Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Tuesday 15 January 2019

Morning (Time: 2 hours 30 minutes) | Paper Reference **WMA02/01**

Core Mathematics C34

Advanced

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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Pearson

(34 Jan 2019.

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1. (a) Express $7\sin 2\theta - 2\cos 2\theta$ in the form $R\sin(2\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90°$. Give the exact value of R and give the value of α to 2 decimal places.

(3)

(b) Hence solve, for $0 \leqslant \theta < 90°$, the equation

$$7\sin 2\theta - 2\cos 2\theta = 4$$

giving your answers in degrees to one decimal place.

(4)

(c) Express $28\sin\theta\cos\theta + 8\sin^2\theta$ in the form $a\sin 2\theta + b\cos 2\theta + c$, where a, b and c are constants to be found.

(3)

(d) Use your answers to part (a) and part (c) to deduce the exact maximum value of $28\sin\theta\cos\theta + 8\sin^2\theta$

(2)

a) $7\sin 2\theta - 2\cos 2\theta$.

$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

$R\sin(2\theta - \alpha) \equiv$

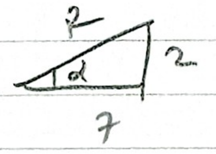
$R[\sin 2\theta\cos\alpha - \cos 2\theta\sin\alpha]$

$7\sin 2\theta - 2\cos 2\theta = R\cos\alpha\sin 2\theta - R\sin\alpha\cos 2\theta$

$R\cos\alpha = 7 \rightarrow \cos\alpha = 7/R$

$R\sin\alpha = 2 \qquad \sin\alpha = 2/R$.

$\tan\alpha = \frac{2}{7}$ $\underline{\alpha = 15.95°}$



$R = \sqrt{7^2 + 2^2}$

$= \underline{\sqrt{53}}$.

(b) $\sqrt{53}\sin(2\theta - 15.95) = 4$.

$\sin(2\theta - 15.95) = \frac{4}{\sqrt{53}}$

$2\theta - 15.95 = 33.33,\ 146.67$.

$2\theta = 49.28,\ 162.62$

$\theta = \underline{24.6°},\ \underline{81.3°}$

(c) $28\sin\theta\cos\theta + 8\sin^2\theta$

$a\sin 2\theta + b\cos 2\theta + c$.

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = 1 - 2\sin^2\theta$.

$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$

Leave blank

Question 2 continued

$a(2\sin\theta\cos\theta)+b(1-2\sin^2\theta)$

$=2a\sin\theta\cos\theta+b-2b\sin^2\theta$

$28\sin\theta\cos\theta+8\sin^2\theta$
can be written as;

$\frac{28}{2}\sin 2\theta+\frac{8}{2}(1-\cos 2\theta)$

$=14\sin 2\theta+4-4\cos 2\theta$

$\Rightarrow 14\sin 2\theta-4\cos 2\theta+4$

$a=14 \quad b=-4 \quad c=4$

(d) $28\sin\theta\cos\theta+8\sin^2\theta$

$\equiv 14\sin 2\theta-4\cos 2\theta+4$

$2(7\sin 2\theta-2\cos 2\theta)+4$

$\equiv 2R\sin(2\theta-\alpha)+4$

$2\sqrt{53}\,\underbrace{\sin(2\theta-15.95)}_{\text{max value of this is } 1}+4$

$=2\sqrt{53}+4=\underline{\underline{18.56}}$

Leave blank

2. Given that

$$\frac{3x^2 + 4x - 7}{(x+1)(x-3)} \equiv A + \frac{B}{x+1} + \frac{C}{x-3}$$

(a) find the values of the constants A, B and C.

(4)

(b) Hence, or otherwise, find the series expansion of

$$\frac{3x^2 + 4x - 7}{(x+1)(x-3)} \qquad |x| < 1$$

in ascending powers of x, up to and including the term in x^2

Give each coefficient as a simplified fraction.

(6)

$A + \frac{B}{x+1} + \frac{C}{x-3}$

$\frac{A(x+1)(x-3) + B(x-3) + C(x+1)}{(x+1)(x-3)}$

$A(x+1)(x-3) + B(x-3) + C(x+1)$

$= 3x^2 + 4x - 7$.

let $x = -1$

$-4B = 3(-1)^2 + 4(-1) - 7$

$B = 2$

let $x = 3$.

$4C = 3(3)^2 + 4(3) - 7$

$4C = 32$

$C = 8$

let $x = 0$.

$A(1)(-3) + 2(-3) + 8(1) = -7$.

$-3A - 6 + 8 = -7$.

