=0$

$3x^2+2x-5=0$

$(3x+5)(x-1)=0$

$\therefore x=-5/3 \quad x=1$

for A at $x=1$

$$f(x)=\frac{6(1)+2}{3(1)+5} = \frac{8}{8} = 1$$

$\therefore$ point $A \rightarrow (1,1)$

For B

$\cancel{f(}$

$$f(x)=\frac{6(-5/3)+2}{3\left(\frac{-5}{3}\right)^2+5} = \frac{-3}{5}$$

$\therefore$ point B $\left(\frac{-5}{3}, \frac{-3}{5}\right)$

(c) $A: (1,1) \rightarrow$ max

$y=f(2x)+4$.

max point $= \left(1\times\frac{1}{2}, 1+4\right)$

$= \left(\frac{1}{2}, 5\right)$

d) Range $\frac{-3}{5} \leq y \leq \frac{2}{5}$

as yint $x=0$

$$\therefore \frac{6(0)+2}{3(0)^2+5} = \frac{2}{5}$$

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9. (a) Using the formula for $\sin(A+B)$ and the relevant double angle formulae, find an identity for $\sin 3x$, giving your answer in the form

$$\sin(3x) \equiv P\sin x + Q\sin^3 x$$

where P and Q are constants to be determined.

(4)

(b) Hence, showing each step of your working, evaluate

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin 3x \cos x \, dx$$

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(4)

a) $\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(2x+x) = \sin 2x\cos x + \cos 2x\sin x$

$\sin 2x = 2\sin x\cos x$

$\cos 2x = 1 - 2\sin^2 x$

$\sin(3x) = (2\sin x\cos x)\cdot\cos x + (1-2\sin^2 x)\sin x$

$= 2\sin x\cos^2 x + \sin x - 2\sin^3 x$

$\cos^2 x = 1 - \sin^2 x$

$2\sin x(1-\sin^2 x) + \sin x - 2\sin^3 x$

$= 2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x$

$\sin(3x) = 3\sin x - 4\sin^3 x$

$\underline{P = 3} \quad \underline{Q = -4}$

(b) $\int_{\pi/6}^{\pi/2} (3\sin x - 4\sin^3 x)\cos x \, dx$

let $u = \sin x$ $\quad \frac{du}{dx} = \cos x$

$d = \cos x dx$

$u_1 = \sin(\pi/6) = 1/2$

$u_2 = \sin(\pi/2) = 1$

$\int_{1/2}^{1} (3u - 4u^3)\, du$

$= \left[\frac{3u^2}{2} - \frac{4u^4}{4}\right]_{1/2}^{1}$

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Question 9 continued

$$\left[\frac{3}{2}(1) - 1\right] - \left[\frac{3}{2}\left(\frac{1}{4}\right) - \left(\frac{1}{2}\right)^4\right]$$

$$= \frac{3}{16}$$

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10. (a) Use the binomial series to find the expansion of

$$\frac{1}{(2+3x)^3} \qquad |x| < \frac{2}{3}$$

in ascending powers of x, up to and including the term in x^2, giving each term as a simplified fraction.

(5)

(b) Hence or otherwise, find the coefficient of x^2 in the series expansion of

(i) $\dfrac{1}{(2+6x)^3} \qquad |x| < \dfrac{1}{3}$

(ii) $\dfrac{4-x}{(2+3x)^3} \qquad |x| < \dfrac{2}{3}$

(4)

a) $\dfrac{1}{(2+3x)^3} = (2+3x)^{-3} = \dfrac{1}{2^3}\left(1+\dfrac{3}{2}x\right)^{-3}$

$= \dfrac{1}{8}\left(1+\dfrac{3}{2}x\right)^{-3}$

$= \dfrac{1}{8}\left[1 + (-3)\times\left(\dfrac{3}{2}x\right) + \dfrac{(-3)(-4)}{2!}\left(\dfrac{3}{2}x\right)^2\right]$

$= \dfrac{1}{8}\left[1 + -\dfrac{9}{2}x + \dfrac{3\times 4}{2}\times\dfrac{9}{4}x^2\right]$

$\dfrac{1}{8} - \dfrac{9}{16}x + \dfrac{27}{16}x^2$

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Question 10 continued

bi) $= \dfrac{1}{(2+6x)^3}$ $\quad 6x = 2(3x)$

$\dfrac{27}{16}(2x)^2 = \dfrac{27}{16} \times 4x^2 = \dfrac{27}{4}x^2 \quad \therefore$ co-eff $= \dfrac{27}{4}$

ii) $\dfrac{4-x}{(2+3x)^3} = (4-x)\left[\dfrac{1}{8} - \dfrac{9}{16}x + \dfrac{27}{16}x^2\right]$

$\left(4 \times \dfrac{27}{16} + \dfrac{9}{16}\right)x^2 = \dfrac{117}{16}x^2 \quad \therefore$ co-eff $= \dfrac{117}{16}$.