$-3A = -9$

$A = 3$

$A = 3$, $B = 2$ $C = 8$

(b) $\frac{1}{(x+1)} = (x+1)^{-1} = 1 - x + x^2 + \ldots$

$\frac{1}{(x-3)} = (x-3)^{-1} = \frac{-1}{3}\left(1 - \frac{x}{3}\right)^{-1}$

$= \frac{-1}{3}\left(1 + \frac{1}{3}x + \frac{1}{9}x^2 \ldots\right)$

$\frac{3x^2+4x-7}{(x+1)(x-3)} = 3 + 2(1 - x + x^2)$

$\frac{-8}{3}\left(1 + \frac{1}{3}x + \frac{1}{9}x^2\right)$

Leave blank

Question 2 continued

$$3+2-2x+2x^2-\frac{8}{3}-\frac{8}{9}x-\frac{8}{27}x^2$$

$$=\left(3+2-\frac{8}{3}\right)-2x-\frac{8}{9}x+2x^2-\frac{8}{27}x^2$$

$$=\frac{7}{3}-\frac{26}{9}x+\frac{46}{27}x^2$$

Turn over

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3. The function f is defined by

$$f: x \mapsto 2x^2 + 3kx + k^2 \qquad x \in \mathbb{R}, -4k \leqslant x \leqslant 0$$

where k is a positive constant.

(a) Find, in terms of k, the range of f.

(4)

The function g is defined by

$$g: x \mapsto 2k - 3x \qquad x \in \mathbb{R}$$

Given that $gf(-2) = -12$

(b) find the possible values of k.

(4)

(a) $f(-4k)$

$= 2(-4k)^2 + 3k(-4k) + k^2$

$= 32k^2 - 12k^2 + k^2$

$f(-4k) = 21k^2$

$f(0) = k^2$

$\frac{df(x)}{dx} = 0$

$f(x) = 2x^2 + 3kx + k^2$

$\frac{df(x)}{dx} = 4x + 3k + 0 = 0$

$x = -\frac{3k}{4}$ → Min value of $f(x)$

$f\left(-\frac{3k}{4}\right)$

$= 2\left(-\frac{3}{4}k\right)^2 + 3k\left(-\frac{3k}{4}\right) + k^2$

$= 2 \times \frac{9}{16}k^2 - \frac{9}{4}k^2 + k^2$

$= -\frac{1}{8}k^2$

$\therefore -\frac{1}{8}k^2 \leq f(x) \leq 21k^2$

(b) $f(-2)$

$= 2(-2)^2 + 3k(-2) + k^2$

$= +8 - 6k + k^2$

~~$2(8 - 6k + k^2)$~~

$2k - 3(8 - 6k + k^2)$

$= 2k - 24 + 18k - 3k^2 = -12$

$-3k^2 + 20k - 12 = 0$

$k = 6, \; k = 2/3$

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4. The curve C has equation

$$81y^3 + 64x^2y + 256x = 0$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

(b) Hence find the coordinates of the points on C where $\frac{dy}{dx} = 0$

(6)

(a) $243y^2 \cdot \frac{dy}{dx} + 64x^2 \cdot \frac{dy}{dx} + 128xy + 256 = 0.$ (Implicit diff)

$243y^2 \cdot \frac{dy}{dx} + 64x^2 \cdot \frac{dy}{dx} = -128xy - 256.$

$\frac{dy}{dx}(243y^2 + 64x^2) = -128xy - 256$

$\frac{dy}{dx} = \frac{-128xy - 256}{243y^2 + 64x^2}$

(b) $\frac{dy}{dx} = 0$ $\quad -128xy - 256 = 0$

$\cancel{x = -2}$ $\quad -128xy = 256$

$x = -\frac{2}{y}.$

when $x = -\frac{2}{y}$

$81y^3 + 64\left(\frac{-2}{y}\right)^2 \cdot y + 256\left(\frac{-2}{y}\right) = 0.$

$81y^3 + \frac{256}{y} - \frac{512}{y} = 0$

$81y^3 = \frac{256}{y}.$

$y^4 = \frac{256}{81}$ $\quad y = \pm\frac{4}{3}$

$\therefore$ when $y = \pm 4/3$

$x = \mp 3/2.$

$(-3/2, 4/3)$ $\quad (3/2, -4/3)$

Leave blank

5. The angle x and the angle y are such that

$$\tan x = m \quad \text{and} \quad 4\tan y = 8m + 5$$

where m is a constant.

Given that $16\sec^2 x + 16\sec^2 y = 537$

(a) find the two possible values of m.

(4)

Given that the angle x and the angle y are acute, find the exact value of

(b) $\sin x$

(2)

(c) $\cot y$

(2)

(a) $\sec^2 x = 1 + \tan^2 x$

$\sec^2 x = 1 + m^2$

$\sec^2 y = 1 + \tan^2 y$

$= 1 + \left(\frac{8m+5}{4}\right)^2$

$= 1 + \frac{64m^2 + 80m + 25}{16}$

$16\sec^2 y = 16 + (64m^2 + 80m + 25)$

$\therefore 16(1+m^2) + 16 + 64m^2 + 80m + 25 = 537$

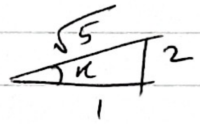
$= 16 + 16m^2 + 16 + 64m^2 + 80m + 25 - 537 = 0$

$80m^2 + 80m - 480 = 0$

$m = 2$ or -3

(b) Since angle is acute $m = 2$

$\tan x = 2$



$\therefore \sin x = \frac{2}{\sqrt{5}}$

(c) $\frac{1}{\tan y} = \cot y$

$\tan y = \frac{8(2)+5}{4}$

$\tan y = \frac{21}{4}$

$\cot y = \frac{1}{\frac{21}{4}} = \frac{4}{21}$

6. Relative to a fixed origin O, the points A, B and C have coordinates $(2, 1, 9)$, $(5, 2, 7)$ and $(4, -3, 3)$ respectively.

The line l passes through the points A and B.

(a) Find a vector equation for the line l. **(2)**

(b) Find, in degrees, the acute angle between the line l and the line AC. **(3)**

The point D lies on the line l such that angle ACD is 90°

(c) Find the coordinates of D. **(4)**

(d) Find the exact area of triangle ADC, giving your answer as a fully simplified surd. **(2)**

$\pm \overrightarrow{AB} = \pm [\vec{B} - \vec{A}]$

$= \pm \begin{bmatrix}5\\2\\7\end{bmatrix} - \begin{pmatrix}2\\1\\9\end{pmatrix}$

$= \pm \begin{pmatrix}3\\1\\-2\end{pmatrix}$

$\therefore r = \begin{pmatrix}2\\1\\9\end{pmatrix} + \lambda \begin{pmatrix}3\\1\\-2\end{pmatrix}$

(b) $\overrightarrow{AC} = \pm(\vec{C} - \vec{A})$

$= \pm \begin{pmatrix}4\\-3\\3\end{pmatrix} - \begin{pmatrix}2\\1\\9\end{pmatrix}$

$= \pm \begin{pmatrix}2\\-4\\-6\end{pmatrix}$

$\overrightarrow{AB} \cdot \overrightarrow{AC} = |AB||AC|\cos\theta$

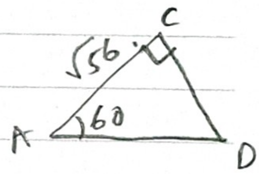
$\begin{pmatrix}3\\1\\-2\end{pmatrix} \cdot \begin{pmatrix}2\\-4\\-6\end{pmatrix} = \sqrt{9+1+4} \times \sqrt{4+16+36} \times \cos\theta$

$= 6-4+12 = \sqrt{14} \times \sqrt{56} \cos\theta$

$\theta = \cos^{-1}\left(\frac{14}{\sqrt{14}\times\sqrt{56}}\right)$

$\underline{\underline{\theta = 60^\circ}}$

(c) $\tan 60 = \frac{CD}{AC}$



$CD = \sqrt{56} \times \tan 60$

$\underline{CD = 2\sqrt{42}}$

$\overrightarrow{CD} = \overrightarrow{AB} - \overrightarrow{OC}$

$\begin{pmatrix}2\\1\\9\end{pmatrix} + \lambda\begin{pmatrix}3\\1\\-2\end{pmatrix} - \begin{pmatrix}4\\-3\\-3\end{pmatrix}$

length of CD

$(2+3\lambda-4)^2 + (1+\lambda+3)^2 + (9+(-2\lambda)-3)^2 = (2\sqrt{42})^2$

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Question 6 continued

$(-2+3\lambda)^2+(4+\lambda)^2$

$+(6-2\lambda)^2 = (2\sqrt{42})^2$

$4-12\lambda+9\lambda^2+16+8\lambda+\lambda^2$
$+36-24\lambda+4\lambda^2 = 2^2\times 42$

$14\lambda^2-28\lambda-112=0$

$\underline{\underline{\lambda=4}}$

$\overrightarrow{OD} = \begin{pmatrix}2\\1\\9\end{pmatrix}+4\begin{pmatrix}3\\1\\-2\end{pmatrix}$

$= \underline{\underline{\begin{pmatrix}14\\5\\1\end{pmatrix}}}$

(d) Area $= \frac{1}{2} AC \times CD$

$= \frac{1}{2}\times\sqrt{56}\times 2\sqrt{42}$

$= \underline{\underline{28\sqrt{3}}}$

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7.