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11. (a) Given

$$\frac{9}{t^2(t-3)} \equiv \frac{A}{t} + \frac{B}{t^2} + \frac{C}{(t-3)}$$

find the value of the constants A, B and C.

(3)

(b)

$$I = \int_4^{12} \frac{9}{t^2(t-3)}\,dt$$

Find the exact value of I, giving your answer in the form $\ln(a) - b$, where a and b are positive constants.

(6)

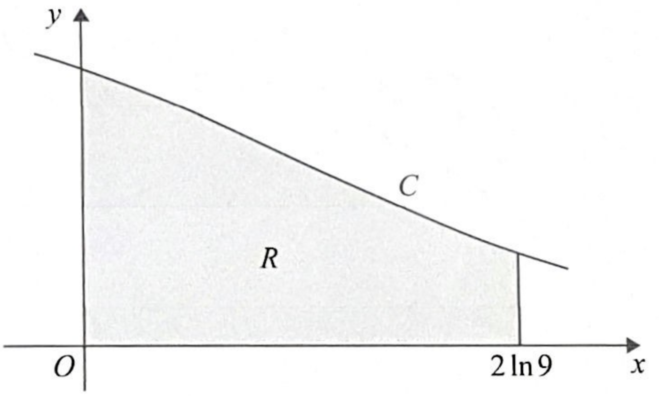

Figure 3

Figure 3 shows a sketch of part of the curve C with parametric equations

$$x = 2\ln(t-3), \qquad y = \frac{6}{t} \qquad t > 3$$

The finite region R, shown shaded in Figure 3, is bounded by the curve C, the y-axis, the x-axis and the line with equation $x = 2\ln 9$

The region R is rotated 360° about the x-axis to form a solid of revolution.

(c) Show that the exact volume of the solid generated is

$$k \times I$$

where k is a constant to be found.

(3)

(a) $9 = At(t-3) + B(t-3) + Ct^2$

at $t=0 \rightarrow 9 = B(0-3)$ $\underline{B=-3}$

$t=3$ $\quad 9 = C(3)^2$ $\underline{C=1}$

$At^2 + Ct^2 = 0 \times t^2$

$A + C = 0 \qquad A + 1 = 0$

$\underline{A = -1}$

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Question 11 continued

b) $I = \int_4^{12} \frac{9}{t^2(t-3)} dt$

$= \int_4^{12} \left(\frac{-1}{t} + \frac{-3}{t^2} + \frac{1}{(t-3)} \right) dt$

$= \left[-\ln t - \frac{3}{-t} + \ln(t-3) \right]_4^{12}$

$= \left[\frac{3}{t} + \ln\left(\frac{t-3}{t}\right) \right]_4^{12}$

$= \frac{3}{12} + \ln\left(\frac{12-3}{12}\right) - \frac{3}{4} - \ln\left(\frac{4-3}{4}\right)$

$= \frac{3-9}{12} + \ln\frac{9}{12} - \ln\frac{1}{4}$

$\frac{-6}{12} + \ln \frac{9/12}{1/4}$

$= \frac{-1}{2} + \ln \frac{9}{\cancel{12}\,3} \times \cancel{4}$

$I = \frac{-1}{2} + \ln 3 = \ln 3 - \frac{1}{2}$

$a = 3 \quad b = 1/2$

(c) $V = \int_0^{2\ln 9} \pi y^2 \, dx$.

$x = 2\ln(t-3)$ and $y = \frac{6}{t}$

$\frac{dx}{dt} = \frac{2}{(t-3)}$

$dx = \frac{2}{(t-3)} dt$

$x = 0 \rightarrow 0 = 2\ln(t-3)$

$t - 3 = 1 \quad \underline{t = 4}$

$x = 2\ln 9$

$\cancel{2}\ln 9 = \cancel{2}\ln(t-3)$.

$t - 3 = 9$.

$\underline{\underline{t = 12}}$

$V = \int_4^{12} \pi\left(\frac{6}{t}\right)^2 \times \frac{2}{(t-3)} dt$.

$\int_4^{12} \frac{\pi \times 36 \times 2}{t^2(t-3)} dt = \int_4^{12} \frac{\pi \times 4 \times 9 \times 2}{t^2(t-3)} dt$

$= \int_4^{12} \frac{8\pi \times 9}{t^2(t-3)} dt = 8\pi \int_4^{12} \frac{9}{t^2(t-3)} dt$

$= 8\pi \times I \quad \underline{\underline{K = 8\pi}}$

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DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

P 5 5 8 0 1 A 0 3 5 4 8

12. Relative to a fixed origin O,

the point A has position vector $(2\mathbf{i} - 3\mathbf{j} - 2\mathbf{k})$

the point B has position vector $(3\mathbf{i} + 2\mathbf{j} + 5\mathbf{k})$

the point C has position vector $(2\mathbf{i} + 4\mathbf{j} - 3\mathbf{k})$

The line l passes through the points A and B.