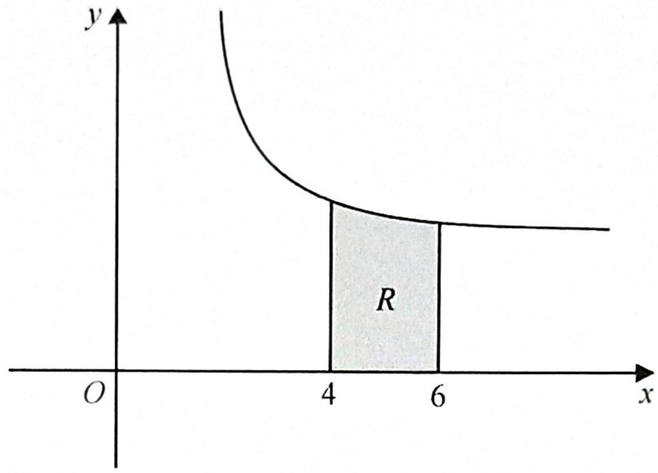

Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = \frac{x+7}{\sqrt{2x-3}} \qquad x > \frac{3}{2}$$

The region R, shown shaded in Figure 1, is bounded by the curve, the line with equation $x = 4$, the x-axis and the line with equation $x = 6$

(a) Use the trapezium rule with 4 strips of equal width to find an estimate for the area of R, giving your answer to 2 decimal places.

(4)

(b) Using the substitution $u = 2x - 3$, or otherwise, use calculus to find the exact area of R, giving your answer in the form $a + b\sqrt{5}$, where a and b are constants to be found.

(7)

(a) $6 - 4 = 2$

$\frac{2}{4} = \underline{0.5} \to h$

$x = 4 \quad y = 4.92$

$x = 4.5 \quad y = 4.69$

$x = 5 \quad y = 4.54$

$x = 5.5 \quad y = 4.42$

$x = 6 \quad y = 4.33.$

$\frac{1}{2} \times 0.5 \times \left(4.92 + 4.33 + 2(4.54 + 4.42 + 4.69)\right)$

$= \underline{\underline{9.14}}$

(b) $u = 2x - 3$

$\frac{du}{dx} = 2.$

$dx = \frac{du}{2}.$

at $x = 4 \quad u = 5$

$x = 6 \quad \underline{\underline{u = 9.}}$

Leave blank

Question 7 continued

$$\int_{a}^{b} y \, dx.$$

$$= \int \frac{x+7}{\sqrt{2x-3}} \, dx$$

$$= \int \frac{\frac{u+3}{2}+7}{\sqrt{u}} \times \frac{du}{2}$$

$$= \frac{1}{2} \int \frac{\frac{u+3}{2}+7}{\sqrt{u}} \, du$$

$$= \frac{1}{2} \int \frac{\frac{u}{2}+\frac{17}{2}}{\sqrt{u}} \, du$$

$$= \frac{1}{2} \int u^{-1/2} \left(\frac{u}{2} + \frac{17}{2} \right) du$$

$$= \frac{1}{2} \times \frac{1}{2} \int \left(u^{1/2} + 17u^{-1/2} \right) du$$

$$= \frac{1}{4} \int_{5}^{9} u^{1/2} + 17u^{-1/2} \, du$$

$$= \frac{1}{4} \left[\frac{u^{3/2}}{3/2} + \frac{17u^{1/2}}{1/2} \right]_{5}^{9}$$

$$= \frac{1}{4} \left\{ \left[\frac{2}{3}(9)^{3/2} + 34(9)^{1/2} \right] - \left[\frac{2}{3}(5)^{3/2} + 34(5)^{1/2} \right] \right.$$

$$= \frac{1}{4} \left[120 - \frac{112}{3} \sqrt{5} \right]$$

$$= 30 - \frac{28}{3} \sqrt{5}$$

$a = 30$

$b = -\frac{28}{3}.$

Turn over

Leave blank

8. A curve has parametric equations

$$x = t^2 - t, \qquad y = \frac{4t}{1-t} \qquad t \neq 1$$

(a) Find $\dfrac{dy}{dx}$ in terms of t, giving your answer as a simplified fraction.

(4)

(b) Find an equation for the tangent to the curve at the point P where $t = -1$, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(4)

The tangent to the curve at P cuts the curve at the point Q.

(c) Use algebra to find the coordinates of Q.

(5)

(a) $\dfrac{dy}{dx} = \dfrac{dy}{dt} \times \dfrac{dt}{dx}$

$x = t^2 - t$

$\dfrac{dx}{dt} = 2t - 1$

$\dfrac{dt}{dx} = \dfrac{1}{2t-1}$ -- 1

$y = \dfrac{4t}{1-t} = 4t(1-t)^{-1}$

$4t \times -1 \times -1(1-t)^{-2} + 4(1-t)^{-1}$

$= 4t(1-t)^{2} + 4(1-t)$

$\therefore \dfrac{dy}{dt} = \dfrac{4t}{(1-t)^2} + \dfrac{4}{(1-t)}$

$\dfrac{4(1-t)+4t}{(1-t)^2} = \dfrac{4}{(1-t)^2}$

$\therefore \dfrac{dy}{dx} = \dfrac{4}{(1-t)^2} \times \dfrac{1}{(2t-1)}$

$= \dfrac{4}{(1-t)^2(2t-1)}$

$= \dfrac{4}{2t^3 - 5t^2 + 4t - 1}$

(b) $\dfrac{dy}{dx}$ at $t = -1$

$= \dfrac{4}{2(-1)^3 - 5(-1)^2 + 4(-1) - 1}$

$= -\dfrac{1}{3}$

when $t = -1$ $x = 2$

$y = -2$

$\therefore y + 2 = -\frac{1}{3}(x - 2)$

$3y + 6 = -x + 2$

$3y + x + 4 = 0$

Leave blank

Question 8 continued

(c) $x+3y+4=0$.

$x=t^2-t \quad y=\frac{4t}{1-t}$.

$(t^2-t)+3\left(\frac{4t}{1-t}\right)+4=0$

$(t^2-t)(1-t)+3(4t)+4(1-t)=0$

$=t^2-t^3-t+t^2+12t-4-4t=0$

$-t^3+2t^2+7t+4=0$

$t^3-2t^2-7t-4=0$.

$\underline{t=4} \quad \underline{t=-1}$ n/a

When $t=4 \quad x=4^2-4$
$=\underline{12}$

$y=\frac{4\times4}{1-4}==-\frac{16}{3}$

$\therefore Q=\left(12,-\frac{16}{3}\right)$.

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9.

(a) Find $\int x\sin 2x\,dx$

(3)

(b) Find $\int (x+\sin 2x)^2\,dx$

(4)