(a) Find the vector $\overrightarrow{AB}$.

(2)

(b) Find a vector equation for the line l.

(2)

(c) Show that the size of the angle CAB is 62.8°, to one decimal place.

(4)

(d) Hence find the area of triangle CAB, giving your answer to 3 significant figures.

(2)

The point D lies on the line l. Given that the area of triangle CAD is twice the area of triangle CAB,

(e) find the two possible position vectors of point D.

(3)

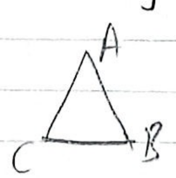
(a) $\overrightarrow{AB} = \vec{B} - \vec{A} = \overrightarrow{OB} - \overrightarrow{OA}$

$(3i+2j+5k) - (2i-3j-2k)$

$\overrightarrow{AB} = (i+5j+7k)$

b) $r = \overrightarrow{OA} + \lambda\overrightarrow{AB}$

$\Rightarrow \quad = (2i-3j-2k) + \lambda(i+5j+7k)$



(c) Angle $\underline{CAB}$

$\overrightarrow{AB} \cdot \overrightarrow{AC} = |AB| \cdot |AC| \cdot \cos(CAB)$

$\overrightarrow{AB} = (i+5j+7k)$

$|\overrightarrow{AB}| = \sqrt{1^2+5^2+7^2} = \underline{\sqrt{75}}$

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = (2i+4j-3k)$

$- (2i-3j-2k)$

$= 0i+7j-k$

$|\overrightarrow{AC}| = \sqrt{7^2+(-1)^2} = \underline{\underline{\sqrt{50}}}$

$\overrightarrow{AB} \cdot \overrightarrow{AC} = 0 + 5\times7 + 7(-1)$

$= 35 - 7 = \underline{\underline{28}}$

Leave blank

Question 12 continued

$$\cos\theta = \frac{\pm 28}{\sqrt{75}\times\sqrt{50}}$$

$\theta = 62.79$

$\approx 62.8^\circ$

d) $\text{Area} = \frac{1}{2} \times |AC| \times |AB| \times \sin\theta$

$= \frac{1}{2}\sqrt{75}\times\sqrt{50}\times\sin 62.8$

$= 27.2$

e) $l = (2i - 3j - 2k) \pm \lambda(i + 5j + 7k)$

$\Delta = 2\Delta$

$\therefore \lambda = 2$

$\vec{D} = (2i - 3j - 2k) \pm 2(i + 5j + 7k)$

$\vec{D} = (2+2)i + (-3+10)j + (-2+14)k$

$\vec{D} = 4i + 7j + 12k$

or $\vec{D} = (2-2)i + (-3-10j) + (-2-14)k$

$\vec{D} = 0i - 13j - 16k$

Leave blank

13.

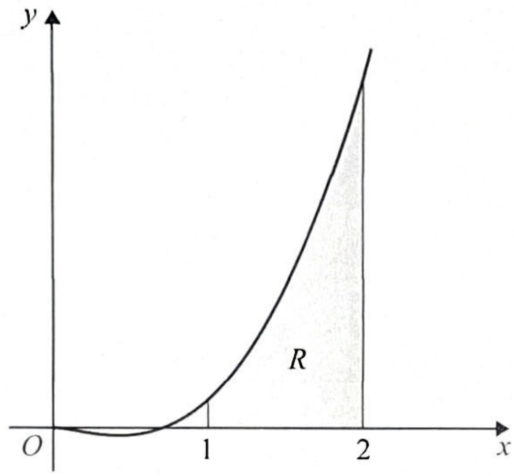


Figure 4

Figure 4 shows a sketch of the curve with equation $y = 12x^2 \ln(2x^2)$, $x > 0$

The finite region R, shown shaded in Figure 4, is bounded by the curve, the line with equation $x = 1$, the x-axis and the line with equation $x = 2$

The table below shows corresponding values of x and y for $y = 12x^2 \ln(2x^2)$, with the values of y given to 3 significant figures.

+0.25 +0.25

x	1	1.25	1.5	1.75	2
y	8.32	21.4	40.6	66.6	99.8

(a) Use the trapezium rule, with all the values of y, to obtain an estimate for the area of R, giving your answer to 2 significant figures.

(3)

(b) Use the substitution $u = x^2$ to show that the area of R is given by

$$\int_1^4 6u^{\frac{1}{2}} \ln(2u)\, du$$

(3)

(c) Hence, using calculus, find the exact area of R, writing your answer in the form $a + b\ln 2$, where a and b are constants to be found.