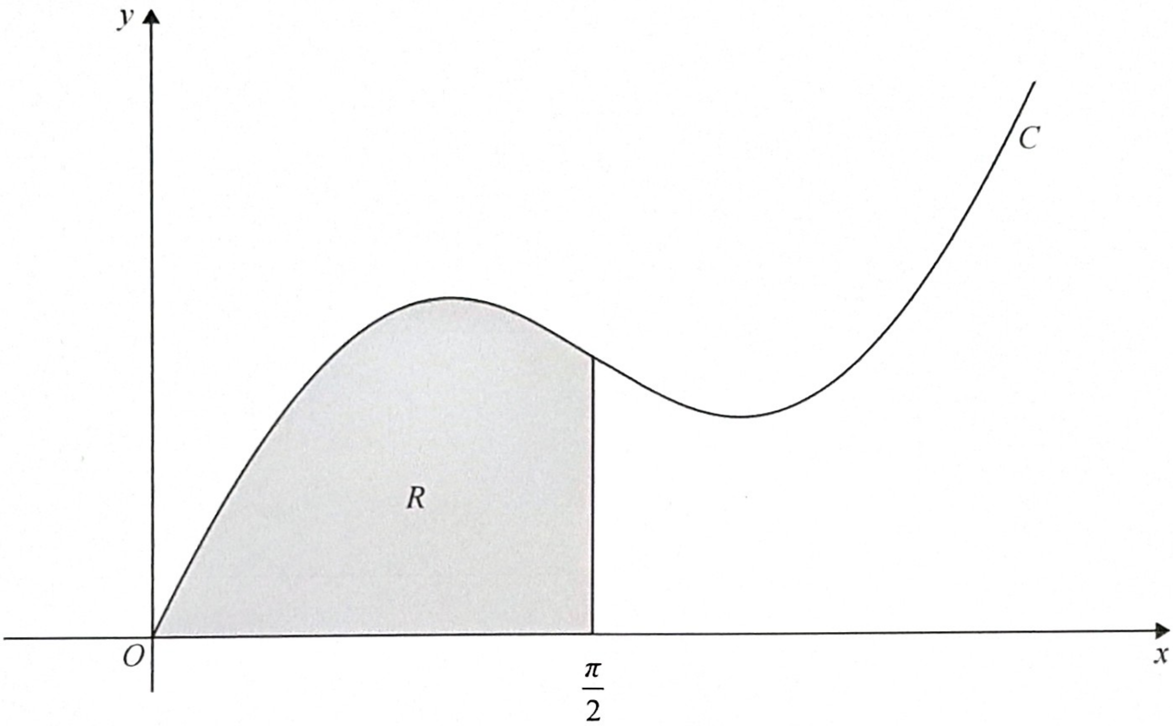

Figure 2

Figure 2 shows a sketch of part of the curve C with equation $y = x + \sin 2x$.

The region R, shown shaded in Figure 2, is bounded by C, the x-axis and the line with equation $x = \frac{\pi}{2}$

The region R is rotated through 2π radians about the x-axis to form a solid of revolution.

(c) Find the exact value for the volume of this solid, giving your answer as a single, simplified fraction.

(3)

a) $\int x \sin 2x$

$u \qquad \frac{dv}{dx}$

$u = x \qquad v = -\frac{1}{2}\cos 2x$

$\frac{du}{dx} = 1 \qquad \frac{dv}{dx} = \sin 2x$

$uv - \int v\,du$

$= x \cdot -\frac{1}{2}\cos 2x - \int -\frac{1}{2}\cos 2x\,dx$

$= -\frac{x}{2}\cos 2x + \frac{1}{2}\int \cos 2x$.

Leave blank

Question 9 continued

$= -\frac{x}{2}\cos 2x + \frac{1}{2} \times \frac{\sin 2x}{2}$

$= -\frac{x}{2}\cos 2x + \frac{\sin 2x}{4} + c$

(b) $\int (x+\sin 2x)^2 \, dx$

$= \int x^2 + 2x\sin 2x + \sin^2 2x \, dx$

$\int x^2 \, dx = \frac{x^3}{3}$

$2\int x\sin 2x \, dx = 2\times\left[-\frac{x}{2}\cos 2x + \frac{\sin 2x}{4}\right]$

$= \frac{1}{2}\sin 2x - x\cos 2x$

$\int \sin^2(2x) \, dx$

$\sin^2 x = \frac{1}{2}(1-\cos 2x)$

$\therefore \sin^2(2x) = \frac{1}{2}(1-\cos 4x)$

$\frac{1}{2}\int 1-\cos 4x \, dx$

$= \frac{1}{2}\left[x - \frac{\sin 4x}{4}\right] + c$

$= \frac{1}{2}x - \frac{1}{8}\sin 4x + c$

$\therefore$ Final answer

$= \frac{x^3}{3} + \frac{1}{2}\sin 2x - x\cos 2x + \frac{1}{2}x - \frac{1}{8}\sin 4x + C$

(c) $\pi\int_0^{\pi/2} y^2 \, dx$

$= \pi\int_0^{\pi/2} (x+\sin 2x)^2 \, dx$

$= \pi\left[\frac{x^3}{3} + \frac{1}{2}\sin 2x - x\cos 2x + \frac{1}{2}x - \frac{1}{8}\sin 4x\right]_0^{\pi/2}$

$\pi\left[\frac{(\pi/2)^3}{3} + \frac{1}{2}\sin(2\times\pi/2) - \frac{\pi}{2}\cos\left(2\times\frac{\pi}{2}\right) + \frac{1}{2}(\pi/2) - \frac{1}{8}\sin 4(\pi/2)\right] - 0$

$= \pi\left[\pi^3/24 + 0 + \frac{\pi}{2} + \frac{\pi}{4} - 0\right]$

$= \pi\left[\frac{\pi^3 + 18\pi}{24}\right] = \frac{\pi^2(\pi^2+18)}{24}$

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10.

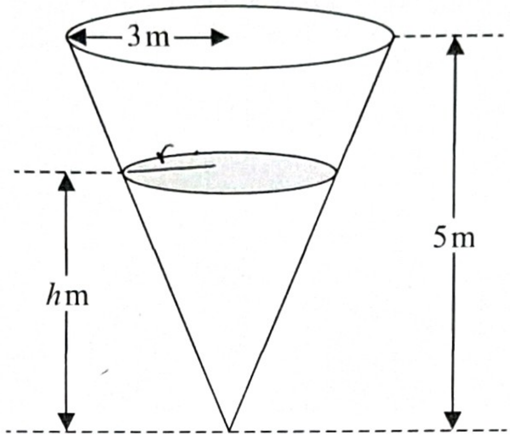


Figure 3

Figure 3 shows a container in the shape of an inverted right circular cone which contains some water.

The cone has an internal radius of 3 m and a vertical height of 5 m as shown in Figure 3.

At time t seconds, the height of the water is h metres, the volume of the water is V m^3 and water is leaking from a hole in the bottom of the container at a constant rate of 0.02 m^3 s^{-1}

[*The volume of a cone of radius r and height h is* $\frac{1}{3}\pi r^2 h$.]

$\frac{dv}{dt} = -0.02$

(a) Show that, while the water is leaking,

$$h^2 \frac{dh}{dt} = -\frac{1}{k\pi}$$

where k is a constant to be found.

(5)

Given that the container is initially full of water,

(b) express h in terms of t.

(3)

(c) Find the time taken for the container to empty, giving your answer to the nearest minute.

(2)

(a) $\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dv}{dt}$

$V = \frac{1}{3}\pi r^2 h$.

using similar triangles.

$\frac{r}{3} = \frac{h}{5}$.

$r = \frac{3h}{5}$.

$\therefore V = \frac{1}{3}\pi\left(\frac{3h}{5}\right)^2 \times h$

$V = \frac{1}{3}\pi \times \frac{9h^3}{25}$

$V = \frac{3h^3\pi}{25}$

Leave blank

Question 5 continued

$\frac{dv}{dh} = \frac{3}{25}\pi \times 3h^2$

$= \frac{9\pi h^2}{25}$

$\therefore \frac{dh}{dv} = \frac{25}{9\pi h^2}$

$\therefore \frac{dh}{dt} = \frac{25}{9\pi h^2} \times -0.02$

$h^2 \frac{dh}{dt} = \frac{-1}{18\pi}$

$\therefore k = 18$

(b) $\int \frac{dv}{dt} = \int -0.02$

$v = -0.02t + c$

at $t=0$ $r=3$ $h=5$

$v = \frac{1}{3}\pi r^2 h = \frac{1}{\cancel{3}}\pi \times \cancel{9}^{3} \times 5$

$= 15\pi$

$15\pi = -0.02 \times 0 + c$

$c = 15\pi$

$\therefore v = -0.02t + 15\pi$

$\frac{3}{25}\pi h^3 = -0.02t + 15\pi$

$h^3 = \frac{25}{3\pi}(-0.02t + 15\pi)$

$h^3 = \frac{2 \times -0.02t}{3\pi} + \frac{25 \times \cancel{15}^{5}\cancel{\pi}}{\cancel{3}\cancel{\pi}}$

$h^3 = \frac{-t}{6\pi} + 125$

$h = \sqrt[3]{125 - \frac{t}{6\pi}}$

(c) $h = 0$ $t = ?$

$0 = \sqrt[3]{125 - \frac{t}{6\pi}}$

$125 = \frac{t}{6\pi}$

$t = 2356.19\text{sec}$

$t = 39\text{min}$

Turn over

Leave blank

11. (a) Given that $0 \leqslant f(x) \leqslant \pi$, sketch the graph of $y = f(x)$ where

$$f(x) = \arccos(x - 1), \qquad 0 \leqslant x \leqslant 2$$

(2)

The equation $\arccos(x - 1) - \tan x = 0$ has a single root α.

(b) Show that $0.9 < \alpha < 1.1$

(2)

The iteration formula

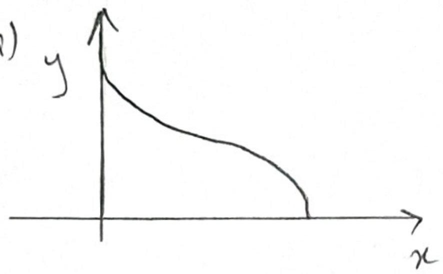
$$x_{n+1} = \arctan(\arccos(x_n - 1))$$

can be used to find an approximation for α.

(c) Taking $x_0 = 1.1$ find, to 3 decimal places, the values of x_1 and x_2

(2)

(a)



(b) $\arccos(x-1) - \tan x = 0$

Let $x = 0.9$.

$\arccos(0.9-1) - \tan 0.9 = 0.4108$

$x = 1.1$

$\arccos(1.1-1) - \tan 1.1 = -0.4941$

$\therefore$ change in sign and continuity of the graph in the region indicates

$-0.9 < \alpha < 1.1$

(c) $x_0 = 1.1$

$x_{n+1} = \arctan(\arccos(x_n - 1))$

$x_{0+1} = x_1 = \arctan(\arccos(1.1-1))$

$= 0.974$

$x_2 = \arctan(\arccos(0.974-1))$

$= 1.011$

Leave blank

12. Given that k is a positive constant,

(a) sketch the graph with equation

$$y = 2|x| - k$$

Show on your sketch the coordinates of each point at which the graph crosses the x-axis and the y-axis.