(6)

$h = 0.25$

$\text{Area} = \frac{0.25}{2}\left[8.32 + 99.8 + 2(21.4 + 40.6 + 66.6)\right]$

$= 45.665$

Area $= \underline{\underline{46}}$ (2sf)

Leave blank

Question 13 continued

b) $u = x^2$

Area of $R = \int_1^2 y\,dx$

$y = 12x^2 \ln(2x^2)$

$u = x^2 \rightarrow du = 2x\,dx$

$x = 1 \rightarrow u = 1$

$x = 2 \rightarrow u = 4$

$R = \int_1^4 12u \ln 2u \times dx$

$= \int_1^4 12u \ln 2u \times \frac{du}{2x} = \int_1^4 12u \ln u \times \frac{du}{2\sqrt{u}}$

Area $= \int_1^4 6u^{1/2} \ln 2u\, du$

as req.

$\int_1^4 6u^{1/2} \ln 2u\, du.$

$\int uv\,dx = u\int v\,dx - \int u'.(\int v\,dx))\,dx$

or $uv - \int v\,du$

$u = \ln 2u \quad u' = \frac{2}{2u} = \frac{1}{u}$

$\int 6u^{1/2}\,du = 6 \times \frac{1}{3/2} \times u^{3/2}$

$= \cancel{6}^{2} \times \frac{2}{\cancel{3}} \times u^{3/2} = 4u^{3/2}$

$= \int_1^4 6u^{1/2} \ln 2u\, du$

$= \ln 2u \times 4u^{3/2} - \int 4u^{3/2} \times \frac{1}{u}\,du$

$= 4u^{3/2} \ln 2u - \int 4u^{1/2}\,du$

$= 4u^{3/2} \ln 2u - 4 \times \frac{2}{3} u^{3/2}$

$= \left[4u^{3/2} \ln 2u - \frac{8}{3} u^{3/2}\right]_1^4$

$\left[4(4)^{3/2} \ln(2\times4) - \frac{8}{3} \times 4^{3/2}\right]$

$- \left[4(1)^{3/2} \times \ln 2 - \frac{8}{3} \times (1)^{3/2}\right]$

$= \cancel{38}.$

$= (32\ln 8 - 4\ln 2) - \frac{56}{3}$

$= 32\ln 2^3 - 4\ln 2 - \frac{56}{3}$

$= 96\ln 2 - 4\ln 2 - 56/3$

$= 92\ln 2 - \frac{56}{3}$

$a = -\frac{56}{3} \quad b = 92$

Turn over

14.

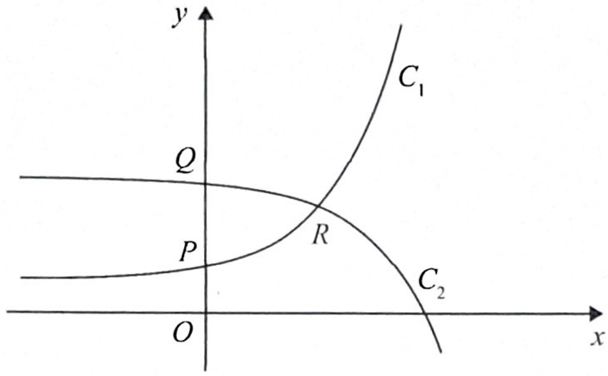


Figure 5

Figure 5 shows a sketch of the curves C_1 and C_2

C_1 has equation $y = 3 + e^{x+1} \quad x \in \mathbb{R}$

C_2 has equation $y = 10 - e^{x} \quad x \in \mathbb{R}$

Given that C_1 and C_2 cut the y-axis at the points P and Q respectively,

(a) find the exact distance PQ.

(2)

C_1 and C_2 intersect at the point R.

(b) Find the exact coordinates of R.

R at $y_{C_1} = y_{C_2}$

(5)

(a) $x_{prq} = 0$

$y_P = 3 + e^{0+1} = 3 + e$

$y_Q = 10 - e^0 = 10 - 1 = \underline{9}$

Distance PQ

$= y_Q - y_P = 9 - (3+e)$

$Pq = \underline{\underline{6 - e}}$

(b) $3 + e^{x+1} = 10 - e^x$

$e^{x+1} + e^x = 10 - 3 = 7$

$\cancel{e^{x+1}}\, e^x(e+1) = 7$

$e^x = \frac{7}{e+1} \qquad x = \ln\left(\frac{7}{e+1}\right)$

$y = 10 - e^x = 10 - e^{\left(\ln\left(\frac{7}{e+1}\right)\right)}$

$= 10 - \frac{7}{e+1} = \frac{10(e+1) - 7}{e+1}$

$= \frac{10e + 3}{e+1}$

$R = \left(\ln\left(\frac{7}{e+1}\right), \frac{10e+3}{(e+1)}\right)$