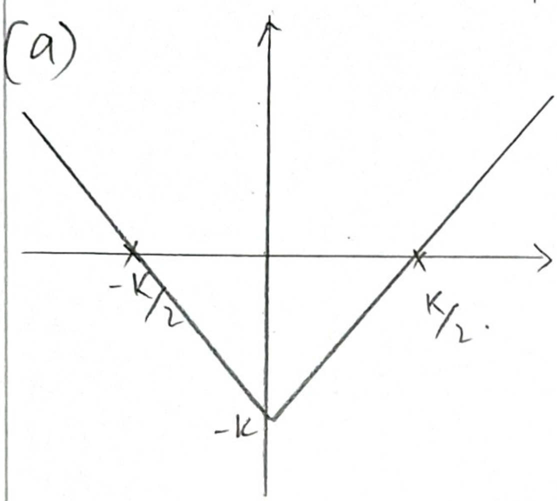
(2)

(b) Find, in terms of k, the values of x for which

$$2|x| - k = \frac{1}{2}x + \frac{1}{4}k$$

(3)

(a)



(b) $2|x| - k = \frac{1}{2}x + \frac{1}{4}k$

(+) $2x - k = \frac{1}{2}x + \frac{1}{4}k$

$2x - \frac{1}{2}x = k + \frac{1}{4}k$

$\frac{3}{2}x = \frac{5}{4}k$

$x = \frac{5}{6}k$

(−) ~~2x~~

$-2x - k = \frac{1}{2}x + \frac{1}{4}k$

$-2x - \frac{1}{2}x = \frac{1}{4}k + k$

$-\frac{5}{2}x = \frac{5}{4}k$

$x = -\frac{1}{2}k$

Leave blank

13. A scientist is studying a population of insects. The number of insects, N, in the population, t days after the start of the study is modelled by the equation

$$N = \frac{240}{1 + ke^{-\frac{t}{16}}}$$

where k is a constant.

Given that there were 50 insects at the start of the study,

(a) find the value of k

(2)

(b) use the model to find the value of t when $N = 100$

(3)

(c) Show that

$$\frac{dN}{dt} = \frac{1}{p}N - \frac{1}{q}N^2$$

where p and q are integers to be found.

(5)

(a) $t = 0 \quad N = 50$

$50 = \dfrac{240}{1+ke^0}$

$50 = \dfrac{240}{1+k}$

$1+k = \dfrac{24}{5}$

$k = \dfrac{19}{5}$

(b) $N = 100 \quad t = ?$

$100 = \dfrac{240}{1+\frac{19}{5}e^{-t/16}}$

$1+\dfrac{19}{5}e^{-t/16} = \dfrac{12}{5}$

$\dfrac{19}{5}e^{-t/16} = \dfrac{7}{5}$

$e^{-t/16} = \dfrac{7}{19}$

$-\dfrac{t}{16} = \ln\left(\dfrac{7}{19}\right)$

$t = -16 \times$ ~~Ans~~ $\ln\left(\dfrac{7}{19}\right)$

$= 15.97$

$= 16.$ OR $-16\ln\left(\dfrac{7}{19}\right)$

Leave blank

Question 13 continued

c) $N = 240(1+ke^{-t/16})^{-1}$

$\frac{dN}{dt} = 240 \times -1(1+ke^{-t/16})^{-2}$
$\times ke^{-t/16} \times \frac{-1}{16}$

$= \frac{15ke^{-t/16}}{(1+ke^{-t/16})^2}$

$= \frac{A+B(1+ke^{-t/16})}{(1+ke^{-t/16})^2}$

$15ke^{-t/16} = (A+B) + Bke^{-t/16}$

$B = 15$

$A+B=0$
$A=-15$

$\therefore \frac{dN}{dt} = \frac{-15}{(1+ke^{-t/16})^2} + \frac{15}{(1+ke^{-t/16})}$

$N = \frac{240}{(1+ke^{-t/16})} \rightarrow \frac{1}{(1+ke^{-t/16})} = \frac{N}{240}$

$\frac{dN}{dt} = -15 \times \left(\frac{N}{240}\right)^2 + 15 \times \left(\frac{N}{240}\right)$

$\frac{dN}{dt} = \frac{1}{16}N - \frac{1}{3840}N^2$

$p = 16$ $q = 3840$

Turn over